Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Unstiffened Panels of Ship Structures

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ABSTRACT

The main objective of ship structural design is to ensure safety, functional, and performance requirements of the structural element for target reliability levels and for specified time period. As this must be accomplished under conditions of uncertainty, probabilistic analyses are necessary in the development of such reliability-based design of unstiffened panels for ship structures. The load and resistance factor design (LRFD) format was developed in this paper for unstiffened panels. Partial safety factors were determined to account for the uncertainties in strength and load effects. In developing these factors, Monte Carlo simulation was utilized to assess the probabilistic characteristics of strength models by generating basic random variables that define the strength and substituting them in these models; and the First-Order Reliability Method (FORM) was used to determine the partial safety factors based on prescribed probabilistic characteristics of load effects. Also, strength factors were computed for a set of load factors to meet a target reliability level.

1. INTRODUCTION

Ship panels or plates are important components in ship structures, and therefore they should be designed for a set of failure modes that govern their strength. Plate elements, in general, are parts of stiffened panels for which their strengths need to be predicted. However, a

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global failure of a stiffened panel can be partially controlled by designing the strength of plate elements between stiffeners. To evaluate the strength of an unstiffened plate element it is necessary to review various strength predicting models and to study their applicability and limitations for different loading conditions acting on the element. The uncertainties that are associated with a numerical analysis are generally a result of experimental approximation or numerical inaccuracies, which can be reduced by some procedures. On the hand, the uncertainty associated with a strength design model is different and cannot be eliminated because it results from not accounting for some variables that influence the strength. For this reason, the uncertainty and the bias of a design equation should be assessed and evaluated by comparing its predictions with more accurate ones. An advanced prediction model should account for more variables than the one that is being assessed for use in load and resistance factor design (LRFD) guidelines. Probably the most important parameter that effect plate strength is the slenderness ratio *B* (Soares 1988). In ship plates *B*, which is a non-dimensional parameter, can take a value between 1 and 5. These values of *B* correspond to a reduction of plate strength from yield strength F_y to 0.4 F_y. The aspect ratio a/b has less effect on plate strength than *B*. Most ship plates have $a/b > 1.0$. Typical plate strength changes by 5% as the aspect ratio varies from 0.6 to 1.0 (Frieze et al 1977). Other parameters that can affect plate strength are its boundary condition and material imperfections. A study conducted by Soares in 1988, which is based on experimental results of Moxham (1971), shows that in the range of slenderness ratios between 2.5 and 3.5, clamped (fixed) plates are between 15% and 30% stronger than simply-supported plates. Wherever possible, the different types of biases resulting from these models were computed. In doing so, these prediction models were classified as follows (Atua and Ayyub 1996): (1) prediction models that can be used by the LRFD guidelines, (2) advanced prediction models that can be used for various analytical purposes, (3) some experimental results from model testing, and (4) some real measurements based on field data during the service life of a ship. Furthermore, the relationships and uncertainty analyses for these models are required. The relationships can be defined in terms of biases (bias factors). These bias factors can be expressed as given by the following expressions:

$$B_{21} = \frac{\text{Advanced predicted value}}{\text{Rules value}} \tag{1}$$

$$B_{32} = \frac{\text{Experimental value}}{\text{Advanced predicted value}} \tag{2}$$

$$B_{43} = \frac{\text{Real value}}{\text{Experimental value}} \tag{3}$$

$$B_{41} = \frac{\text{Real value}}{\text{Rules value}} = B_{21}B_{32}B_{43} \tag{4}$$

2. DESIGN LOADS AND LOAD COMBINATIONS

Primary structural loads on a ship are due to its own weight, cargo, buoyancy, and operation in a random environment, i.e., the sea. The loads acting on the ship's hull girder can be categorized into three main types that are used in this paper: (a) stillwater loads, (b) wave loads, and (c) dynamic loads. The load effect of concern herein is bending moment exerted on the ship hull girder.

Stillwater loads can be predicted and evaluated with a proper consideration of variability in weight distribution along the ship length, variability in its cargo loading conditions, and buoyancy. Both wave loads and dynamic loads are related and affected by many factors such as ship characteristics, speed, heading of ship at sea, and sea state (waves heights). Waves height is a random variable that requires statistical and extreme analyses of ship response data collected over a period of time in order to estimate maximum wave-induced and dynamic bending moments that the ship might encounter during its life. The statistical representation of sea waves allows the use of statistical models to predict the maximum wave loads in ship's life.

Procedures for computing design wave loads for a ship's hull girder based on spectral analysis can be found in numerous references pertaining to ship structures such as Hughes (1988), Sikora (1983) and, Ayyub et al. (2002b).

2.1 Design Loads

The design loads that are of concern in this study for developing reliability-based design for unstiffened panels of ship structures are those loads resulting from ship hull girder bending and their combinations. As indicated earlier, the loads acting on the ship's hull girder can be categorized into three main types: stillwater loads, wave loads, and dynamic loads. Each of these types of loads are presented subsequently under its own heading.

2.1.1 Stillwater Loads

The calm water or stillwater loading should be investigated in design processes although it rarely governs the design of a ship on its own. The ship is balanced on the draft load waterline with the longitudinal center of gravity aligned with the longitudinal center of buoyancy in the same vertical plan. Then, the hull girder loads are developed based on the differences between the weights and the buoyancy distributions along the ship's length. The net load generates shear and bending moments on the hull girders. The resulting values from this procedure are to be considered the design (nominal) values in the LRFD format for the stillwater shear forces and bending moments on the hull girder.

2.1.2 Wave-induced Bending Moment

Wave-induced bending moment is treated as a random variable dependent on ship's principal characteristics, environmental influences, and operational conditions. Spectral and extreme analyses (see Ayyub et al. 2002b) can be used to determine the extreme values and the load spectra of this load type during the design life of the ship. The outcome of this analysis can be in the form of vertical or horizontal longitudinal bending moments or stresses on the hull girder. Computer programs have been developed and are available to perform these calculations for different ships based on their types, sizes, and operational conditions (Sikora et al. 1983).

2.1.3 Dynamic Bending Moment

Spectral and extreme analyses can be used to obtain the combined wave-induced and dynamic load effects on the hull girder. Computer programs can be used for this purpose as provided by in Sikora et al. (1983). The average peak-to-peak whipping bending moments (in ft-ton) for fine bow ships is described in Sikora (1983) as

$$M_{WH} = 0.0022\, LBP^2\, B \quad \text{for} \quad LBP^2\, B < 5\text{x}10^6 \tag{5}$$

and

$$M_{WH} = 5.4 LLBP\sqrt{B} \quad \text{for} \quad LBP^2\, B < 5\text{x}10^6 \tag{6}$$

where M_{WH} = mean value of peak-to-peak whipping bending moment, LBP = length between perpendiculars of the ship (in ft), and B = molded breadth of the ship (in ft). For ships with bow flare or with flat bottom (such as auxiliaries and cargo ships), the whipping bending moments can be determined (in ft-ton) using (Sikora 1989)

$$M_{WH} = 0.0022\, LBP^2\, B \tag{7}$$

The lifetime extreme value of whipping bending moments for a ship was defined as the whipping bending moment value with a one percent chance of being exceeded in its lifetime and is given by

$$M_{WH_e} = 4.6 M_{WH} \tag{8}$$

where M_{WHe} = extreme value of whipping bending moment in ton-ft.

2.1.4 Combined Wave-induced and Dynamic Bending Moment

Spectral and extreme analyses can be used to determine the design value of the combined wave-induced and dynamic bending moments on a ship hull girder during its design life (Sikora et al. 1983).

2.2 Load Combinations and Ratios

Reliability-based structural design of unstiffened panels as presented in this paper is based on two load combinations that are associated with correlation factors as presented in the subsequent sections (Mansour et al. 1984).

2.2.1 Stillwater and Vertical Wave-induced Bending Moments

The load effect (stress) on unstiffened panel element due to combinations of stillwater and vertical wave-induced bending moments is given by

$$f_c = f_{SW} + k_{WD} f_{WD} \tag{9}$$

where f_{SW} = stress due to stillwater bending moment, f_{WD} = stress due to wave-induced bending moment, f_c = un-factored combined stress, k_W = correlation factor for wave-induced bending moment and can be set equal to one (Mansour et al. 1984).

2.2.2 Stillwater, Vertical Wave-induced, and Dynamic Bending Moments

The load effect on unstiffened panel element due to combinations of stillwater, vertical wave-induced and dynamic bending moments is given by

$$f_c = f_{SW} + k_W (f_W + k_D f_D) \tag{10}$$

where f_{SW} = stress due to stillwater bending moment, f_W = stress due to waves bending moment, f_D = stress due to dynamic bending moment, f_c = un-factored combined load, and k_D = correlation factor between wave-induced and dynamic bending moments. The correlation factor k_D is given by the following two cases of hogging and sagging conditions (Mansour et al. 1984):

a. Hogging Conditions:

$$k_D = Exp\left[\frac{53080}{\left(158LBP^{-0.2} + 14.2LBP^{0.3}\right)LBP}\right] \tag{11}$$

b. Sagging Condition:

$$k_D = Exp\left[\frac{21200}{\left(158LBP^{-0.2} + 14.2LBP^{0.3}\right)LBP}\right] \tag{12}$$

where LBP = length between perpendiculars for a ship in ft. Values of k_D for LBP ranging from 300 to 1000 ft can be obtained either from Table 1 or from the graphical chart provided in Figure 1.

3. LIMIT STATES AND DESIGN STRENGTH

The unstiffened panel of ship structure for all stations should meet one of the following conditions; the selection of the appropriate equation depends on the availability of information as required by these equations:

For uniaxial compression,

Limit State 1:

$$\phi_M f_u \geq \gamma_{SW} f_{SW} + \gamma_{WD} k_{WD} f_{WD} \tag{13}$$

Limit State 2:

$$\phi_M f_u \geq \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_D) \tag{14}$$

For biaxial compression,

$$\left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right)^2 - \eta_b \left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)\left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right) \leq 1 \tag{15}$$

$$\left(\frac{f_{2x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{2y}}{\phi_{R_{uy}} R_{uy}}\right)^2 - \eta_b \left(\frac{f_{2x}}{\phi_{R_{ux}} R_{ux}}\right)\left(\frac{f_{2y}}{\phi_{R_{uy}} R_{uy}}\right) \leq 1 \tag{16}$$

For biaxial compression and edge shear,

$$\left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right)^2 + \left(\frac{f_{1\tau}}{\phi_{R_{u\tau}} R_{u\tau}}\right)^2 \leq 1 \tag{17}$$

where

f_u = ultimate strength (compressive stress) for uniaxially stiffened panel

$\phi_{R_u}, \phi_{R_{ux}}, \phi_{R_{uy}}$ = strength reduction factors for ultimate strength capacity of a plate, the ultimate strength capacity R_u, R_{ux}, and R_{uy} depends on the loading conditions for the plate (i.e., uniaxial, edge shear, etc.), see Section 3.1

$\phi_{R_{u\tau}}$ = strength reduction factor for plates in shear

R_u, R_{ux}, R_{uy} = ultimate strength capacity of a plate, the ultimate strength capacity R_u, R_{ux}, and R_{uy} depends on the loading conditions for the plate, see Section 3.1

$R_{u\tau}$ = ultimate load capacity of plate in shear.

f_{1x} = $\gamma_{SW} f_{SWx} + k_{WD}\gamma_{WD} f_{WDx}$, magnification of the applied stress in the *x*-direction for limit state 1

f_{2x} = $\gamma_{SW} f_{SWx} + k_W (\gamma_W f_{Wx} + k_D \gamma_D f_{Dx})$, magnification of the applied stress in the *x*-direction for limit state 2

f_{1y} = $\gamma_{SW} f_{SWy} + k_{WD}\gamma_{WD} f_{WDy}$, magnification of the applied stress in the *y*-direction for limit state 1

f_{2y} = $\gamma_{SW} f_{SWy} + k_W (\gamma_W f_{Wy} + k_D \gamma_D f_{Dy})$, magnification of the applied stress in the *y*-direction for limit state 2

$f_{1\tau}$ = $\gamma_{SW} f_{SW\tau} + k_{WD}\gamma_{WD} f_{WD\tau}$, magnification of the applied stress in the τ-direction for limit state 1

$f_{2\tau}$ = $\gamma_{SW} f_{SW\tau} + k_{W}(\gamma_{W} f_{W\tau} + k_{D}\gamma_{D} f_{D\tau})$, magnification of the applied stress in the τ-direction for limit state 2

γ_{SW} = load factor for the stress due to stillwater bending moment

f_{SW} = stress due to stillwater bending moment

k_{WD} = combined wave-induced and dynamic bending moment factor

γ_{WD} = load factor for the stress due combined wave-induced and dynamic bending moment

f_{WD} = stress due to combined wave-induced and dynamic bending moments

k_{W} = load combination factor, can be taken as 1.0

γ_{W} = load factor for the stress due waves bending moment

f_{W} = stress due to waves bending moment

k_{D} = load combination factor, can be taken as 0.7

γ_{D} = load factor for the stress due to dynamic bending moment

f_{D} = stress due to dynamic bending moment

$$\eta_b = \begin{cases} 0.25 & \text{if} \quad \alpha \geq 3.0 \\ 0.25 - \left(\dfrac{\alpha - 3}{2}\right)\left[3.2e^{-0.35B} - 2.25\right] & \text{if } 1.0 < \alpha < 3.0 \\ 3.2e^{-0.35B} - 2 & \text{if} \quad \alpha = 1.0 \end{cases}$$

where α = aspect ratio of plate (*a*/*b*), and *B* = plate slenderness ratio.

The nominal (i.e., design) values of the strength and load components should satisfy these formats in order to achieve specified target reliability levels. The nominal strength for unstiffened panels (plates) can be determined as described in subsequent sections.

3.1 Design Strength for Unstiffened Panel

The design strength of unstiffened panels (plates) can be computed using formulas that correspond appropriately to their loading conditions. This section provides a summary of these formulas. They shall be used appropriately based on the loading conditions of the plate between stiffeners. Both serviceability and strength limit states are provided herein although only the strength limit states were considered in the paper for computing strength reduction factors.

3.1.1 Uniaxial Compression

The ultimate strength f_u of plates under uniaxial compression stress shall be computed from one of the following two cases (Bleich 1952 and Faulkner 1975):

1. for $a/b > 1.0$

$$f_u = \begin{cases} F_y\sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ F_y\left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & \text{if } 1.0 \leq B < 3.5 \\ F_y & \text{if } B < 1.0 \end{cases} \tag{18}$$

2. for $a/b < 1.0$

$$f_u = F_y\left[\alpha C_u + 0.08(1-\alpha)\left(1+\frac{1}{B^2}\right)\right]^2 \leq F_y \tag{19}$$

where

F_y = yield strength (stress) of plate

a = length or span of plate

b = distance between longitudinal stiffeners,

B $= \dfrac{b}{t}\sqrt{\dfrac{F_y}{E}}$, plate slenderness ratio

α $= \dfrac{a}{b}$, aspect ratio of plate

t = thickness of the plate

E = the modulus of elasticity

ν = Poisson's ratio

and

$$C_u = \begin{cases} \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & \text{if } 1.0 \leq B < 3.5 \\ 1.0 & \text{if } B < 1.0 \end{cases} \tag{20}$$

3.1.2 Edge Shear

According to Basler (1963), the ultimate strength $f_{u\tau}$ of plates under pure edge shear stress can be computed as

$$f_{u\tau} = F_{cr\tau} + F_{P\tau} \tag{21}$$

where $F_{cr\tau}$ = critical or buckling stress and $F_{P\tau}$ = post-buckling strength using tension field action. The buckling strength can be computed based on one of the following three conditions that correspond to shear yield, inelastic buckling, and elastic buckling:

$$F_{cr\tau} = \begin{cases} F_{y\tau} & \text{if } B \leq \dfrac{\sqrt{k_\tau \dfrac{\pi^2 F_y F_{pr}}{12(1-\nu^2)}}}{F_{y\tau}} \\ \sqrt{k_\tau \dfrac{\pi^2 F_y F_{pr}}{12(1-\nu^2)B^2}} & \text{if } \dfrac{\sqrt{k_\tau \dfrac{\pi^2 F_y F_{pr}}{12(1-\nu^2)}}}{F_{y\tau}} < B \leq \sqrt{k_\tau \dfrac{\pi^2 F_y}{12(1-\nu^2)F_{pr}}} \\ k_\tau \dfrac{\pi^2 F_y}{12(1-\nu^2)B^2} & \text{if } B > \sqrt{k_\tau \dfrac{\pi^2 F_y}{12(1-\nu^2)F_{pr}}} \end{cases} \tag{22}$$

where $F_{y\tau}$ = yield stress in shear and F_{pr} = proportional limit in shear which can be taken as $0.8F_{y\tau}$. The buckling coefficient k_τ can be obtained from Figure 2 or from the following two expressions depending on whether the plate under pure shear is simply supported or clamped, respectively:

(a) For $\alpha \geq 1.0$,

$$k_\tau = \begin{cases} 5.35 + \dfrac{4.0}{\alpha^2} & \text{for simple supports} \\ 8.98 + \dfrac{5.6}{\alpha^2} & \text{for clamped supports} \end{cases} \tag{23}$$

(b) For, $\alpha \le 1.0$,

$$k_\tau = \begin{cases} 4.0 + \dfrac{5.35}{\alpha^2} & \text{for simple supports} \\ 5.6 + \dfrac{8.98}{\alpha^2} & \text{for clamped supports} \end{cases} \tag{24}$$

The yield stress in shear ($F_{y\tau}$) is given by

$$F_{y\tau} = \frac{F_y}{\sqrt{3}} \tag{25}$$

where F_y = yield stress of plate. The post-buckling shear strength $F_{P\tau}$ is given by

$$F_{P\tau} = \frac{F_y - \sqrt{3}F_{cr\tau}}{2\sqrt{1+\alpha^2}} \tag{26}$$

where α is the aspect ratio of plate (*a/b*). If the aspect ratio α exceeds 3.0, tension field action is not permitted. In this case, the ultimate shear strength of a plate shall be based on elastic and inelastic buckling theory such that

$$f_{u\tau} = F_{cr\tau} \tag{27}$$

where $F_{cr\tau}$ can be computed from Eq. 22.

3.1.3 Lateral Pressure

The ultimate strength f_{up} of plates under lateral pressure is given as (Bruchman and Dinsenbacher 1991)

$$f_{up} = \frac{2.222F_y^2}{EB^2}\left[\left(\frac{\frac{w_u}{b}}{\left[0.00356+0.01988\tanh\left(\frac{B}{60}\sqrt{\frac{E}{F_y}}\right)\right]}\right)^{\frac{1}{3}}+1\right] \tag{28}$$

where F_y = yield strength (stress) of plate, b = distance between longitudinal stiffeners, or plate width, $B = \frac{b}{t}\sqrt{\frac{F_y}{E}}$, slenderness ratio of plate, $\alpha = \frac{a}{b}$, aspect ratio of plate, a = length or span of plate, t = thickness of the plate, E = the modulus of elasticity, and w_u = specified permanent set. Values for the ratio of the permanent set to plate width (w_u/b) or the permanent set to plate thickness (w_u/t) varies with both the material type and the location of a plate within the ship. When using Eq. 28, these values can be obtained from Tables 2 and 3, respectively.

3.1.4 Biaxial Compression

The ultimate strength f_{ux} and f_{uy} of plates under biaxial compression stresses should meet the requirement of following interaction equation (Valsgard 1980 and Frieze et al. 1977):

$$\left(\frac{f_x}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 - \eta_b\left(\frac{f_x}{f_{ux}}\right)\left(\frac{f_y}{f_{uy}}\right) \le 1 \tag{29}$$

where

$$\eta_b = \begin{cases} 0.25 & \text{if} \quad \alpha \ge 3.0 \\ 0.25 - \left(\frac{\alpha - 3}{2}\right)\left[3.2e^{-0.35B} - 2.25\right] & \text{if } 1.0 < \alpha < 3.0 \\ 3.2e^{-0.35B} - 2 & \text{if} \quad \alpha = 1.0 \end{cases} \tag{30}$$

and $\alpha = a/b$, the aspect ratio of plate, f_x = the applied stress in the x-direction, f_y = the applied stress in the y-direction, f_{ux} = the ultimate strength of a plate under compressive normal stress in the x-direction acting alone, and f_{ux} = the ultimate strength of a plate under compressive normal stress in the y-direction acting alone.

The ultimate stresses f_{ux} and f_{uy} can be computed from Eqs. 18 and 19, respectively. It should be noted that when using Eqs. and 18 and 19 for calculating both f_{ux} and f_{uy}, the length of plate (a) is assumed to coincide with the x-direction and the aspect ratio α is greater than unity. If, however, α is less than unity, then f_{ux} and f_{uy} should be interchanged in Eqs. 18 and 19.

3.1.5 Biaxial Compression and Edge Shear

The ultimate strength f_{ux}, f_{uy}, and $f_{u\tau}$ of plates under biaxial compression and edge shear stresses should meet the requirement of following interaction equation as adopted by the API (1993) and the DnV (1977):

$$\left(\frac{f_x}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 + \left(\frac{f_\tau}{f_{u\tau}}\right)^2 \leq 1 \qquad (31)$$

where f_x = the applied stress in the x-direction, f_y = the applied stress in the y-direction, f_τ = the applied shear stress, f_{ux} = the ultimate strength of a plate under compressive normal stress in the x-direction acting alone, f_{ux} = the ultimate strength of a plate under compressive normal stress in the y-direction acting alone, and $f_{u\tau}$ = the ultimate shear stress when the plate is subjected to pure edge shear. The ultimate stresses f_{ux}, f_{uy}, $f_{u\tau}$ can be computed from Eqs. 18, 19, and 21, respectively.

3.1.6 Other Load Combinations with Lateral Pressure

The loading conditions for unstiffened plates that are covered in this section are the combined in-plane and lateral pressure loads. Lateral pressure in combination with the other cases of loading presented in the previous sections can lead to a number of loading conditions that can have an effect on the overall strength of plates. The following cases can be identified:

1. Lateral pressure and uniaxial compression
2. Lateral pressure and biaxial compression
3. Lateral pressure, uniaxial compression, and edge shear
4. Lateral pressure, biaxial compression, and edge shear
5. Lateral pressure and edge shear

The effect of lateral pressure on the ultimate strength of plates subjected to in-plane loads is so complex that there are no simple models (formulas) available to predict the strength of plates under these types of loading. However, there are design charts available for some of these load combinations. For example, large deflection solutions for case 4 (lateral pressure, biaxial compression, and edge shear) exist, but the results cannot be put in the form of a simple formula as those given in the previous sections. Researchers demonstrated that the lateral pressure has negligible effect on both the uniaxial and biaxial compressive strength of plates when b/t is less than 50 (Becker et al. 1970). However, for values of the ratio b/t greater than 50, the lateral pressure can have a negative impact on the biaxial strength (case 2). Also, they pointed out that a clear understanding of the influence of pressure on strength of plates subjected to in-plane loads is lacking and that additional testing and research on the subject is deemed to be appropriate to clarify some of the aspects involved. Therefore, it is recommended to treat lateral pressure as an uncoupled load from other in-plane loads, and to design for them individually and separately (Assakkaf 1998 and Ayyub and Assakkaf 1996).

4. LRFD GUIDELINES FOR UNSTIFFENED PANELS

4.1 Target Reliability Levels

Selecting a target reliability level is required in order to establish reliability-based design guidelines for ship structures such as the unstiffened panels. The selected reliability level determines the probability of failure of the unstiffened panel element. The following three methods can be used to select a target reliability value:

1. Agreeing upon a reasonable value in cases of novel structures without prior history.
2. Calibrating reliability levels implied in currently used design codes.
3. Choosing target reliability level that minimizes total expected costs over the service life of the structure for dealing with design for which failures result in only economic losses and consequences.

The recommended range of target reliability indices for unstiffened panel can be set to range from 3.0 to 4.0 (Mansour et al. 1996).

4.2 Statistical Characteristics of Basic Random Variables

The statistical characteristics of random variables of strength and load models are needed for reliability-based LRFD design and assessment of ship structures including unstiffened panels. The moment methods for calculating partial safety factors (Ang and Tang 1990, Ayyub and McCuen 1997, and Ayyub and White 1978) require full probabilistic characteristics of both strength and load variables in the limit state equation. For example, the relevant strength variables for unstiffened panel element are the material's yield strength (stress) F_y, length of a panel a, and thickness t of plate. While the relevant loads variables are the external pressures due to stillwater bending moment, wave bending moment, and dynamic loads.

The definition of these random variables requires the investigation of their uncertainties and variability. In reliability assessment of any structural system, these uncertainties must be quantified. Furthermore, partial safety factors (PSF's) evaluation for both the strengths and loads in any design equation also requires the characterization of these variables. For example, the first-order reliability method (FORM) as outlined earlier requires the quantification of mean values, standard deviations (or the coefficient of variation (*COV*)), and distribution types of all relevant random variables. They are needed to compute the safety index β or the PSF's. Therefore, complete information on the probability distributions of the basic random variables under consideration must be developed. Quantification of random variables of loads and strength in terms of their means, standard deviations or *COV*'s, and probability distributions can be achieved in two steps: (a) data collection and (b) data analysis. The first step is the task of collecting as many sets of data deemed to be appropriate for representing the random variables under study. The second is concerned with statistically analyzing the collected data to determine the probabilistic characteristics of these variables.

The objective herein is to compile statistical information and data based on literature review on both strength and loads random variables for quantifying the probabilistic characteristics of these variables. The quantification of the probabilistic characteristics of these variables is needed for reliability analysis and design of hull structural components. Tables 4, 5 and 6 provide summaries of the probabilistic characteristics of strength and loads random variables. The information given in these tables is tabulated based on data from a literature review performed in Atua (1998) and Assakkaf (1998).

Tables 7 through 10 provide all the recommended values of information required for establishing reliability-based LRFD guidelines for unstiffened panels of ship structures. This information includes limit state functions for different load combinations; probabilistic characteristics (mean values, *COV*, and distribution type) of random variables involved in these limit state functions. The information also includes mean to nominal values (biases) of these random variables, deterministic values of the probabilistic load-combination factors; mean ratios between different load components, the biases between different values of each of the random variables; and probabilistic characteristics of model and prediction uncertainty parameters. This information is needed to calculate partial safety factors (PSF's) for unstiffened panels using, for example, FORM as discussed in Ayyub et al. (2002a).

4.3 Calculations of Partial Safety Factors

In this section, calculations of partial safety factors (PSF's) of both strength and load components in limit state functions for unstiffened ship plates are presented for demonstration purposes. The first-order reliability method (FORM) as outlined in Ayyub et al. (2002a) was used to develop the partial safety factors. The partial safety factors are defined as the ratio of the value of a variable in a limit state at its most probable failure point to the nominal value. The first section summarizes the methods for calculating partial safety factors. It also gives a brief review of recommended load and load combinations and their probabilistic characteristics used in computing the partial safety factors. The second section describes the development of a program for computing partial safety factors based on FORM as outlined in Ayyub et al (2002a). The final section summarizes the results of partial safety factors calculations for unstiffened panel under unaxial compression.

4.3.1 Performance Functions for Calculating Partial Safety Factors for Unstiffened Panels

Reliability-based design LRFD format involves the ultimate strength capacity of an unstiffened plate element and the load random variable of stillwater, wave-induced, and dynamic bending moments. The partial safety factors format allows transforming the desired reliability index into separate safety factors for each of the design variables in the recommended format. Two recommended limit state formats for unstiffened panels are provided as follows:

<u>Limit State I:</u>

$$g\left(R_u, f_{SW}, f_{WD}\right) = R_u - f_{SW} - k_{WD} f_{WD} \tag{32}$$

<u>Limit State II:</u>

$$g\left(R_u, f_{SW}, f_W, f_D\right) = R_u - f_{SW} - k_W\left(f_W + k_D f_D\right) \tag{33}$$

where g = the limit sate or performance function, f_{SW} = stress due to stillwater bending moment, f_{WD} = stress due to combined wave-induced and dynamic bending moments, f_W = stress due to waves bending moment, k_{WD} = combined wave-induced and dynamic bending moments factor equals unity, k_W = load combination factor equals unity, k_D = load combination factor equals 0.7, and R_u = ultimate strength capacity of an unstiffened plate. The ultimate strength capacity R_u depends on the loading conditions for the plate (i.e., uniaxial, edge shear, etc.) and is given by the design strength models as described in Section 3.1. The two limit states given by Eqs. 32 and 33 are referred to as limit state 1 and 2, respectively.

The load and load combinations probabilistic characteristics are shown in Table 8. The recommended mean and nominal load factors based on hull-girder bending are given in Tables 11 and 12, respectively. The recommended values for the load components in Table 8 were used to develop the partial safety factors for the loads and the strength models, while the recommended load factors of Tables 11 and 12 were used to calibrate the strength factors based on the recommended load factors.

4.3.2 Development of FORM-based Partial Safety Factors

The generalized FORM was selected to calculate the partial safety factors for the formats as given in Eqs. 32 and 33 due to the existence of non-normal basic random variables in the corresponding limit states for unstiffened panels. The generalized form of the limit state function can be set in any computational tool to the following form:

$$g(X) = C_1 X_1^{n1} X_2^{n2} + C_2 X_3^{n3} X_4^{n4} + C_3 X_5^{n5} X_6^{n6} + C_4 X_7^{n7} X_8^{n8} + C_5 X_9^{n9} X_{10}^{n10} X_{11}^{n11} \qquad (34)$$

where $g(X)$ = performance function, C_i = deterministic coefficient, X_i = probabilistic basic random variables, n = real-valued power.

By equating the reliability index, β, with the target reliability index, β_o, the partial safety factors are computed. The strength variables in the limit state at the design point is given by

$$R_u^* = f_{SW}^* + k_{WD} f_{WD}^* \qquad (35)$$

$$R_u^* = f_{SW}^* + k_W f_W^* + k_W k_D f_D^* \qquad (36)$$

Then the partial safety factors are computed as follows:

$$\phi_{R_u} = \frac{R_u^*}{R_{u_n}^*} \tag{37}$$

$$\gamma_{f_{SW}} = \frac{f_{SW}^*}{f_{SW_n}^*} \tag{38}$$

$$\gamma_{f_{WD}} = \frac{f_{WD}^*}{f_{WD_n}} \tag{39}$$

$$\gamma_{f_W} = \frac{f_W^*}{f_{W_n}} \tag{40}$$

$$\gamma_{f_D} = \frac{f_D^*}{f_{D_n}} \tag{41}$$

where the subscript n means nominal value.

The partial safety factors calculations are iterative in nature to search for the PSF's that satisfy the target reliability level, β_O. In this iterative procedure, only one input variable is varied. In the limit states of Eqs. 32 and 33, this variable is the mean value of the ultimate capacity R_u of a plate.

4.3.3 Results of the Partial Safety Factors Calculations

In this example, results of partial safety factors calculations for unstiffened panel under uniaxial compression are demonstrated. Similar results can be achieved for unstiffened panel under various type of loading (i.e., edge shear, lateral pressure, etc.) as described in Section 3.1. The two formats for limit states as given by Eqs. 32 and 33 were selected for the development of partial safety factors (PSF's). The ultimate strength f_u of plates under uniaxial compression stress is given by Eqs.18 and 19. The recommended range of target reliability index β_0 for unstiffened plates under uniaxial compression stress was set to be from 3.0 to 4.0. These values are used in calculating the PSF's for both the strength and the loads.

Limit State I:

The limit state function for plates under uniaxial compression is given by

$$g = f_u - f_{SW} - k_{WD} f_{WD} \tag{42}$$

where f_u is the ultimate strength for plates under uniaxial compression as defined by Eq. 18 and 19, f_{SW} = stress due to stillwater bending moment, f_{WD} = stress due to combined wave-induced and dynamic bending moments, and k_{WD} = combined wave-induced and dynamic bending moment factor set equal to unity (Mansour et al. 1996). The mean values of stillwater and combined wave-induced and dynamic stresses are given in the form of a ratio of f_{SW}/f_{WD} as shown in Table 13. The table also shows the ranges of the target reliability index β_0 and the uncertainty (*COV*) in the strength f_u. The probabilistic characteristics for both the strength and the loads are summarized in Table 14. The results of the partial safety factors using FORM (ASM) are provided in Table 15. Calibration (recalculation) of the strength factor for a recommended set of load factors is given in Tables 16 and 17.

Limit State II:

The limit state function for plates under uniaxial compression is given by

$$g = f_u - f_{SW} - k_W\left(f_W + k_D f_D\right) \tag{43}$$

where f_u is the ultimate strength for plates under uniaxial compression as defined by Eq. 18 and 19, f_{SW} = stress due to stillwater bending moment, f_W = stress due to waves bending moment, f_D = stress due to dynamic bending moment, k_W = load combination factor equals 1.0 (Mansour et al. 1996), and k_D = load combination factor equals 0.7 (Mansour et al. 1996). The mean values of stillwater, waves, and dynamic stresses are given in the form of a ratio of f_{SW}/f_W as shown in Table 18. The table also shows the ranges of the target index β and the uncertainty (*COV*) in the strength f_u. The probabilistic characteristics for both the strength and the loads are summarized in Table 19. The results of the partial safety factors using FORM (ASM) are provided in Table 20. Calibration (recalculation) of the strength factor for a recommended set of load factors is given in Tables 21 and 22.

4.4 Sample LRFD Guidelines

This section provides sample reliability-based LRFD guidelines for unstiffened panels of ship structures. The guidelines, as demonstrated herein, consist of limit state expressions, partial safety factors for both the strength and the loads, and a range of target reliability levels. Unstiffened plate elements of ship structure for all stations should meet one of the limit states as given by Eqs 13 through 17 in Section 3.

The ultimate strength capacity R_u depends on the loading conditions for the unstiffened plate element (i.e., uniaxial, edge shear, etc.) and is given by the design strength models as described in Section 3.1. The two limit states given by Eqs. 13 and 14 are referred to as limit state 1 and 2, respectively.

The nominal (i.e., design) values of the strength and load components shall satisfy these formats in order to achieve specified target reliability levels. The strength factors are provided in Table 23 in accordance with the following parameters: (1) target reliability level ranging from 3.0 to 4.0, (2) the type of load combinations as shown in the table, and (3) ultimate strength prediction for unstiffened panel as provided in section 3.1. The target reliability should be selected based on the ship type and usage. Then, the corresponding factor can be looked up from Table 23 based on the strength model under consideration. The load factors that can be used in conjunction with strength factors are provided in Table 24.

5. EXAMPLES DESIGN

The following two examples demonstrate the use of LRFD-based partial safety in the limit state equation for designing and checking the adequacy of unstiffened panels of ships:

EXAMPLE 1. Plate Design

Given: A 48”x 24”x *t* unstiffened plate element is to be designed at the bottom deck of a ship to withstand a uniaxial compression stress due to environmental bending moment loads acting on the ship. The stresses due to the environmental loads are estimated to have the following values: 12 ksi due to stillwater bending, 4.8 ksi due to waves bending, and 1.8 ksi due to dynamic bending. If the yield strength of steel is 34 ksi, design the thickness *t* of the plate assuming target level of 3.0.

Solution: For unstiffened panel under uniaxial compression, the strength is given by Eq. 18 as

$$f_u = \begin{cases} F_y\sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ F_y\left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & \text{if } 1.0 \leq B < 3.5 \\ F_y & \text{if } B < 1.0 \end{cases}$$

Assume that t = 0.25 in., therefore

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} = \frac{24}{0.25}\sqrt{\frac{34}{29000}} = 3.29$$

and

$$f_u = F_y\left(\frac{2.25}{B} - \frac{1.25}{B^2}\right) = 34\left(\frac{2.25}{3.29} - \frac{1.25}{(3.29)^2}\right) = 19.33\,\text{ksi}$$

The design of the plate should meet the requirement of the LRFD guidelines (see Section 4.4) as given in Tables 23 and 24 for the limit state under consideration and the appropriate partial safety factors for $\beta_0 = 3.0$, that is,

$$\phi f_u \geq \gamma_{SW} f_{SW} + k_W\left(\gamma_W f_W + \gamma_D k_D f_D\right)$$

$\phi f_u = 0.83(19.33) = 16.04$ ksi

$\gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_{SW}) = (1.05)(12) + (1)[1.4(4.8) + (1.1)(0.7)(1.8)]$

$= 20.7$ ksi

$(\phi f_u = 16.04 \text{ ksi}) < 20.7$ ksi **unacceptable**

Try a value of t = 0.350 in., therefore

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} = \frac{24}{0.35}\sqrt{\frac{34}{29000}} = 2.3$$

and

$$f_u = F_y\left(\frac{2.25}{B} - \frac{1.25}{B^2}\right) = 34\left(\frac{2.25}{2.3} - \frac{1.25}{(2.3)^2}\right) = 25.23\,\text{ksi}$$

$\phi f_u = 0.83(25.23) = 20.94$ ksi

$\gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_{SW}) = (1.05)(12) + (1)[1.4(4.8) + (1.1)(0.7)(1.8)]$

$= 20.7$ ksi

$(\phi f_u = 20.94 \text{ ksi}) > 20.7$ ksi **acceptable**

Hence, select **PL: 48 x 24 x 0.350**

EXAMPLE 2. Adequacy Checking

Given: Suppose that the plate of Example 1 is to be checked for the effect of lateral pressure. Would this plate be adequate to withstand the lateral pressure generated by the environmental loads?

Solution: For unstiffened panel under pure lateral pressure, the strength is given by Eq. 28 as

$$f_{up} = \frac{2222F_y^2}{EB^2}\left[\left(\frac{\frac{w_u}{b}}{\left[0.00356 + 0.01988\tanh\left(\frac{B}{60}\sqrt{\frac{E}{F_y}}\right)\right]}\right)^{\frac{1}{3}} + 1\right]$$

For MS Steel, and Lower Shell/Tank, Table 3 gives

$$\frac{w_u}{b} = 0.009$$

With $B = 3.0$ as computed in Example 1, therefore,

$$f_{up} = \frac{2222(34)^2}{29000(3)^2}\left[\left(\frac{0.009}{\left[0.00356 + 0.01988\tanh\left(\frac{3}{60}\sqrt{\frac{29000}{34}}\right)\right]}\right)^{\frac{1}{3}} + 1\right] = 17.21\,\text{ksi}$$

The design of the plate should meet the requirement of the LRFD guidelines (see Section 4.4) as given in Tables 23 and 24 for the limit state under consideration and the appropriate partial safety factors for $\beta_0 = 3.0$, that is,

$$\phi f_{up} \geq \gamma_{SW} f_{SW} + k_W\left(\gamma_W f_W + \gamma_D k_D f_D\right)$$

$\phi f_{up} = 0.47(17.21) = 8.09$ ksi

$\gamma_{SW} f_{SW} + k_W\,(\gamma_W f_W + \gamma_D\, k_D f_{SW}) = (1.05)(12) + (1)\,[1.4\,(4.8) + (1.1)\,(0.7)\,(1.8)]$

$= 20.7$ ksi

($\phi f_u = 8.09$ ksi) < 20.7 ksi **unacceptable**

Hence, the plate will not be adequate for lateral pressure. A new plate should be designed.

6. SUMMARY AND CONCLUSIONS

Future design guidelines for unstiffened panels of ship structures will be developed using reliability methods and they will be expressed in a special and practical format such as the Load and Resistance Factor Design (LRFD). The LRFD guidelines for unstiffened panels, which are based on structural reliability theory, can be built on previous and currently used specifications for ships, buildings, bridges, and offshore structures. This paper provides methods for and demonstrates the development of LRFD guidelines for ship unstiffened plate elements subjected to various types of loading. These design methods were developed according to the following requirements: (1) spectral analysis of wave loads, (2) building on conventional codes, (3) nominal strength and load values, and (4) achieving target reliability levels.

The First-Order Reliability Method (FORM) was used to develop the LRFD-based partial safety factors (PSF's) for selected limit states and for various types of loading acting on unstiffened panel element. These factors were determined to account for the uncertainties in strength and load effects. FORM was used to determine these factors based on prescribed probabilistic characteristic of strength and load effects. Also, strength factors were computed for a set of load factors to meet selected target reliability levels for demonstration purposes. The resulting LRFD guidelines are demonstrated in this paper using design examples.

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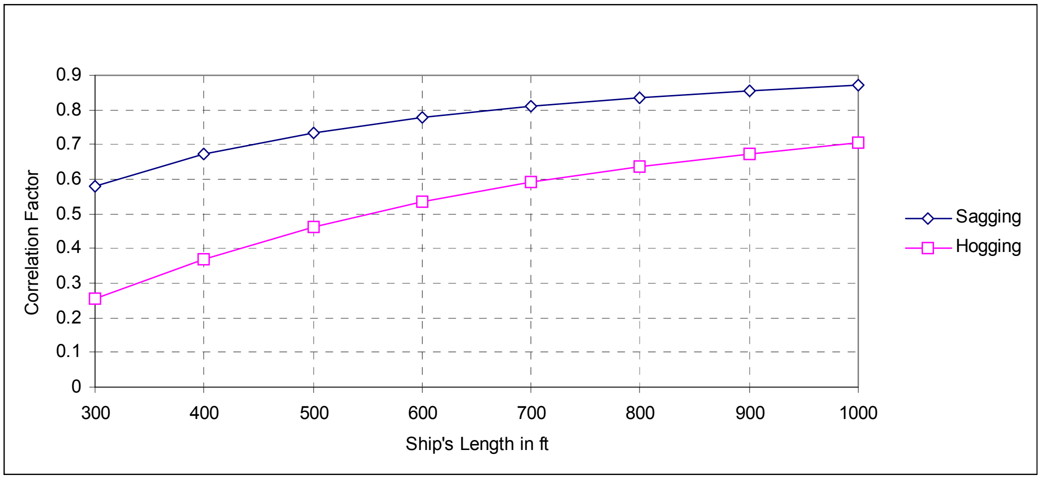

Figure 1. Correlation Coefficient of Whipping Bending Moment (k_D) for 300 < LBP < 1000 ft (Mansour et al. 1984 and Ayyub et al. 1995)

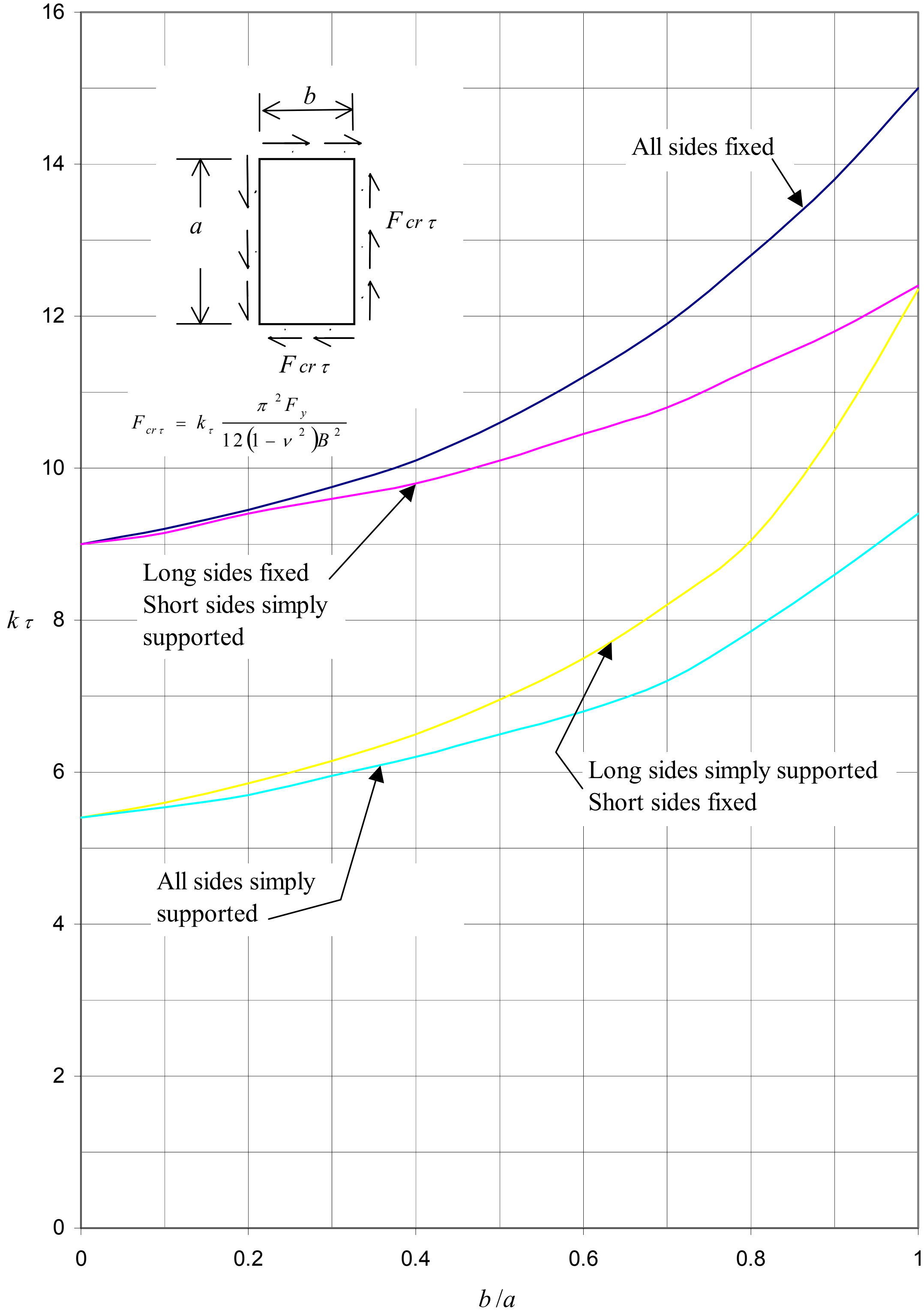

Figure 2. Buckling Coefficient k_τ of Plates in Shear as a Function of *b/a*

Table 1. Correlation Coefficient of Whipping Bending Moment (k_D) for *LBP* between 300 and 1000 ft (Mansour at al. 1984 and Atua 1998)

Length (ft)	300	400	500	600	700	800	900	1000
$k_{D(sag)}$	0.5779	0.672	0.734	0.778	0.810	0.835	0.854	0.870
$k_{D(hog)}$	0.2539	0.369	0.461	0.533	0.591	0.637	0.675	0.706

Table 2. Ranges of the Ratio w_u/b

Aluminum or Steel Type	Yield Strength F_y (ksi)	Top Side			Lower Shell/Tank			Flooding/Damage Control		
		Min	Recommended	Max	Min	Recommended	Max	Min	Recommended	Max
AL5086	28	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.009	0.011
AL5456	33	0.000	0.000	0.000	0.001	0.001	0.001	0.021	0.032	0.038
MS	34	0.000	0.000	0.000	0.006	0.009	0.011	0.085	0.128	0.155
HTS	47	0.000	0.000	0.000	0.004	0.006	0.008	0.065	0.098	0.119
HY80	80	0.000	0.000	0.000	0.001	0.001	0.002	0.014	0.021	0.025
HY100	100	0.000	0.000	0.000	0.000	0.000	0.001	0.013	0.019	0.023

Table 3. Ranges of the Ratio w_u/t

Aluminum or Steel Type	Yield Strength F_y (ksi)	Top Side			Lower Shell/Tank			Flooding/Damage Control		
		Min	Recommended	Max	Min	Recommended	Max	Min	Recommended	Max
AL5086	28	0.000	0.000	0.000	0.001	0.005	0.010	0.186	0.821	1.687
AL5456	33	0.000	0.000	0.000	0.015	0.066	0.135	0.632	2.792	5.741
MS	34	0.000	0.002	0.022	0.181	0.801	1.647	2.552	11.282	23.20
HTS	47	0.000	0.001	0.006	0.125	0.553	1.138	1.958	8.658	17.80
HY80	80	0.000	0.000	0.000	0.026	0.114	0.233	0.412	1.822	3.746
HY100	100	0.000	0.000	0.000	0.008	0.037	0.076	0.383	1.692	3.478

Table 4a. Probabilistic Characteristic of Strength Basic Random Variables for Unstiffened Panels (Atua et al. 1996 and Assakkaf 1998)

Variable	Nominal Value	Statistical Information		
		Mean	Standard Deviation	Distribution Type
Thickness of Plate (in)	*t*	*t*	0.02	normal
Length of Plate (in)	*a*	*a*	0.11	normal
Width of plate (in)	*b*	*b*	0.09	normal

Table 4b. Probabilistic Characteristic of Strength Basic Random Variables for Unstiffened panels (Atua 1998 and Assakkaf 1998)

Variable	Nominal Value	Statistical Information		
		Mean	*COV*	Distribution Type
Ordinary Strength F_y (ksi)	F_y	1.11 F_y	0.07	lognormal
High Strength F_y (ksi)	F_y	1.22 F_y	0.09	lognormal
F_u (ksi)	F_u	1.05 F_u	0.05	normal
E (ksi)	E	1.024 E	0.02	normal
ν	0.3	0.3	0	

OS = Ordinary Steel, HS = Higher Strength Steel, na = not available

Table 5a. Ranges for Statistics of Strength Basic Random Variables for Unstiffened panels (Atua et al. 1996 and Assakkaf 1998)

Random Variable		Bias Information	
		Mean	Standard Deviation
Thickness of Plate t (in)	Minimum	t	0.00520
	Recommended	t	0.01720
	Maximum	t	0.04170
Length of Plate a (in)	Minimum	a	na
	Recommended	a	0.10600
	Maximum	a	na
Width of Plate b (in)	Minimum	b	na
	Recommended	b	0.09300
	Maximum	b	na

Table 5b. Ranges for Statistics of Strength Basic Random Variables for Unstiffened Panels (Atua 1998 and Assakkaf 1998)

Random Variable		Statistical Information		
		Mean	*COV*	Bias
OS F_y (ksi)	Minimum	33.8	0.03	1.000
	Recommended	**37.3**	**0.07**	**1.110**
	Maximum	44.0	0.12	1.220
HS F_y (ksi)	Minimum	39.6	0.07	1.100
	Recommended	**49.6**	**0.09**	**1.220**
	Maximum	66.0	0.10	1.350
F_u (ksi)	Minimum	59.3	0.02	1.007
	Recommended	**61.6**	**0.05**	**1.046**
	Maximum	64.3	0.09	1.090
E (ksi)	Minimum	28,980	0.01	1.000
	Recommended	**29,696**	**0.02**	**1.024**
	Maximum	30,200	0.06	1.076

OS = Ordinary Steel, HS = Higher Strength Steel, na = not available

Table 6. Probabilistic Characteristics of Load Random Variables (Atua 1998)

Random Variable	Distribution Type	Mean to Nominal Ratio	Coefficient of Variation
Stillwater Bending Moment M_{SW}	Normal	0.4 to 0.6 for commercial ships, and 0.7 for naval vessels	0.3 to 0.9 for commercial ships, and 0.15 for naval vessels
Life-time Extreme Wave-induced Bending Moment M_W	Largest extreme value (type I)	1.0	0.1 to 0.2
Whipping Bending Moment M_D	Extreme value (type I) exponential	Mean value can be determined using formulae based on spectral analysis	0.2 to 0.3
Springing Bending Moment M_{SP}	Extreme value (type I)	1.0	0.3
Hydrostatic pressure due to stillwater, P_{SW}	Normal	0.4 to 0.6 for commercial ships, and 0.7 for naval vessels	0.15
Hydrostatic pressure due to waves, P_W	Largest extreme value (type I)	1.0	0.15
Hydrostatic pressure due to dynamic effects, P_D	Largest extreme value (type I)	1.0	0.25
Hydrostatic pressure due to combined waves and dynamic loads, P_{WD}	Weibull	1.0	0.25

Table 7. Recommended Total Bias and Coefficients of Variation (*COV*) for the Strength of Unstiffened Panel (Assakkaf 1998)

Loading Case	Distribution Type	Total Bias B_T	*COV* (%)
Uniaxial Compression	Lognormal	1.16	18
Edge Shear	Lognormal	1.13	20
Uniform Lateral Pressure	Lognormal	1.11	17
Biaxial Compression	Lognormal	1.10	20
Biaxial Compression and Edge Shear	Lognormal	1.06	11

Table 8. Recommended Total Bias and Coefficients of Variation (*COV*) for Basic Loads Acting on a Ship (Atua and Ayyub 1996)

Type of Load	Abbreviation	Distribution Type	Total Bias B_T	*COV* (%)
Stillwater Bending Moment	M_{SW}	Normal	1.0	15
Wave-induced and Dynamic Bending Moment	M_{WD}	Weibull	1.0	25
Waves Bending Moment	M_W	Type I Largest	1.0	15
Dynamic Bending Moment due to Whipping	M_D	Type I Largest	0.97	25

Table 9. Recommendations for Ratios of Different Load Components

Ratio	Recommended Value	Reference
$\overline{M}_{SW} / \overline{M}_W$	0.25 to 0.35	Mansour et al. (1996)
$\overline{M}_D / \overline{M}_W$	0.25 to 0.35	Mansour et al. (1996)
$\overline{M}_{WD} / \overline{M}_W$	1.0 to 1.35	Assumed values

Table 10. Recommendations for Combination Factors for Load Components

Factor	Deterministic Value	References and Comments
k_W	1.0	Sikora et al. (1983) and Atua et al. (1996)
k_D	$EXP\left[\frac{53080}{\left(158LBP^{-0.2}+14.2LBP^{0.3}\right)LBP}\right]$ (Hogging) $EXP\left[\frac{21200}{\left(158LBP^{-0.2}+14.2LBP^{0.3}\right)LBP}\right]$ (Sagging)	Reference (Sikora et al. 1983) Ranging from 0.35 to 0.65 for *LBP* = (400 to 800) ft Ranging from 0.65 to 0.85 for *LBP* = (400 to 800) ft
k_{WD}	1.0	Assumed value as defined in Sikora et al. (1983)

Table 11. Recommended Mean Load Factors

β	γ_{fS}	γ_{fWD}	γ_{fW}	γ_{fD}
3.0	1.05	1.30	1.20	1.05
3.5	1.05	1.35	1.25	1.05
4.0	1.05	1.40	1.30	1.05

Table 12. Recommended Nominal Load Factors

β	γ_{fS}	γ_{fWD}	γ_{fW}	γ_{fD}
3.0	1.05	1.30	1.20	1.05
3.5	1.05	1.35	1.25	1.05
4.0	1.05	1.40	1.30	1.05

Table 13. Ranges of Key Parameters for Limit State I

Parameter	Ranges
β_0	3.0, 3.5, and 4.0
$COV\ (f_u)$	0.18
f_{SW}/f_{WD}	0.1, 0.2, and 0.3

Table 14. Probabilistic Characteristic of Strength and Loads for Limit State I

Random Variable	*COV* (recommended)	Distribution Type	Total Bias
f_u	0.18 (0.18)	Lognormal	1.16
f_{SW}	0.15 (0.15)	Normal	1.0
f_{WD}	0.22 to 0.29 (0.25)	Weibull	1.0

Table 15. Results of Partial Safety Factors Calculations using FORM (ASM) for Limit State I

β_0	Mean of f_u	ϕ_u	γ_{SW}	γ_{WD}
3.0	2.56	0.64	1.04	1.43
3.5	2.85	0.59	1.04	1.47
4.0	2.17	0.54	1.05	1.50

Table 16. Recommended Mean Strength and Load Factors for Limit State I

β_0	ϕ_u	γ_{SW}	γ_{WD}
3.0	0.65	1.05	1.45
3.5	0.60	1.05	1.50
4.0	0.55	1.05	1.55

Table 17. Recommended Nominal Strength and Load Factors for Limit State I

β_0	ϕ_u	γ_{SW}	γ_{WD}
3.0	0.75	1.05	1.45
3.5	0.70	1.05	1.50
4.0	0.64	1.05	1.55

Table 18. Ranges of Key Parameters for Limit State II

Parameter	Ranges
β_0	3.0, 3.5, and 4.0
$COV\ (f_u)$	0.18
f_{SW}/f_W	0.2, 0.3, and 0.4
f_D/f_W	0.25, 0.30, and 0.35

Table 19. Probabilistic Characteristic of Strength and Loads for Limit State II

Random Variable	*COV* (recommended)	Distribution Type	Total Bias
f_u	0.18 (0.18)	Lognormal	1.16
f_{SW}	0.15 (0.15)	Normal	1.0
f_W	0.1 to 0.2 (0.15)	Type I Largest	1.0
f_D	0.2 to 0.3 (0.25)	Type I Largest	1.0

Table 20. Results of Partial Safety Factors Calculations using FORM (ASM) for Limit State II

β_0	Mean of f_u	ϕ_u	γ_{SW}	γ_W	γ_D
3.0	2.87	0.65	1.05	1.33	1.05
3.5	3.23	0.62	1.05	1.45	1.06
4.0	3.46	0.59	1.05	1.59	1.06

Table 21. Recommended Mean Strength and Load Factors for Limit State II

β_0	ϕ_u	γ_{SW}	γ_W	γ_D
3.0	0.71	1.05	1.40	1.10
3.5	0.68	1.05	1.55	1.10
4.0	0.68	1.05	1.70	1.10

Table 22. Recommended Nominal Strength and Load Factors for Limit State II

β_0	ϕ_u	γ_{SW}	γ_W	γ_D
3.0	0.83	1.05	1.40	1.10
3.5	0.79	1.05	1.55	1.10
4.0	0.79	1.05	1.70	1.10

Table 23. Nominal Strength Factors for Unstiffened Panels

Loading Condition	Load Combination	Strength Factors, ϕ					
		Target Reliability Index, β_0					
		3.0		3.5		4.0	
		ϕ	ϕ_τ	ϕ	ϕ_τ	ϕ	ϕ_τ
Uniaxial Compression	$\phi_u f_u \ge \gamma_{SW} f_{SW} + k_{WD}\gamma_{WD} f_{WD}$	0.75	N/A	0.70	N/A	0.64	N/A
	$\phi_u f_u \ge \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + k_D \gamma_D f_D)$	0.83	N/A	0.79	N/A	0.79	N/A
Edge Shear	$\phi_{u\tau} f_{u\tau} \ge \gamma_{SW} f_{SW} + k_{WD}\gamma_{WD} f_{WD}$	N/A	0.70	N/A	0.64	N/A	0.59
	$\phi_{u\tau} f_{u\tau} \ge \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + k_D \gamma_D f_D)$	N/A	0.77	N/A	0.73	N/A	0.68
Lateral Pressure	$\phi_{up} f_{up} \ge \gamma_{SW} f_{SW} + k_{WD}\gamma_{WD} f_{WD}$	0.39	N/A	0.36	N/A	N/A	0.34
	$\phi_{up} f_{up} \ge \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + k_D \gamma_D f_D)$	0.47	N/A	0.46	N/A	0.44	N/A
Biaxial Compression	$\left(\frac{f_{1x}}{\phi_{ux} f_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{uy} f_{uy}}\right)^2 - \eta_b \left(\frac{f_{1x}}{\phi_{ux} f_{ux}}\right)\left(\frac{f_{1y}}{\phi_{uy} f_{uy}}\right) \le 1$	0.54	N/A	0.40	N/A	0.29	N/A
	$\left(\frac{f_{2x}}{\phi_{ux} f_{ux}}\right)^2 + \left(\frac{f_{2y}}{\phi_{uy} f_{uy}}\right)^2 - \eta_b \left(\frac{f_{2x}}{\phi_{ux} f_{ux}}\right)\left(\frac{f_{2y}}{\phi_{uy} f_{uy}}\right) \le 1$	0.61	N/A	0.51	N/A	0.42	N/A
Biaxial Compression and Edge Shear	$\left(\frac{f_{1x}}{\phi_{ux} f_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{uy} f_{uy}}\right)^2 + \left(\frac{f_{1\tau}}{\phi_{u\tau} f_{u\tau}}\right)^2 \le 1$	0.68	0.70	0.60	0.64	0.53	0.59
	$\left(\frac{f_{2x}}{\phi_{ux} f_{ux}}\right)^2 + \left(\frac{f_{2y}}{\phi_{uy} f_{uy}}\right)^2 + \left(\frac{f_{2\tau}}{\phi_{u\tau} f_{u\tau}}\right)^2 \le 1$	0.84	0.77	0.82	0.73	0.80	0.68

Note: f_1 and f_2 are the magnified applied stresses in x, y, and τ-direction; the subscripts refer to limit state 1 and 2, respectively, according to Eqs. 44 and 45; N/A = not applicable

Table 24. Nominal Load Factors

Target Reliability Index (β_0)	Load Factors			
	γ_{SW}	γ_W	γ_D	γ_{WD}
3.0	1.05	1.40	1.10	1.45
3.5	1.05	1.55	1.10	1.50
4.0	1.05	1.70	1.10	1.55