# Strength Models for Developing LRFD Rules for Unstiffened Panels of Marine Structures

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## Abstract

The objective of this paper is to summarize development in strength prediction models for unstiffened panels (plates) that are suitable for Load and Resistance Factor Design (LRFD) for marine structures. Monte Carlo Simulation was utilized to assess the biases and uncertainties for the models. Recommendations for the use of the models and their biases in LRFD development are provided. The first-order reliability method (FORM) was used to demonstrate the development of partial safety factors for a selected limit state.

## Introduction

Plates are important components in ship structures, and therefore they should be designed for a set of failure modes that govern their strength. Plates elements, in general, are parts of stiffened panels for which their strengths need to be predicted. However, a global failure of stiffened panels can be partially controlled by designing the strength of plate elements between stiffeners. To evaluate the strength of an unstiffened plate element it is necessary to review various strength prediction models and to study their applicability and limitations for different loading conditions acting on the element. The uncertainties that are associated with a numerical analysis are generally a result of experimental approximation or numerical inaccuracies that can be reduced by some procedures. However, the uncertainties that are associated with a strength design model is different and cannot be eliminated because it results from not accounting for some variables which can have strong influence on the strength. For this reason, the uncertainty and the bias of a design equation should be assessed and evaluated by comparing its predictions with more accurate ones. Wherever possible, the different types of biases resulting from these models were computed. In doing so, these prediction models were classified as follows (Atua and Ayyub 1996): (1) prediction models that can be used by the LRFD rules, (2) advanced prediction models that can be used for various analytical purposes, (3) some experimental results from model testing, and (4) some real measurements based on field data during the service life of a ship. Furthermore, the relationships and uncertainty analyses for these models are required. The relationships can be defined in terms of biases (bias factors). These bias factors are given by $B_{21} = \frac{\text{Advanced predicted value}}{\text{Rules value}}$,

$B_{32} = \frac{\text{Experimental value}}{\text{Advanced predicted value}}$, $B_{43} = \frac{\text{Real value}}{\text{Experimental value}}$, and $B_{41} = \frac{\text{Real value}}{\text{Rules value}} = B_{21}B_{32}B_{43}$

In this paper, only the strength models that are deemed suitable for LRFD design format are highlighted and investigated.

## Strength Model for Plates under Uniaxial Compression

As described by Mansour (1986), the ultimate strength $f_u$ which causes collapse of plates between stiffeners is given by one of the following two cases (Bleich 1952 and Faulkner 1975):

(a) For *a/b* > 1.0

$$f_u = \begin{cases} F_y \sqrt{\dfrac{p^2}{3(1-n^2)B^2}} & \text{if } B \geq 3.5 \\ F_y \left( \dfrac{2.25}{B} - \dfrac{1.25}{B^2} \right) & \text{if } 1.0 \leq B < 3.5 \\ F_y & \text{if } B < 1.0 \end{cases} \tag{1}$$

(b) For *a/b* < 1.0

$$f_u = F_y \left[ aC_u + 0.08(1-a)\left(1 + \frac{1}{B^2}\right) \right]^2 \leq F_y \tag{2}$$

where $F_y$ = yield strength (stress) of plate, *a* = length or span of plate, *b* = distance between longitudinal stiffeners, $B = \frac{b}{t}\sqrt{\frac{F_y}{E}}$, $a = \frac{a}{b}$, *t* = thickness of the plate, *E* = the modulus of elasticity, ν = Poisson's ratio, and

$$C_u = \begin{cases} \sqrt{\dfrac{p^2}{3(1-n^2)B^2}} & \text{if } B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & \text{if } 1.0 \leq B < 3.5 \\ 1.0 & \text{if } B < 1.0 \end{cases} \tag{3}$$

The bias factors and their probabilistic characteristics are provided in Table 1. Variability of the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$) was investigated and the results are plotted as shown in Figure 1, which suggests that a normal or lognormal distribution is suitable for the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$). However, according to the Chi-square test, the lognormal model is more suitable for the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$) than the normal model.

Table 1. Bias Factors for Uniaxial Compression

| Bias Factor | Mean Bias | COV of Bias | Bias Value | |
|---|---|---|---|---|
| | | | Mean | COV |
| $B_{21}$ | na | na | na | na |
| $B_{32}$ | na | na | na | na |
| $B_{43}$ | 1.13 to 1.88 | 0.02 to 0.04 | 1.47 | 0.03 |
| $B_{\text{Real/Rules}}$ | 0.79 to 1.37 | 0.02 to 0.05 | 1.04 | 0.04 |

na = not available

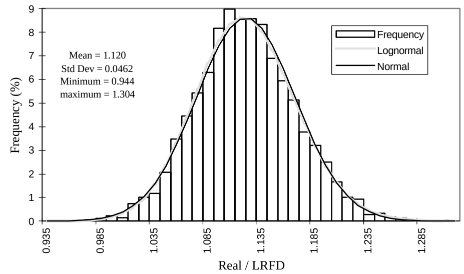


Figure 1. Bias Ratio Uniaxial Compression

## Strength Model for Plates under Pure Edge Shear

According to Basler (1963), the ultimate shear stress is given by

$$f_{ut} = F_{crt} + F_{Pt} \tag{4}$$

where $F_{crt}$ = critical or buckling stress, and $F_{Pt}$ = post-buckling strength using tension field action. The COV associated with this equation based on simulation is 0.08. The buckling strength is given by

$$F_{crt} = \begin{cases} F_{yt} & \text{if } B \le \dfrac{\sqrt{k_t \dfrac{p^2 F_y F_{Pr}}{12(1-n^2)}}}{F_{yt}} \\ \sqrt{k_t \dfrac{p^2 F_y F_{Pr}}{12(1-n^2)B^2}} & \text{if } \dfrac{\sqrt{k_t \dfrac{p^2 F_y F_{Pr}}{12(1-n^2)}}}{F_{yt}} < B \le \sqrt{k_t \dfrac{p^2 F_y}{12(1-n^2)F_{Pr}}} \\ k_t \dfrac{p^2 F_y}{12(1-n^2)B^2} & \text{if } B > \sqrt{k_t \dfrac{p^2 F_y}{12(1-n^2)F_{Pr}}} \end{cases} \tag{5}$$

where $F_{yt}$ = yield stress in shear, and $F_{Pr}$ = proportional limit in shear which can be taken as $0.8F_{yt}$. The buckling coefficient $k_t$ , depending on whether the plate under pure shear is simply-supported or clamped, can be computed from the following two expressions:
(a) For $a \ge 1.0$,

$$k_t = \begin{cases} 5.35 + \dfrac{4.0}{a^2} & \text{for simple supports} \\ 8.98 + \dfrac{5.6}{a^2} & \text{for clamped supports} \end{cases} \tag{6}$$

(b) For, $a \le 1.0$,

$$k_t = \begin{cases} 4.0 + \dfrac{5.35}{a^2} & \text{for simple supports} \\ 5.6 + \dfrac{8.98}{a^2} & \text{for clamped supports} \end{cases} \tag{7}$$

The COV associated with Basler's model (Eq. 5) based on simulation is 0.07. The yield stress in shear ($F_{yt}$) can be taken as $F_y / \sqrt{3}$ , where $F_y$ = yield stress of plate. The post buckling shear strength $F_{Pt}$ is given by $F_{Pt} = \frac{F_y - \sqrt{3}F_{crt}}{2\sqrt{1+a^2}}$, where $a$ is the aspect ratio of plate ($a/b$). The bias factors and their probabilistic characteristics are provided in Table 2. Variability of the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$) was investigated and the results suggest that a normal or lognormal distribution can be used for the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$).

Table 2. Bias Factors for Strength (Post Buckling) of Plates under Edge Shear Stress

| Bias Factor | Mean Bias | COV of Bias | Bias Value | |
|---|---|---|---|---|
| | | | Mean | COV |
| $B_{21}$ | na | na | na | na |
| $B_{32}$ | na | na | na | na |
| $B_{43}$ | 0.94 to 1.23 | 0.002 to 0.06 | 1.05 | 0.016 |
| $B_{\text{Real/Rules}}$ | 0.91 to 1.77 | 0.002 to 0.38 | 1.09 | 0.052 |

na = not available

## Strength Model for Plates under Uniform Lateral Pressure

Based on finite element methods, some researchers suggested the following model for plates under uniform lateral pressure (Ayyub and Assakkaf 1997):

$$f_{up} = \frac{2.222F_y^2}{EB^2}\left[\left(\frac{w_u}{b\left[0.00356 + 0.01988\tanh\left(\frac{B}{60}\sqrt{\frac{E}{F_y}}\right)\right]}\right)^{\frac{1}{3}} + 1\right] \tag{8}$$

The COV for this model based on simulation is 0.12. The bias factors and their probabilistic characteristics are provided in Table 3. Variability of the bias ratio ($B_{\text{Real}}/B_{\text{LRFD}}$) was investigated and the results suggest that a lognormal distribution model.

Table 3. Bias Factors for Uniform Pressure

| Bias Factor | Mean Bias | COV of Bias | Bias Value | |
|---|---|---|---|---|
| | | | Mean | COV |
| $B_{21}$ | 1.43 to 2.72 | 0.001 to 0.069 | 1.692 | 0.01 |
| $B_{32}$ | 0.54 to 0.74 | 0.002 to 0.045 | 0.697 | 0.01 |
| $B_{43}$ | 0.75 to 1.05 | 0.002 to 0.015 | 0.854 | 0.005 |
| $B_{\text{Real/Rules}}$ | 0.78 to 1.55 | 0.003 to 0.038 | 0.998 | 0.012 |

## Strength Model for Plates under Biaxial Compression

This ultimate strength $F_{crx}$ can be obtained from the following interaction equation for which the applied stress $f_y$ is known:

$$\left(\frac{F_{crx}}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 - h_b\left(\frac{F_{crx}}{f_{ux}}\right)\left(\frac{f_y}{f_{uy}}\right) = 1 \tag{9}$$

where

$$h_b = \begin{cases} 0.25 & \text{if} \quad a \geq 3.0 \\ 0.25 - \left(\frac{a-3}{2}\right)\left[3.2e^{-0.35B} - 2.25\right] & \text{if } 1.0 < a < 3.0 \\ 3.2e^{-0.35B} - 2 & \text{if} \quad a = 1.0 \end{cases} \tag{10}$$

and $\alpha$ is the aspect ratio of plate ($a/b$), $f_y$ is the applied stress in the $y$-direction, $f_{ux}$ is the ultimate strength of a plate under compressive normal stress in the $x$-direction acting alone, and $f_{ux}$ is the ultimate strength of a plate under compressive normal stress in the $y$-direction acting alone. The ultimate stresses are defined according to Eq. 1.

## Strength Model for Plates under Biaxial Compression and Edge Shear

This ultimate axial strength $F_{crx}$, as described by Mansour (1986), is given by

$$\left(\frac{F_{crx}}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 + \left(\frac{f_t}{f_{ut}}\right)^2 = 1 \tag{11}$$

where $f_y$ is the applied stress in the $y$-direction, $f_t$ is the applied shear stress, $f_{ux}$ is the ultimate strength of a plate under compressive normal stress in the $x$-direction acting alone, $f_{uy}$ is the ultimate strength of a plate under compressive normal stress in the $y$-direction acting alone, and $f_{ut}$ is the ultimate shear stress when the plate is subjected to pure edge shear. These ultimate stresses are defined according to the strength models given by Eqs. 1 and 2.

## Other Load Combinations

The lateral pressure in combination with the other cases of loading presented in the previous sections can lead to a number of loading conditions that can have an effect on the overall strength of plates. These combinations are not covered in this paper. Additional information is provided by Mansour (1976), Becker et al (1970), and Faulkner (1979).

## Example Partial Safety Factors

The limit state for this case is given by

$$g = f_u - f_s - k_w(f_w + k_D f_D) \tag{12}$$

where $f_u$ = ultimate uniaxial strength as defined by Eq. 1, $f_s$ = stress due to stillwater bending moment, $f_w$ = stress due to waves bending moment, $f_D$ = stress due to dynamic bending moment, $k_w$ = load combination factor equals 1.0, and $k_D$ = load combination factor equals 0.7. The partial safety factors for the above limit state equation (Eq. 1) were developed using a target reliability index $\beta$ of 3.0. The first-order reliability method requires the probabilistic characteristics of $f_u$, $f_s$, $f_w$ and $f_D$. The partial safety factors for a target reliability level of 3.0 are summarized in Tables 4 and 5. The ratios of means for strength/wave ranges are summarized in Table 6. Calibration on the strength factor $\phi_u$ for a given set of prescribed recommended load factors (such as $\gamma_s$ = 1.05, $\gamma_w$ = 1.2, and $\gamma_D$ = 1.05) are provided in Table 7. Recommended mean and nominal partial safety factors for both the strength and load effects are given in Tables 8 and 9. The following LRFD format can be used:

$$\phi_u f_u \leq \gamma_s f_s + k_w(\gamma_w f_w + k_D \gamma_D f_D) \tag{13}$$

Table 4. Partial Safety factors ($b$ = 3.0)

| | $f_u$ | $g_s$ | $g_w$ | $g_D$ |
|---|---|---|---|---|
| Minimum | 0.893886 | 1.034425 | 1.554748 | 1.039628 |
| Mean | 0.93574 | 1.051914 | 1.616088 | 1.061957 |
| Maximum | 0.9740 | 1.069720 | 1.667869 | 1.08549 |

Table 5. Strength Mean Value ($b$ = 3.0)

| | Minimum | Mean | Maximum |
|---|---|---|---|
| $m_{fu}$ | 2.11200 | 2.30652 | 2.51402 |

Table 6. Strength Reduction Factors for $g_s$ = 1.05, $g_w$ = 1.2, and $g_D$ = 1.05 with $b$ = 3.0

| | Minimum | Mean | Maximum |
|---|---|---|---|
| $m_{fu}$ | 0.72524 | 0.75244 | 0.78058 |

Table 7. Bias Factors

| $f_u$ | $g_s$ | $g_w$ | $g_D$ |
|---|---|---|---|
| 1.16 | 0.7 | 1.0 | 1.0 |

Table 8. Recommended Mean Factors

| $f_u$ | $g_s$ | $g_w$ | $g_D$ |
|---|---|---|---|
| 0.75 | 1.05 | 1.2 | 1.05 |

Table 9. Recommended Nominal Factors

| $f_u$ | $g_s$ | $g_w$ | $g_D$ |
|---|---|---|---|
| 0.87 | 0.75 | 1.2 | 1.05 |

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