Reliability-based Design of Unstiffened Panels for Ship Structures

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Abstract

The main objective of structural design is to insure safety, function, and performance of an engineering system for target reliability levels and for specified time period. As this must be accomplished under conditions of uncertainty, probabilistic analyses are necessary in the development of such probability-based design of unstiffened panels for ship structures. The load and resistance factor design (LRFD) format was developed in this paper for unstiffened panels. Partial safety factors were determined to account for the uncertainties in strength and load effects. In developing these factors, Monte Carlo simulation was utilized to assess the probabilistic characteristics of strength models by generating basic random variables that define the strength and substituting them in these models; and the First-Order Reliability Method (FORM) was used to determine the partial safety factors based on prescribed probabilistic characteristics of load effects. Also, strength factors were computed for a set of load factors to meet a target reliability level.

1: Introduction

In recent years, structural design has been moving toward a more rational and probability-based design procedure referred to as limit states design. Such a design procedure takes into account more information than deterministic methods in the design of structural components. This information includes uncertainties in the strength of various structural elements, in loads, and modeling errors in analysis procedures. Probability-based design formats are more flexible and rational than working stress formats because they provide consistent levels of safety over various types of structures. In probability-based limit states design, probabilistic methods are used to guide the selection of strength (resistance) factors and load factors which account for the variabilities in the individual resistances and loads and give the desired overall level of safety. The load and resistance factors (or called partial safety factors) are different for each type of load and resistance. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with strength, the lower the corresponding strength factor.

Designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods without explicitly performing probabilistic analysis. For this reason, design criteria should be as simple as possible. Moreover, they should be developed in a form that is familiar to the users or designers and should produce desired levels of uniformity in safety among different types of structures without departing drastically from existing current practice. There is no unique format for a design criterion. A criterion can be developed on a probability bases in any format. In general, the basic approach to develop a reliability-based strength standard is first to determine the relative reliability of design based on current practice. This relative reliability can be expressed in terms of either a probability of failure or a safety index. The safety index for structural components normally varies between 2 and 6 [5]. By performing such reliability analyses for many structures, representative values of target safety index can be selected reflecting the average reliability of current designs. Based on these values and by using reliability analysis again, it is possible to select partial safety factors for the loads and the strength which can be used as a basis for developing the design requirements.

For the purpose of designing code provisions, the most common format is the use of load amplification factors and resistance reduction factors (partial safety factors), as represented by

$$\phi R \geq \sum_{i=1}^{n} \gamma_i L_i \tag{1}$$

where ϕ = the resistance R reduction factor; γ_i = the partial load amplification factor; and L_i = the load effect. In fact, the American Institute of Steel Construction

(AISC) and other industries in this area have implemented this format. Also, a recommendation for the use of this format is given by the National Institute of Standards and Technology [4].

The First-Order Reliability Method (FORM) is commonly used to estimate the partial safety factors ϕ and γ_i for a specified target safety index β. In this paper, this method was used to determine the partial safety factors for simply-supported plates under uniform uniaxial compression stress.

2: First-Order Reliability Method (FORM)

The First-Order Reliability Method is a convenient tool to assess the reliability of a structural system. It also provides a means for calculating the partial safety factors ϕ and γ_i that appear in Eq. 1 for a specified target reliability level β. The simplicity of the First-Order Reliability Method (FORM) stems from the fact that this method, beside the requirement that the distribution types must be known, requires only the first and second moments; namely the mean values and the standard deviations of the respective random variables. Knowledge of the joint probability density function (PDF) of the design basic variables is not needed as in the case of the direct integration method for calculating the safety index β. Even if the joint PDF of the basic random variables is known, the computation of β by the direct integration method can be a very difficult task.

In design practice, there are usually two types of limit states: the ultimate limit states and the serviceability limit state. Both types can be represented by the following performance function:

$$g(\boldsymbol{X}) = g(X_1, X_2, \ldots, X_n) \qquad (2)$$

in which $\boldsymbol{X}$ is a vector of basic random variables (X_1, X_2, ..., X_n) for the strengths and the loads. The performance function $g(\boldsymbol{X})$ is sometimes called the limit state function. It relates the random variables for the limit state of interest. The limit state is defined when $g(\boldsymbol{X}) = 0$, and therefore, failure occurs when $g(\boldsymbol{X}) < 0$.

As indicated earlier, the basic approach to develop a reliability-based strength standard is to determine the relative reliability of designs based on current practice. In order to do that, reliability assessment of existing structural components is needed to estimate a representative value of the safety index β. The First-Order-Reliability Method is very well suited to perform such a reliability assessment. The following are computational steps, as outlined by Ayyub and McCuen [2], for determining β using FORM method:

1. Assume a design points x_i^* and obtain $x_i^{'*}$ using the following equation:

$$x_i^{'*} = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}} \qquad (3)$$

where $x_i^{'*} = -\alpha_i^* \beta$, μ_{X_i} = mean value of the basic random variable, and σ_{X_i} = standard deviation of the basic random variable. The mean values of the basic random variables can be used as initial values for the design points. The notation x^* and $x^{'*}$ are used respectively for the design point in the regular coordinates and in the reduced coordinates.

2. Evaluate the equivalent normal distributions for the non-normal basic random variables at the design point using the following equations:

$$\mu_X^{\mathrm{N}} = x^* - \Phi^{-1}\left(\mathrm{F}_X(x^*)\right)\sigma_X^{\mathrm{N}} \qquad (4a)$$

and

$$\sigma_X^{\mathrm{N}} = \frac{\phi\left(\Phi^{-1}\left(\mathrm{F}_X(x^*)\right)\right)}{\mathrm{f}_X(x^*)} \qquad (4b)$$

where μ_X^{N} = mean of the equivalent normal distribution, σ_X^{N} = standard deviation of the equivalent normal distribution, $\mathrm{F}_X(x^*)$ = original cumulative distribution function (CDF) of X_i evaluated at the design point, $\mathrm{f}_X(x^*)$ = original probability density function (PDF) of X_i evaluated at the design point, $\Phi(\cdot)$ = CDF of the standard normal distribution, and $\phi(\cdot)$ = PDF of the standard normal distribution.

3. Compute the directional cosines (α_i^*, i = 1,2, ..., n) using the following equations:

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial x_i^{'}}\right)_*}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i^{'}}\right)_*^2}} \qquad \text{for } i = 1, 2, \ldots, n \qquad (5)$$

where

$$\left(\frac{\partial g}{\partial x_i^{'}}\right)_* = \left(\frac{\partial g}{\partial x_i}\right)_* \sigma_{X_i}^{\mathrm{N}} \qquad (6)$$

4. With α_i^*, $\mu_{X_i}^{\mathrm{N}}$, and $\sigma_{X_i}^{\mathrm{N}}$ are now known , the following equation can be solved for the root β:

$$g\left[(\mu_{X_1}^{\mathrm{N}} - \alpha_{X_1}^* \sigma_{X_1}^{\mathrm{N}} \beta), \ldots, (\mu_{X_n}^{\mathrm{N}} - \alpha_{X_n}^* \sigma_{X_n}^{\mathrm{N}} \beta)\right] = 0 \qquad (7)$$

5. Using the β obtained from step 4, a new design point can be obtained from the following equation:

$$x_i^* = \mu_{X_i}^{\mathrm{N}} - \alpha_i^* \sigma_{X_i}^{\mathrm{N}} \beta \qquad (8)$$

6. Repeat steps 1 to 5 until a convergence of β is achieved.

The important relation between the probability of failure and the safety index is given by

$$P_f = 1 - \Phi(\beta) \tag{9}$$

2.1: Partial Safety Factors (PSF)

The first-order reliability method can be used to estimate partial safety factors such those found in the design format of Eq. 1. At the failure point (R^*, L_1^*, …, L_n^*), the limit state of Eq. 1 is given by

$$g = R^* - L_1^* - \ldots - L_n^* = 0 \tag{10}$$

or, in a general form

$$g(\boldsymbol{X}) = g(x_1^*,\ x_2^*, \ldots,\ x_n^*\ ) = 0 \tag{11}$$

For a given target reliability index β, probability distributions and statistics (means and standard deviations) of the load effects, and coefficient of variation of the strength, the mean value of the resistance and the partial safety factors can be determined by the iterative solution of Section 2, namely Eqs. 3 through 8. The mean value of the resistance and the design point can be used to compute the required partial design safety factors as

$$\phi = \frac{R^*}{\mu_R} \tag{12}$$

$$\gamma_i = \frac{L_i^*}{\mu_{L_i}} \tag{13}$$

2.2: Determination of a Strength Factor for a Given Set of Load Factors

In developing design code provisions, it is sometimes necessary to follow the current design practice to insure consistent levels of safety over various types of structures. Calibrations of existing design codes is needed to make the new design formats as simple as possible and to put them in a form that is familiar to the users or designers. Moreover, the partial safety factors for the new codes should provide consistent levels of safety. For a given safety index β and probability characteristics for the resistance and the load effects, the partial safety factors determined by the FORM approach might be different for different failure modes for the same structural component. For this reason, calibration of the calculated partial safety factors (PSF's) is important in order to maintain the same values for all loads at different failure modes. Normally, the calibration is performed on the strength factor ϕ for a given set of load factors. This can be accomplished by the following algorithm:

1. For a given value of the safety index β, probability distributions and statistics of the load variables, and the coefficient of variation for the strength, compute the mean of the strength R using the first-order reliability method as outlined in the Section 2.1.
2. With the mean value for R computed in step 1, the partial safety factor ϕ can be revised as follows:

$$\phi = \frac{\sum_{i=1}^{n} \gamma_i \mu_{L_i}}{\mu_R} \tag{14}$$

where μ_{L_i} and μ_R are the mean values of the loads and strength variables, respectively; and γ_i, i = 1, 2, ..., n, are the given set of load factors.

3: Example: Unstiffened Panel Under Uniaxial Compression

Plates are important components in ship structures, and therefore they should be designed for a set of failure modes such as yielding, buckling, and fatigue of critical connecting components. This example consider only a simply-supported plate with uniaxial compressive stress of a size a and b. The limit state for this case is given by

$$g = F_u - f_S - f_W \tag{15}$$

where F_u = the strength of the plate (stress), f_S = external stress due to stillwater bending, and f_W = external stress due to wave bending. The strength F_u is given by one of the following two cases:

1. For $a/b \geq 1.0$

$$\frac{F_u}{f_{yp}} = \begin{cases} \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & \text{if } 1.0 \leq B < 3.5 \\ 1.0 & \text{if } B < 1.0 \end{cases} \tag{16}$$

2. For $a/b < 1.0$

$$\frac{F_u}{f_{yp}} = \alpha C_u + 0.08(1-\alpha)\left(1 + \frac{1}{B^2}\right)^2 \leq 1.0 \tag{17}$$

where f_{yp} = yield strength (stress) of plate, a = length or span of plate, b = distance between longitudinal stiffeners, and in which $B = \frac{b}{t}\sqrt{\frac{f_{yp}}{E}}$, $\alpha = \frac{a}{b}$, t =

thickness of the plate, E = the modulus of elasticity, ν = Poisson's ratio, and

$$C_u = \begin{cases} \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & if\ B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & if\ 1.0 \leq B < 3.5 \\ 1.0 & if\ B < 1.0 \end{cases} \qquad (18)$$

The probabilistic characteristics of the strength F_u was assessed based on the underlying basic random variables that define F_u. These variables are a, b, t, f_{yp}, and E. Monte Carlo simulation was utilized to assess the probabilistic characteristics of the strength, F_u by generating a, b, t, f_{yp} , and E, and then feeding the generated values in the strength equation to obtain F_u values. This process was repeated for ranges of selected key parameters as shown in Table 1a. Additional information and assumptions were needed for the probabilistic characteristics of the basic random variables. This information and assumptions are provided in Table 1b. Poisson's ratio ν was assumed to be deterministic and thus, a value of 0.3 was considered in this example.

Table 1a. Ranges of Key Parameters

Mean	Range
a/b	0.4, 0.6, 0.8, 2, 3, and 4
b/t	50, 100, and 150
t (inch)	0.25, 0.375, and 0.5

Table 1b. Probabilistic Characteristics of Basic Random Variables

Variable	Nominal Value	Statistical Information			Bias or Error Information		
		Mean	COV	Dist. Type	Mean	Std. Dev.	Dist. Type
t (inch)					0	0.01563	Normal
b (inch)					0	0.125	Normal
a (inch)					0	0.125	Normal
f_{yp} (ksi)	34000	35700	0.07	Normal	1.05		
E (ksi)	29500	29500	0.05	Normal			

The above strength basic random variables were assumed to have normal probability distributions. The results of the simulation were expressed in the form of mean to nominal ratio of F_u, the coefficient of variation (COV) of F_u, and the distribution type of F_u. The number of simulation cycles was set at 100 which is adequate for all practical purposes based on the charts provided in Fig. 1 for a typical set of an estimated mean, coefficient of variation, and the coefficient of variation of the sample mean for F_u. The results of the simulation of F_u are summarized in Tables 2, and 3. The distribution type for F_u was determined to be either normal or lognormal. A lognormal probability distribution for R was used in this study. The strength F_u has a mean to nominal ratio of about 1.03. This ratio will be needed to revise the resulting strength reduction factor by multiplying it by 1.03. The maximum and minimum strength ratios were found to be 1.043, and 1.006, respectively. The maximum and minimum coefficient of variation (COV) of strength were found to be 0.08, and 0.04, respectively.

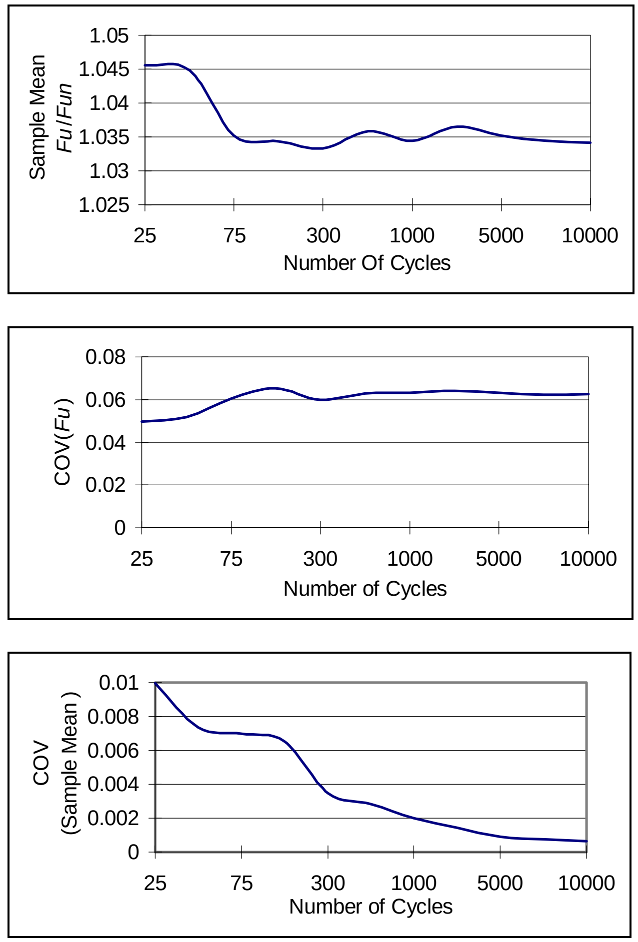

Figure 1. Effect of Simulation Cycles on Sample Mean for F_u/F_{un} , COV of F_u, and COV of Sample Mean for F_u/F_{un}

3.1. Calculation of Partial Safety Factors

The partial safety factors for the limit state equation (Eq. 15) were developed using a target reliability index β of 3.0. This equation provides a strength minus load effect expression of the limit state. The First-Order Reliability Method (FORM) as discussed in Section 2.1 requires the probabilistic characteristics of F_u, f_S, and f_W. The stillwater load effect f_S is due to stillwater bending that can be assumed to follow a normal distribution with a coefficient of variation of 0.2. The wave load effect f_W is due to waves that can be assumed to follow an extreme value distribution (Type I, largest) with a coefficient of variation of 0.1. The mean values of stillwater and waves are considered in the study in the form of a ratio of wave/stillwater loads that ranges from 1.5 to 1.7.

Table 2. Mean to Nominal Strength Ratio (F_u / F_{un}) using 100 Simulation Cycles

a/b	*t* (in)	*b/t* = 50	100	150
2	0.250	1.0329002	1.018011	1.0243729
	0.375	1.0384686	1.0232304	1.0267309
	0.500	1.0415835	1.0297619	1.0278817
3	0.250	1.0322676	1.0249275	1.0217597
	0.375	1.0401574	1.0250572	1.0098963
	0.500	1.0412124	1.0211235	1.0274101
4	0.250	1.0431215	1.0061456	1.0300229
	0.375	1.0359757	1.0296331	1.020356
	0.500	1.0317967	1.0351399	1.0212485
0.4	0.250	1.0316134	1.0367885	1.0379335
	0.375	1.0286892	1.0322983	1.0271185
	0.500	1.037029	1.0313444	1.031872
0.6	0.250	1.0292437	1.0245135	1.0282429
	0.375	1.0317401	1.0328774	1.0324076
	0.500	1.0404428	1.0317212	1.0346454
0.8	0.250	1.0232164	1.0128006	1.0191385
	0.375	1.0402862	1.0119037	1.0141044
	0.500	1.0397768	1.0348356	1.0209614

Table 3. Coefficient of Variation of Strength (F_u) using 100 Simulation Cycles

a/b	*t* (in)	*b/t* = 50	100	150
2	0.250	0.0584253	0.0790815	0.0694034
	0.375	0.0607941	0.0510484	0.0572355
	0.500	0.0527346	0.0475373	0.0553382
3	0.250	0.0576359	0.0793697	0.0693333
	0.375	0.0542866	0.0533326	0.0585843
	0.500	0.0489141	0.0546104	0.0511533
4	0.250	0.0668116	0.0763437	0.0707261
	0.375	0.0600205	0.0479042	0.0595471
	0.500	0.0556326	0.0506372	0.0549195
0.4	0.250	0.0705274	0.0744475	0.0706838
	0.375	0.0572604	0.0588016	0.053054
	0.500	0.0523423	0.053527	0.0561629
0.6	0.250	0.0574048	0.050443	0.0485009
	0.375	0.0552815	0.0557277	0.0617511
	0.500	0.054886	0.0576134	0.0467804
0.8	0.250	0.0621478	0.0701532	0.0717148
	0.375	0.0597223	0.0517489	0.058896
	0.500	0.0526932	0.0462986	0.0591769

The simulation results of F_u were used to develop the partial safety factors based on the limit state equation. The partial safety factors were computed for several selected cases that cover the assumed ranges of the parameters *a*, *b*, *t*, f_{yp} and *E*. The ratios of means for strength/stillwater load and the partial safety factors for a target reliability of 3.0 are summarized in Tables 4 and 5, respectively, and in Fig. 2. Based on these results, the following preliminary values for partial safety factors are recommended:

Strength reduction factor (ϕ) = 0.85(1.03) = 0.88
Stillwater load factor (γ_S) = 1.3
Wave load factor (γ_W) = 1.25

Table 4. Ratios of Means for Strength/Stillwater Load

$COV(F_u)$	Ratios of Means for Wave/Stillwater Load: 1.5	1.6	1.7
0.04	3.43035	3.5695	3.70977
0.08	3.6375	3.7817	3.9271

Table 5. Partial Safety Factors (for $COV(F_u)$ of 0.04 and 0.08, respectively)

Partial Safety Factors	Ratios of Means for Wave/Stillwater Load: 1.5	1.6	1.7
Strenght Reduction Factor (ϕ)	0.960338 0.863684	0.961079 0.86526	0.961747 0.86679
Stillwater Load Factor (γ_S)	1.301221 1.28566	1.283616 1.270806	1.267817 1.257081
Wave Load Factor (γ_w)	1.328696 1.237262	1.341832 1.250783	1.352955 1.262827

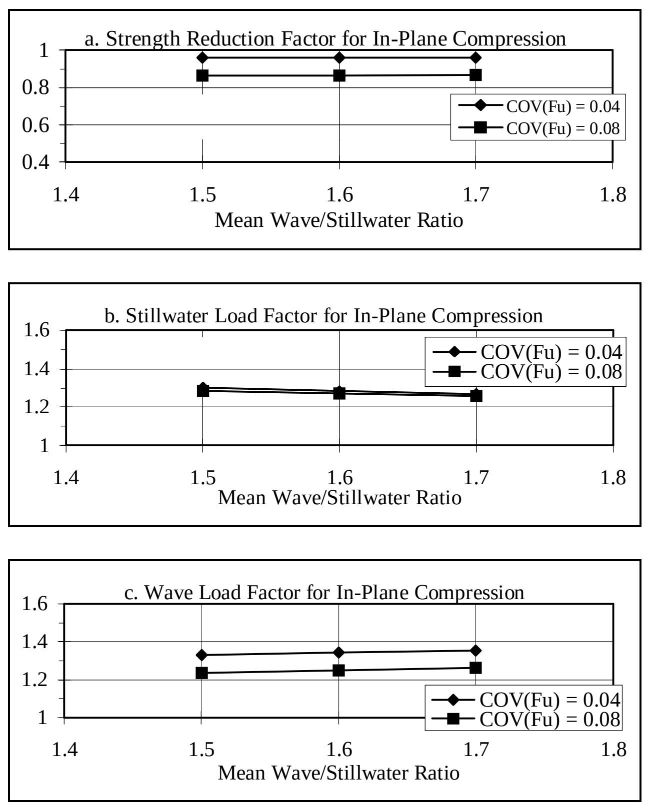

Figure 2. Partial Safety Factors for Plates Under Uniaxial Compression.

3.2: Calculation of Strength Factor For a Given Set of Load Factors

As indicated in Section 2.2 , for a given β and probabilistic characteristics for the strength and the load effects, the partial safety factors determined by the FORM approach might be different for different failure modes. For this reason calibration is often needed on the strength factor ϕ to maintain the same values for all load factors γ's. The following numerical example illustrates the procedure of Section 2.2 for revising the strength factor for a given set of load factors. For instance, given γ_S = 1.3, γ_W = 1.2, and the probabilistic characteristics of the random variables as shown Table 6, the corresponding strength factor ϕ was calculated for a target reliability level β = 3.0. Using FORM as outlined in Section 2.2, the mean of F_u was found to be 3.66. With the mean value known , Eq. 14 gives

$$\phi = \frac{\gamma_S \mu_S + \gamma_W \mu_W}{\mu_{F_u}}(1.03) = \frac{1.3(1) + 1.2(1.6)}{3.66}(1.03) = 0.91$$

Since the strength F_u has a mean to nominal ratio of 1.03, this ratio was needed to revise ϕ by multiplying it by 1.03.

Table 6. Probabilistic Characteristics of Random Variables

Random Variable	Mean	COV	Distribution Type
F_u	not provided	0.06	Lognormal
L_S	1	0.2	Normal
L_W	1.6	0.1	Type I (Largest)

4: Summary and Conclusions

The First-Order Reliability Method (FORM) can be used to assess the reliability of a structural systems as well as to develop and establish partial safety factors. In this study, the FORM method was used to develop partial safety factors for a simply-supported plate (unstiffend panel) under uniaxial compressive stress. The strength model for the plate F_u for this case was established. Then Monte Carlo simulation was utilized to assess the probabilistic characteristics of the strength F_u by generating the basic random variables that define the strength and then feeding the generated values in the strength model for the plate to obtain F_u values. The distribution type of F_u was determined to be lognormal. The maximum and minimum COV values of F_u were found to be 0.08 and 0.04, respectively. The prescribed probabilistic characteristics of the load effects and the simulation results of the strength were used to develop the partial safety factors based on a linear limit state. The partial safety factors were computed for several selected cases that cover the assumed ranges of key parameters that define the strength F_u. Based on these results and for a target reliability level β of 3.0, the following values for partial safety factors were selected:

Strength reduction factor ϕ = 0.88
Stillwater load factor γ_S = 1.30
Wave load factor γ_W = 1.25

The resulting partial safety factors can be used to design plates under uniaxial compressive stresses to meet a strength limit state given by the following design format:

$$\phi F_u \le \gamma_S f_S + \gamma_W f_W \quad (19a)$$

or

$$0.88 F_u \le 1.3 f_S + 1.25 f_W \quad (19b)$$

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