RELIABILITY-BASED DESIGN FOR FATIGUE OF MARINE STRUCTURES

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ABSTRACT

Marine and offshore structures are subjected to fatigue primarily due to the action of seawater waves and the sea environment in general. The load cycles in such an environment can be in the order of million cycles per year.

The objective of this paper is to develop design methods for fatigue of structural details for conventional displacement type surface monohull ships. The methods are based on structural reliability theory and can be either as direct reliability-based design or in a load and resistance factor design (LRFD) format. The resulting design methods are to be referred to as the LRFD fatigue rules for marine structures. They were developed according to the following requirements: (1) spectral analysis of wave loads, (2) building on conventional codes, (3) nominal strength and load values, and (4) achieving target reliability levels. The first-order reliability method (FORM) was used to demonstrate the development of partial safety factors for selected limit states.

1. INTRODUCTION

In recent years, a great deal of attention has been focused on general fatigue cracking of ship structural details because the phenomenon is so vital that structural engineers must consider fatigue strength in their designs, especially for those structural components that are exposed to cyclic loading. The term “fatigue” is commonly used in engineering to describe repeated-load phenomena and their effect on the strength of a structural member. The exact mechanism of a fatigue failure is complex and is not completely understood. Failure by fatigue is a progressive cracking and unless it is detected this cracking can lead to a catastrophic rupture. When a repeated load is large enough to cause a fatigue crack, the crack will start at the point of maximum stress. This maximum stress is usually due a stress concentration (stress raiser). After a fatigue crack is initiated at some microscopic or macroscopic level of stress concentration, the crack itself can act as an additional stress raiser causing crack propagation. The crack grows with each repetition of the load until the effective cross section is reduced to such an extent that the remaining portion will fail with the next application of the load. For a fatigue crack to grow to such an extent to cause rupture, it usually takes thousands or even millions applications of the stress, depending on the magnitude of the load, type of the material used, and on other related factors. A detailed bibliography for fatigue of welds was developed by the University of Tennessee (1985). However, this bibliography does not cover work beyond 1985.

Fatigue must be considered in the design of all-structural and machine components that are subjected to repeated or fluctuating loads. During the useful life of a structural member, the number of loading cycles, which may expected, varies tremendously. For example, a beam supporting a crane may be loaded as many as 2,000,000 times in 25 years to failure, while an automobile crankshaft might be loaded 5,000,000 times for rupture to occur, if the automobile is driven 200,000 miles (Beer and Johnston, 1981). The number of loading cycles required to cause failure of a structural component through cyclic successive loading and reverse loading may be determined experimentally for any given maximum stress level. One common test used to evaluate the fatigue properties of a material is a rotating-beam test (Byars and Snyder, 1975). In this test, the number of completely reversed cycles of bending stress required to cause failure is measured at different stress levels. In one complete cycle, the stress goes from maximum tensile stress, to zero, to maximum compressive stress of the same magnitude as the maximum tensile stress, and then back to the original maximum stress passing the zero stress level. If a series of tests are conducted in this case, using different maximum stress ranges, the resulting data can be plotted as an *S*-*N* curve. For each test, the maximum stress range *S* is plotted against the number of cycles *N*. These test data are usually plotted on semi-log paper,

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and the resulting plot is referred to as an *S-N* curve. Figure 1 shows typical curves for various materials. It is to be noted that from these curves, as the magnitude of the maximum stress range decreases, the number of cycles required causing rupture increases. Also these curves tend to be approximately horizontal lines as a lower limit. When the stress level for a specimen reaches this limit, the specimen does not fail and it is said to have reached the endurance limit (fatigue limit). The endurance limit is then defined as the stress for which failure does not take place (Beer and Johnston, 1981) even for an indefinitely large number of loading cycles. The endurance limit for most engineering material is less than the yield strength. For a low carbon structural steel, the endurance limit is about half of the ultimate strength of the steel.

Fatigue properties for materials are usually determined at high temperatures and also in various corrosive environments. Temperature and environment can play a drastic role in influencing the fatigue properties. For example, in applications in or near seawater, or in other applications where high level of corrosion is expected, a reduction up to 50% in the endurance limit may be anticipated. Also, since fatigue failure may be initiated at any crack or imperfection, the service condition of a specimen has a vital effect on the value of the endurance limit obtained in the test.

The inherent nature of fatigue tests gives rise to a great deal of scatter in the data. For example, if several specimens that have carefully machined and polished, are tested at the same stress level, it certainly not unusual to have a variation of 10 to 20 percent in their fatigue life measured in terms of the number of loading cycles at which the specimen ruptures (Byars and Snyder, 1975). It therefore requires a few tests to correctly identify an S-N curve for a material.

Fatigue cracking of structural details in ship and offshore steel structures due cyclic loading has gained considerable attention in the past few years. Numerous research studies have been conducted in this field on both the theoretical and practical aspects. Consequently, a great deal of papers has been published resulting in various topics relating to fatigue assessment and prediction. In these papers, the macroscopic behavior of materials as well as models for its description is investigated. Due to the extreme complexity in modeling the process of material cracking at the microscopic level, solutions from the microscopic aspect are rarely available or not practically feasible. This is mainly due to the complexity of the damaging process under cyclic loading and the scatter of material properties. Ship and offshore structures are subjected to fatigue primarily due to the action of seawater waves (Byers et al, 1997) and the sea environment in general. The load cycles in such an environment can be in the order of million cycles per year. Fatigue failures in ship and offshore structures can take place at sites of high stress concentration that can be classified into two major categories: (1) baseplate and (2) weldments. The former includes locations of high stress concentration such as openings, sharp re-entry corners, and plate edges. In general, the mechanisms behind these failures are described by the general approaches to fatigue life prediction as discussed in this paper. There are two major approaches for evaluating fatigue life prediction: (1) the *S-N* curve approach and (2) the fracture mechanic (FM) approach. The *S-N* approach is based on experimental measurement of fatigue life in terms of cycles to failure for different loading levels as discussed previously. On the other hand, the fracture mechanic (FM) approach is based on the existence of an initial crack in a stress-free structure. Only the *S-N* approach is emphasized in this paper.

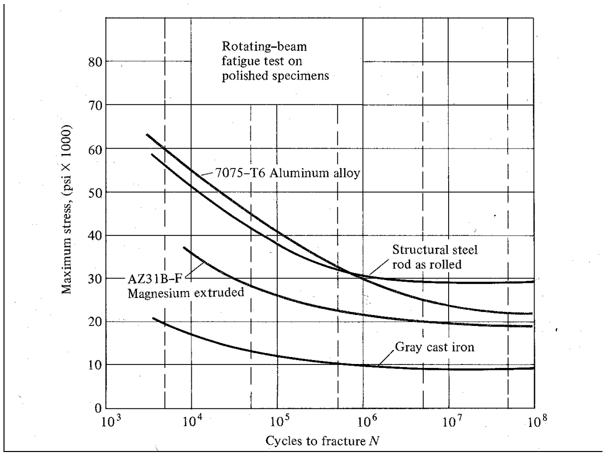

Figure 1. *S-N* Curves for Various Materials (Byars and Snyder, 1975)

2. FATIGUE ANALYSES AND DESIGN APPROACHES

There are two major technical approaches for fatigue analysis and design of welded joints: (1) the fracture mechanics approach and (2) the characteristic *S-N* approach. Both of these approaches are discussed briefly in the subsequent sections with the emphases on the former approach.

2.1 The Fracture Mechanics Approach

The fracture mechanics approach is based on crack growth data. For welded joints it is assumed that the initiation phase is negligible and that life can be predicted using the fracture mechanics method. The fracture mechanics approach is more detailed and it involves examining crack growth and determining the number of load cycles that are needed for small initial defects to grow into cracks large enough to cause fracture. The growth rate is proportional to the stress range. It is expressed in terms of a stress intensity factor *K*, which accounts for the magnitude of the stress, current crack size, and weld and joint details. The basic equation that governs crack growth is given by

$$\frac{da}{dN} = C\Delta K^{m} \qquad (1)$$

where a = crack size, N = number of fatigue cycles, ΔK = range of stress intensity factor, and C and m are crack propagation parameters that come from fracture mechanics. The range of the stress intensity factor is given by Broek (1986) as

$$\Delta K = SY(a)\sqrt{pa} \qquad (2)$$

in which $Y(a)$ is a function of crack geometry. When the crack size a reaches some critical crack size a_{cr}, failure is assumed to have occurred. Although most laboratory testing is typically performed with constant amplitude stress ranges, Eq. 1 is always applied to variable stress range models that ignore sequence effects (Byers et al 1997). Rearranging the variables in Eq. 1, the number of cycles can be computed from

$$N = \frac{1}{CS^{m}} \int_{a_0}^{a} \frac{da}{Y^{m}} \qquad (3)$$

Eqs. 1 and 3 involve a variety of sources of uncertainty (Harris 1995). The crack propagation parameter C in both equations is treated as a random variable (Madsen 1983). However, in more sophisticated models, Eq. 1 is treated as a stochastic differential equation and C is allowed to vary during the crack growth process (Ortiz 1985 and Byers et al 1997). Lin and Yang (1983) treat the crack growth as Markov process, while Ditlevsen (1986) treats it as a first-passage problem.

2.2 The Characteristic *S-N* Approach

The Characteristic *S-N* approach is based on fatigue test data (*S-N* curves) as described in Section 1 and on the assumption that fatigue damage accumulation is a linear phenomenon (Miner's rule). According to Miner's rule, the total fatigue life under a variety of stress ranges is the weighted sum of the individual lives at constant stress *S* as given by the *S-N* curves, with each being weighted according to fractional exposure to that level of stress range (Hughes 1988). Upon crack initiation, cracks propagate based on the fracture mechanics concept as shown in Figure 2.

The fatigue behavior of different types of structural details is generally evaluated in constant-cycle fatigue tests and the results are presented in terms of the nominal applied stresses and the number of cycles of loading that produce failure. The resulting *S-N* curves are usually presented as straight lines on a log-log paper as shown in Figure 3. The basic equation that represents the *S-N* curve is given by

$$N = \frac{A}{S^{m}} \qquad (4)$$

where N = number of cycles to fatigue initiation (failure), A = the intercept of the *S-N* curve at S equals to one, S =constant amplitude stress range at N, and m = slope of the *S-N* curve. Eq. 4 can also be expressed as

$$\log N = \log A - m \log S \qquad (5)$$

where log is to the base 10. The fatigue strength can be computed over a range of lives covered by the straight line if the slope of the line and one point on the line are known. However, only one type of stress cycle and one detail are represented on an individual *S-N* curve (Munse et al 1983). In general, a least-squares analysis of log *N* given *S* is used to produce the *S-N* curve.

Uncertainty in fatigue strength is evidenced by the large scatter in fatigue *S-N* data. The scatter of the data about the mean fatigue line is not the only uncertainty involved in the *S-N* analysis (White and Ayyub 1987).

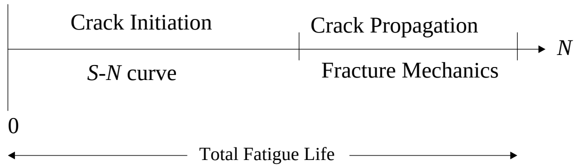

Figure 2. Comparison Between the Characteristic *S-N* Curve and Fracture Mechanic Approach

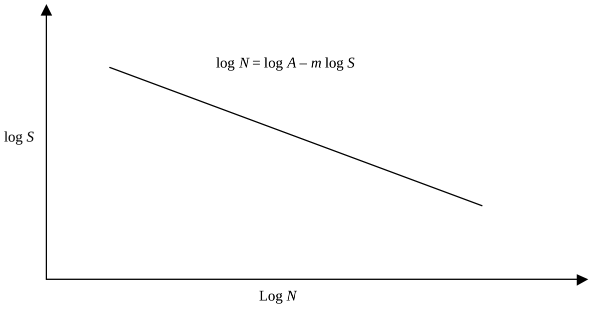

Figure 3. *S-N* Relationship for Fatigue

For this reason, a measure of the total uncertainty in the form of a coefficient of variation (*COV*) in fatigue life is usually developed to include the uncertainty in data, errors in fatigue model, and any uncertainty in the individual stresses and stress effects. According to Ang and Munse (1975), the total coefficient of variation (*COV*) in terms of fatigue life can be given by

$$d_N = \sqrt{d_f^2 + d_A^2 + m^2 d_S^2} \tag{6}$$

where

d_N = total *COV* in terms of cycles to failure

d_f = variation (*COV*) due to errors in fatigue model and utilization of Miner's rule

d_A = uncertainty (*COV*) in mean intercept of the regression line including effects of fabrication, workmanship, and uncertainty in slope

d_S = uncertainty (*COV*) in equivalent stress range including effects of error in stress analysis

m = slope of mean *S*-*N* regression line

Values for d_N and m are obtainable from sets of *S*-*N* curves for the type of detail under consideration. Munse et al (1983) has managed to tabulate such values. Typical values for d_S, d_A, and d_f are 0.1, 0.4, and 0.15, respectively.

Other researchers such as Wirsching (1984) and Ayyub et al (1998) have tackled the same source of uncertainty in a slightly different way. For example, Wirsching (1984) introduces the random variable *B* to represent a bias factor and the random variable Δ to denote fatigue damage at failure. The bias factor *B* is assumed to account for the stress modeling error, while the fatigue damage at failure Δ is to quantify the modeling error associated with Miner's rule, which is presented in the next section. He also suggests that uncertainty in fatigue strength can be accounted for by considering the intercept of the *S*-*N* curve (*A*) as a random variable with the slope of the same *S*-*N* curve (*m*) taking as a constant. Uncertainty in *B*, as described by Wirsching (1984), is assumed to stem from five sources: (1) fabrication and assembly operations, (2) seastate description, (3) wave load prediction, (4) nominal member loads, and (5) estimation of hot spot stress concentration factor.

Ayyub et al (1998), in assessing the fatigue reliability of miter gates components for the U.S. Army Corps of Engineers (USACE), chose to look at the same sources of uncertainty in a slightly different way. He introduces the random variables e' and k_s to account respectively for the uncertainty in the *S*-*N* relationship and fatigue stresses. He also uses a factor Δ similar to that of Wirsching (1984) to account for the uncertainty due to the utilization of linear cumulative damage of Miner's rule. A full coverage of fatigue parameter uncertainties is presented in Section 7.5 (Basic Random Variables) of Assakkaf (1998).

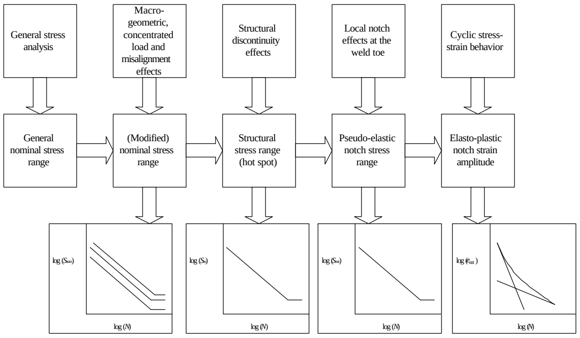

Figure 4. *S*-*N* Approaches for Fatigue Strength Assessment (Niemi 1995)

The choice of appropriate stress history is an important factor in reliability-based design and analysis for fatigue. The question is not really how to determine the stress history, rather, what constitutes an appropriate stress history. According to Moan and Berge (1997) and based on the terminology adapted by the International Institute of Welding (IIW) in 1996, the following four different approaches are classified for stress determination for fatigue design and analysis: (1) the nominal stress approach, (2) the hot spot stress approach, (3) the notch stress approach, and (4) the notch strain approach. Figure 4 shows a schematic of these approaches. Except for the nominal stress approach, the rest are commonly called local stress approach. Probably the most common approaches for determining fatigue stresses in marine industry are the nominal stress and the hot spot approaches. These methods are discussed in the next section. For more detailed description of the notch stress and notch strain approaches, Section 2 (Fatigue and Fracture) of Moan and Berge (1997) provides such a description.

2.2.1 Nominal Stress Versus Hot Spot stress

The nominal stress approach is the simplest one among the others approaches. In this approach, the stress is represented by an average loading of the whole structural detail under study. The nominal stress is the maximum stress due to sectional forces or moments or the combination of the two at the location of possible cracking site in the detail. In this approach, neither the weld toe nor the properties of the material constitutive relations are taken into consideration (Moan and Berge 1997). The *S*-*N* curve resulting from this analysis is unique to the structural detail for which it is established. It is possible to use one such curve to be applied for a range of similar details if there is insignificant variation in their geometry. Most design codes nowadays divide various structural details into different classes and provide standard *S-N* curve for each class. For example, the British Standards (BS 1980) and Norwegian Standards (NS 1984) have nine classifications as shown in Table 1. However, for a more rigorous analysis, each detail must be identified with a specific curve in the menu.

The hot spot stress is defined as the fatigue stress at the toe of the weld, where the stress concentration is the highest and where fatigue cracking is likely to initiate (Mansour et al 1995). The hot spot stress is comprised of membrane and bending shell stress parts, which are linearly distributed over the plate thickness. The hot spot stress analysis takes into account two factors (Moan and Berge 1997): (1) the local increase in membrane stress due to complex structural geometry of welded joint and (2) the information of shell bending stress due to eccentricity. The exact weld toe geometry and nonlinear stress peak due to local notch at the weld toe are disregarded. The hot spot stress is an average nominal stress of the stresses near the weld. The advantage of the hot spot stress method is that only one universal *S-N* curve is required to define fatigue strength for all welds, if such curve exists. The disadvantage is that this approach may require finite element analysis to determine the hot spot stress

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3. RELIABILITY-BASED DESIGN METHODS

3.1 Direct-Reliability-Based Design

A direct reliability-based design requires performing spectral analysis for the loads. The spectral analysis shall be used to develop lifetime fatigue loads spectra by considering the operational conditions and the characteristics of a ship in the sea. The operational conditions are divided into different operation modes according to the combinations of ship speeds, ship headings, and wave heights. The ship characteristics include the length between perpendicular (*LBP*), beam (*B*), and the bow form as shown in Figure 4-1. With the proper identification of the hull girder section modulus (*Z*), the bending moment histograms (moment range versus number of cycles) shall be converted to mean stress range spectra to compute the equivalent stress range $\bar{S}_e$ according to the following equation:

$$\bar{S}_e = \sqrt[m]{\sum_{i=1}^{n_b} f_i S_i^m} \tag{7}$$

where

$\bar{S}_e$ = Miner's mean equivalent stress range

S_i = stress in the i^{th} bolck

f_i = fraction of cycles in the i^{th} stress block

m = slope of *S-N* curve

n_b = number of stress blocks in a stress (loading) histogram

Table 1. Description of Joint Details (BS 1980, NS bbb 1984, and Mansour et al 1995)

Class	Description
B	Plain steel in the as-rolled condition, or with cleaned surfaces, but with no flame cut edges or re-entrant corners. Full penetration butt welds, parallel to the direction of applied stress, with the weld overfill dressed flush with the surface and finish-machined in the direction of stress, and with the weld proved free from significant defects by non-destructive examination
C	Butt or fillet welds, parallel to the direction of applied stress, with the welds made by an automatic submerged or open arc process and with no stop-start positions within the length. Transverse butt welds with the weld overfill dressed flush with the surface and with the weld proved free from significant defects by non-destructive examination.
D	Transverse butt welds with the welds made in the shop either manually or by an automatic process other than submerged arc, provided all runs are made in the flat position.
E	Transverse butt welds that are not class C or D.
F	Load-carrying fillet welds with the joint made with full penetration welds with any undercutting at the corners of the member dressed out by local grinding.
F2	Load-carrying fillet welds with the joint made with partial penetration or fillet welds with any undercutting at the corners of the member dressed out by local grinding.
G	Parent metal at the ends of load-carrying fillet welds which are essentially parallel to the direction of applied stress.
W	Weld metal in load-carrying joints made with fillet or partial penetration welds, with the welds either transverse or parallel to the direction of applied stress (based on nominal shear stress on weld throat area).

The direct reliability-based design for fatigue requires the probabilistic characteristics of the random variables in the performance function equation. It also requires specifying target reliability index b_0 to be compared with a computed b resulting from reliability assessment methods such as first-order reliability method (FORM). The general form for reliability checking used in the rules is given by

$$b \ge b_0 \tag{8}$$

The performance function for fatigue is given by either one of the following two expressions:

$$g_1 = \frac{\Delta A}{k_s^m \bar{S}_e^m} - N_t \tag{9}$$

or

$$\begin{aligned} g_2 &= \log(A) + \log(\Delta) - m\log(\bar{S}_e) \\ &\quad - m\log(k_s) - \log(N_t) \end{aligned} \tag{10}$$

where

Δ = fatigue damage ratio

A = intercept of the *S*-*N* curve

m = slope of the *S*-*N* curve

$\bar{S}_e$ = Miner's mean equivalent stress

k_s = fatigue stress uncertainty factor

N_t = number of loading cycles expected during the life of a structural detail.

The N_t variable is a deterministic quantity that is commonly assigned a value of 10^8 cycles.

3.2 The Load and Resistance Factor Design (LRFD)

An alternative approach for reliability-based design is the use of partial safety factors (PSF's) using a load and resistance factor (LRFD) design format. The PSF's are for both strength and load variables. They are commonly termed strength reduction and load amplification factors. The structural detail or joint element of a ship is to meet one of the following performance functions:

$$g_{S_e} S_e \le \left[\frac{(f_\Delta \Delta)(f_A A)}{g_{k_s}^m k_s^m N_t} \right]^{\frac{1}{m}} \tag{11}$$

$$\begin{aligned} \log(g_{S_e} S_e) &= \frac{\log(f_A A) + \log(f_\Delta \Delta) - m\log(g_{k_s} k_s)}{m} \\ &\quad - \frac{\log(N_t)}{m} \end{aligned} \tag{12}$$

where

$$S_e = \sqrt[m]{\sum_{i=1}^{n_b} f_i S_i^m} \tag{13}$$

S_e = Miner's equivalent stress range

f_Δ = reduction safety factor corresponds to fatigue damage ratio Δ

f_A = reduction safety factor corresponds to the intercept of the *S*-*N* curve

g_{k_s} = amplification safety factor for fatigue stress uncertainty

g_{S_e} = amplification safety factor for Miner's rule equivalent stress range

It is to be noted that the nominal S_e is the best estimate resulting from spectral analysis. The nominal (i.e., design) values of the fatigue variables shall satisfy these formats in order to achieve specified target reliability levels. The probabilistic characteristics and nominal values for the strength and load components were determined based on statistical analysis, recommended values from other specifications, and by professional judgment. These factors are determined using structural reliability methods based on the probabilistic characteristics of the basic random variables for fatigue including statistical and modeling (or prediction) uncertainties. The factors are determined to meet target reliability levels that were selected based on assessing previous designs. This process of developing reliability-based LRFD rules based on implicit reliability levels in current practices is called code calibration.

4. LRFD APPROACH FOR FATIGUE OF MARINE STRUCTURES

As mentioned earlier, the load and resistance factor (LRFD) approach consists of the requirement that a factored (reduced) strength of a structural component is larger than a linear combination of factored (magnified) load effects. In this approach, load effects are increased, and strength is reduced, by multiplying the corresponding characteristic (nominal) values with factors, which are called strength (resistance) and load factors, respectively, or partial safety factors (PSF's). The characteristic value of some quantity is the value that is used in current design practice, and it is usually

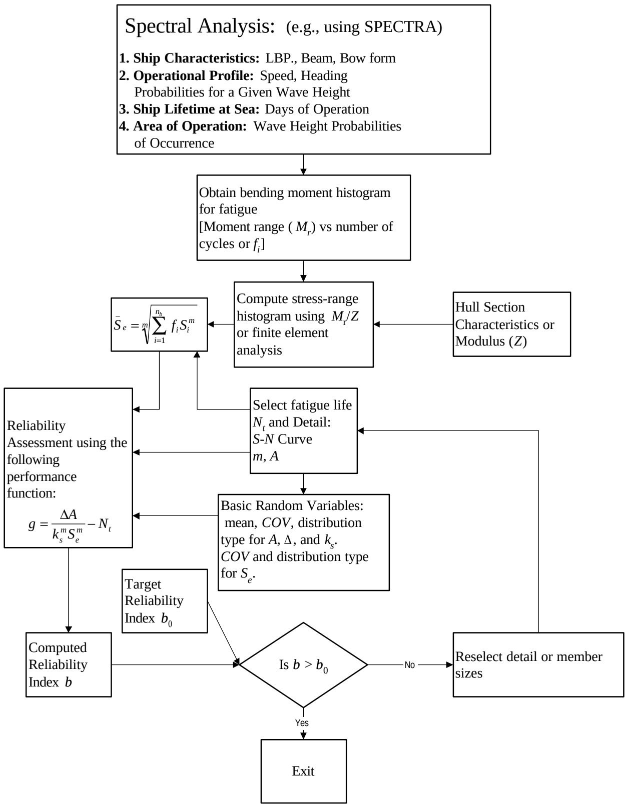

Figure 5. Direct Reliability-based Design and Analysis for Fatigue (Assakkaf 1998)

equal to a certain percentile of the probability distribution of that quantity. The load and strength factors are different for each type of load and strength. The higher the uncertainty associated with a load, the higher the corresponding load factor. These factors are determined probabilistically so that they correspond to a prescribed level of safety. Designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods without explicitly performing probabilistic analysis.

Calculation of partial safety factors (PSF's) for fatigue variables in the limit state function can be accomplished using the first-order reliability methods (FORM). The partial safety factors are defined as the ratio of the value of a variable in a limit state at its most probable failure point (MPFP).

The generalized FORM approach was selected to calculate the partial safety factors due to the existence of non-normal basic random variables in the corresponding limit states for fatigue. Reliability-based design formats for fatigue can be expressed in the following form:

$$g(\Delta, A, k_s, S_e, N_t) = \frac{\Delta A}{k_s^m S_e^m} - N_t \qquad (14)$$

where

$$\bar{S}_e = \sqrt[m]{\sum_{i=1}^{n_b} f_i S_i^m} \qquad (15)$$

Δ = fatigue damage ratio, A = intercept of the S-N curve, m = slope of the S-N curve, $\bar{S}_e$ = Miner's mean equivalent stress, k_s = fatigue stress uncertainty factor, N_t = number of loading cycles expected during the life of a structural detail, n_b = number of stress blocks in a stress (loading) histogram, f_i = fraction of cycles in the i^{th} stress block, and S_i = stress in the i^{th} block. By equating the reliability index, b, with the target reliability index, b_o, the partial safety factors are computed. The strength variables in the limit-state at the design point (MPFP) is given by

$$S_e^* = \left[\frac{\Delta^* A^*}{k_s^{*m} N_t}\right]^{\frac{1}{m}} \qquad (16)$$

By treating S_e, Δ, A, and k_s as random variables, the partial safety factors are computed as follows:

$$f_{S_e} = \frac{S_e^*}{S_{e_n}} \qquad (17)$$

$$g_\Delta = \frac{\Delta^*}{\Delta_n} \qquad (18)$$

$$g_A = \frac{A^*}{A_n} \qquad (19)$$

$$g_{k_s} = \frac{k_s^*}{k_{s_n}} \qquad (20)$$

where the subscript n means nominal value. The variable N_t was treated as a deterministic quantity. However, it can be treated as a random variable, and its partial safety factor can be evaluated accordingly. The uncertainty in A can be attributed to the regression standard error.

4.1 Example 1: Partial Safety Factors for Fatigue

In this example, partial safety factors calculations for two classes of structural detail are illustrated. The probabilistic characteristics of the random variables pertaining to these details are shown in Tables 2 and 3. The first-order reliability method (FORM) was used to develop the partial safety factors. The following performance function is used as defined by Eq. 14:

$$g = \frac{\Delta A}{k_s^m S_e^m} - N_t \tag{21}$$

where A, S_e, Δ, and k_s are random variables, m =slope of S-N curve (deterministic), and $N_t = 10^5$. The partial safety factors are defined as the ratio of the value of a variable in the performance function at its most probable failure point (MPFP) to the nominal value. Summary of the partial safety factors for details B and W of the British standards are shown in Tables 4 and 5, respectively.

Table 2. Statistics of Random Variables (Category B of the British Standards(BS 5400, 1980)

Random Variable	Mean	*COV*	Distribution Type
S_e	27.54 ksi	0.1	Lognormal
Δ	1.0	0.48	Lognormal
A	4.47 E11	0.44	Lognormal
k_s	1.0	0.1	Normal
m	4.0	n/a	n/a
N_t	10^5	n/a	n/a

Table 3. Statistics of Random Variables (Category W of the British Standards, BS 5400)

Random Variable	Mean	*COV*	Distribution Type
S_e	8.21 ksi	0.1	Lognormal
Δ	1.0	0.48	Lognormal
A	2.88 E08	0.44	Lognormal
k_s	1.0	0.1	Normal
m	3.0	n/a	n/a
N_t	10^5	n/a	n/a

Table 4. Partial Safety Factors for Category B of the British Standards (BS 5400)

b	f_D	f_A	g_{ks}	g_S
2.0	0.55	0.60	1.09	1.10
2.5	0.48	0.53	1.11	1.12
3.0	0.42	0.48	1.13	1.15
3.5	0.37	0.43	1.15	1.18
4.0	0.32	0.38	1.17	1.21

Table 5. Partial Safety Factors for Category W of the British Standards (BS 5400, 1980)

b	f_D	f_A	g_{ks}	g_S
2.0	0.52	0.57	1.07	1.08
2.5	0.45	0.50	1.09	1.10
3.0	0.39	0.45	1.11	1.12
3.5	0.34	0.40	1.13	1.15
4.0	0.29	0.35	1.14	1.17

5. DIRECT RELIABILITY-BASED DESIGN AND ANALYSIS APPROACHES FOR FATIGUE OF MARINE STRUCTURES

As mentioned earlier, the direct reliability-based design requires performing spectral analysis for the loads as described in Section 3.1. The spectral analysis is used to develop lifetime fatigue loads spectra by considering the operational conditions and the characteristics of a ship in the sea. The operational conditions are divided into different operation modes according to the combinations of ship speeds, ship headings, and wave heights. The ship characteristics include the length between perpendicular (*LBP*), beam (*B*), and the bow form as. In performing such spectral analysis, it is possible to generate bending moment histograms (moment range versus number of cycles). With the proper identification of the hull girder section modulus, these moment range spectra can be easily converted to stress range spectra. The stress range spectra are used to compute the equivalent stress range S_e as given by Eq. 15.

The reliability-based design and analysis for fatigue requires the probabilistic characteristics of the random variables in the performance function equation. It also requires specifying target reliability index b_0 to be compared with a computed b resulting from reliability assessment methods such as FORM. The performance function for fatigue is given by the following expression:

$$g_1 = \frac{\Delta A}{k_s^m \bar{S}_e^m} - N_t \tag{22}$$

where

$$\bar{S}_e = \sqrt[m]{\sum_{i=1}^{n_b} f_i S_i^m} \tag{23}$$

and Δ = fatigue damage ratio, A= intercept of the S-N curve, m = slope of the S-N curve, $\bar{S}_e$ = Miner's mean equivalent stress , k_s = fatigue stress uncertainty factor, N_t = number of loading cycles expected during the life of a structural detail, n_b = number of stress blocks in a stress (loading) histogram, f_i = fraction of cycles in the

i^{th} stress block, and S_i = stress in the i^{th} block. With this information at hand, it is possible to develop a methodology for reliability checking expressions and design procedures for fatigue details. This methodology is presented in the next two sections. The methodology consists of two parts: (1) reliability checking, and (2) reliability-based design stress procedure.

5.1 Reliability Checking Procedure

The following steps summarize the procedure needed to perform reliability checking on an existing ship structural fatigue detail (see Figure 5):

1. For given ship characteristics (i.e., LBP, Beam, hull section modulus, etc.), operational profiles (i.e., speed, heading), ship lifetime at sea, and area of operation, stress range spectra can be generated using for example the program SPECTRA (Assakkaf 1998).
2. With the generation of stress range spectra, the Miner's mean equivalent stress range $\bar{S}_e$ can be computed using Eq. 23.
3. At this stage, a target reliability index b_0, a ship structural detail, and design life N_t should be selected.
4. The probabilistic characteristics of fatigue variables (Δ, A, k_s) in the performance function of equation Eq. 22 are evaluated in this step. Also, the *COV* of S_e and its distribution type are needed in this step. Section 7.5 of Assakkaf (1998) can be consulted for guidance.
5. Once all the variables are identified and computed in steps 1 through 4, the first-order reliability method (FORM) is used to compute the safety (reliability) index b.

The safety index b computed in step 5 is compared with the target reliability index b_0. If b is greater than b_0, this mean the structural detail under study is adequate, otherwise steps 3 to 6 should be repeated.

5.2 Reliability-Based Design Stress

The following steps provide a procedure for computing the design stress for a ship structural detail:

1. A target reliability index b_0, a ship structural detail, and design life N_t should be selected.
2. The probabilistic characteristics of fatigue variables (Δ, A, k_s) in the performance function equation (Eq. 22) are evaluated in this step.
3. For the selected target reliability index b_0, probability distributions and statistics (means *COV*'s) of the fatigue variables (Δ, A, k_s), and the coefficient of variation of the stress range S_e, the mean value of S_e (i.e., $\bar{S}_e$) is computed based on the iterative solution of FORM. The mean stress value ($\bar{S}_e$) is the design stress.

5.3 Example 2: Direct Reliability-Based Design for Fatigue of Marine Structures

In this example, a direct reliability-based procedure is used. This procedure is used to perform safety checking by evaluating the reliability indices based on selected pairs of m and A that correspond to certain fatigue details of interest, and identifying the details that meet or exceed the specified target reliability of 2.5. The performance function as defined in Eq. 22 is used in this example, where Δ, A, k_s, and S_e are random variables, and $N_t = 10^5$. The probabilistic characteristics of the random variables that are used for each detail in this example are provided in Tables 6 through 9. Summaries of the results based on this approach are shown in Table 10. An alternative procedure is to determine the design stress (mean of S_e) for each detail as outlined in Section 5.2. For target reliability b_0 of 2.5, probabilistic distributions and statistics of fatigue random variables for each detail, and the coefficient of variation of S_e, the mean design stress can be evaluated for each detail. The results based on this approach are summarized in Table 11.

Table 6. Probabilistic Characteristics of Random Variables for Detail No. 5 of Munse (1983)

Random Variable	Mean	*COV*	Distribution Type
S_e	6.96 ksi	0.10	Lognormal
Δ	1.0	0.48	Lognormal
A	4.47 E09	0.40	Lognormal
k_s	1.0	0.10	Normal
m	3.278	na	na
b	2.5	na	na

na = not applicable

Table 7. Probabilistic Characteristics of Random Variables for Detail No. 7(P) of Munse (1983)

Random Variable	Mean	*COV*	Distribution Type
S_e	7.95 ksi	0.10	Lognormal
Δ	1.0	0.48	Lognormal
A	2.88 E11	0.40	Lognormal
k_s	1.0	0.10	Normal
m	4.172	na	na
b	2.5	na	na

na = not applicable

Table 8. Probabilistic Characteristics of Random Variables for Detail # 27(S) of Munse (1983)

Random Variable	Mean	*COV*	Distribution Type
S_e	9.13 ksi	0.10	Lognormal
Δ	1.0	0.48	Lognormal
A	1.15 E12	0.40	Lognormal
k_s	1.0	0.10	Normal
m	5.277	na	na
b	2.5	na	na

na = not applicable

Table 9. Probabilistic Characteristics of Random Variables for Class B Detail (BS)

Random Variable	Mean	*COV*	Distribution Type
S_e	27.54 ksi	0.10	Lognormal
Δ	1.0	0.48	Lognormal
A	4.47 E11	0.44	Lognormal
k_s	1.0	0.10	Normal
m	4.0	na	na
b	2.5	na	na

na = not applicable

Table 10. Results of Reliability Checking for Fatigue Design (Target b = 2.5)

Detail No.	m	Mean A ($\bar{A}$)	$\bar{S}_e$	Computed b	Reliability Checking
5	3.28	4.47 E09	6.96	5.6	acceptable
7(P)	4.17	2.88 E11	7.95	7.5	acceptable
27(S)	5.28	1.15 E12	9.13	4.8	acceptable
Class B	4.0	4.47 E11	27.5	2.3	unacceptable

na = not applicable

Table 11. Results Using Direct Reliability-Based Fatigue Design (Target b = 2.5)

Selected Detail	Computed Mean Value of S_e ($\bar{S}_e$)
5	14.10
7(P)	20.71
27(S)	13.57
Class B	26.27

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