Chapter 19

Reliability-Based Structural Design

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19.1 INTRODUCTION

19.1.1 Structural Design

The main objective of structural design is to insure safety, functional, and performance requirements of an engineering system for target reliability levels and a specified time period. As this must be accomplished under conditions of uncertainty, probabilistic analyses are needed in the development of such reliability-based design of panels and fatigue details of ship structures. The reliability-based structural design formats are more flexible and rational than their counterparts, the working stress formats, because they provide consistent levels of safety over various types of structural components. Such a design procedure takes into account more information than the deterministic methods in the design of ship structural components. This information includes uncertainties in the strength of various ship structural elements, in loads, and modeling errors in analysis procedures.

Uncertainties in an engineering system can be mainly attributed to ambiguity and vagueness is defining the variables and parameters of the system and their relations. The ambiguity component is generally due to noncognitive sources (1). These noncognitive sources include:

- model uncertainties, which result from simplifying assumptions in analytical and prediction models,
- statistical uncertainties of the parameters and variables, and
- physical randomness.

The vagueness sources, on the other hand, include:

- human factors,
- the definition of certain variables or parameters, for example, structural performance (failure or survival), quality, and skill and experience of construction workers and engineers, and
- defining the interrelationships among the parameters of the problem.

Reliability and risk considerations are vital to the analysis and design of an engineering system. The reliability of the system can be stated in reference to some performance criteria. The need for reliability analysis stems from the fact that there is a presence of uncertainty in the definition, understanding, modeling, and behavior prediction of the model (models) that describes the system. The objective of the analysis is the assurance of some level of reliability. Because there are numerous sources of uncertainties associated with an engineered system, the absolute safety cannot be guaranteed. However, a likelihood of unacceptable performance can be limited to a reasonable level. Estimation of this likelihood, even when used to compare various design alternatives, is an important task for a practicing engineer.

The design, analysis, and planning of any engineering system require the basic concept that the supply should be greater or at least satisfy the demand. Depending on the type of problem at hand, different terminology is used to describe this concept. For example, in structural engineering the supply can be expressed in terms of the resistance (strength) of the system (or component, that is, a beam), and the demand can be expressed in terms of the applied loads, load combinations, and their effects (that is, dead and live loads). In hydrology engineering, the height and location of a dam to be built across a river may represent the capacity (supply). On the other hand, annual rainfall, catchments areas, vegetation, and other rivers or streams flowing into the river may represent demand (2).

The notion here is no matter how the supply and demand are presented or modeled, a variety of engineering problems must satisfy this concept. Ship structural design must provide for adequate safety and proper functioning of a structural element regardless of what concept of design is used. Structural elements must have adequate strength to permit proper functioning during their intended service life.

19.1.2 Need for Reliability-Based Ship Design

In recent years, reliability-based design and analysis for ship structures has received increasing interest. Numerous efforts have been made to implement the theory or at least develop the basis for the analyses of some aspects of design stages. As it is common with other industries and classification societies, we see that reliability and risk methodologies are at least being considered. Examples of such efforts are the recent works of the U.S. Navy (USN), the American Bureau of Shipping (ABS), and others to develop reliability-based standards and guidelines for such design approaches.

Such design approaches take into account more information than deterministic methods in the design of ship structural components. This information includes uncertainties in the strength of various structural elements, in loads and load combinations, and modeling errors in analysis procedures. Probability-based design formats are more flexible and rational than their counterparts the working stress formats because they provide consistent levels of safety over various types of structures. In probability-based limit-state design, probabilistic methods are used to guide the selection of strength (resistance) factors and load factors, which account for the variability in the individual resistance and loads and give the desired overall level of reliability. The load and resistance factors (or called partial safety factors) are different for each type of load and resistance. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with strength, the lower the corresponding strength factor.

Ship designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods without explicitly performing probabilistic analysis. For designing code provisions, the most common format is the use of load amplification factors and resistance reduction factors (partial safety factors), as represented by

$$\phi R \geq \sum_{i=1}^{n} \gamma_i L_i \qquad [1]$$

where:

ϕ = the resistance R reduction factor
γ_i = the partial load amplification factor
L_i = the load effect

In fact, the American Institute of Steel Construction (AISC) and other classification societies in this area have implemented this format. Also, a recommendation for the use of this format is given by the National Institute of Standards and Technology (3). The AISC (4) has introduced the Load and Resistance Factor Design (LRFD) Specifications in 1986 after the adoption of several American, Canadian, and European organizations of reliability-based design specifications. The development of the AISC LRFD code was based on a probability-based model, calibration with the 1978 AISC Allowable Stress Design (ASD) Specifications, and expert sound engineering judgment based on previous design experiences. In developing the specifications, it was necessary to change the design practice from working stress to limit stress, and from allowable stress to ultimate strength, which was reliability-based.

Currently, the American Association of State Highway and Transportation Officials (AASHTO) Specifications have been revised to an LRFD format. The National Cooperative Highway Research Program (NCHRP) has published the third Draft of LRFD Specifications and Commentary in 1992 entitled *Development of Comprehensive Bridge Specifications and Commentary*. The AASHTO LRFD (1) code closely follows much of the AISC code. Many of the individuals that were instrumental in the development of the AISC LRFD code were involved with the AASHTO effort.

Other marine and offshore classification societies that are in the process of revising, or have already revised and updated their codes to LRFD format include the U.S. Navy (USN), the American Bureau of Shipping (ABS), the American Petroleum Institute (API), the Association of American Railroads (AAR), Lloyd's Register (LR), and Det Norske Veritas (DnV).

As we will see in the subsequent sections, the First-Order Reliability Method (FORM) can be used to evaluate the partial safety factors ϕ and γ_i (appearing in equation 1) for a specified target levels of reliability. This method was used to determine the partial safety factors associated with the recommended strength models for ship structural components as demonstrated in this chapter.

19.2 SHIP STRUCTURAL COMPONENTS

19.2.1 Hull Girder

One of the fundamental concepts of engineering is that of a system, which can be anything from a simple beam or detail to complicated multilevel subsystems. A ship obviously falls into the category of a relatively large and complex system. The ship consists of several subsystems, which are essential to the integrity of the whole system. Examples of these subsystems are the hull girders, unstiffened and stiffened panels, and structural fatigue details. Probably the most essential part of a ship design is the hull girder system or model. Environmental loads, either static or dynamic, that are due to sea environment and ship's motion are functions of the hull shape. However, much of these loads are relatively independent of the substructures (subsystems) such as unstiffened and stiffened plate elements, that is, they are not affected by the structural layout and shape or by scantlings. Therefore, the design of the hull girder is the first step toward designing the other substructures of a ship because much of the overall load effects on the hull girder can be used for designing these substructures or subsystems.

In a large structure, such as a hull girder, both the loading and the response are extremely complex, and therefore, the response analysis must be performed in two stages (5), 1) an analysis of the overall structure, and 2) a separate and more detail analyses of different substructures.

Many of the load effects from the overall analysis constitute the loads and boundary conditions at the substructure level. The overall structure of a ship is essentially a floating beam (box girder) that internally stiffened and subdivided, and in which the decks and bottom structure are flanges and the side shell and any longitudinal bulkheads are the webs. External forces and moments on a hull girder are those forces or moments that are applied on a beam such as vertical shear force (f_y), longitudinal bending moment in the ship's vertical and horizontal planes (M_y and M_z), and longitudinal twisting moment M_x. The most significant of all these forces and moments is the vertical bending moment of the hull girder about the z-axis as shown in Figure 19.1. This load affect is due primarily to the unequal distribution of the weight (W) of the ship and buoyancy (B_F) along the length of the ship due to waves as shown in Figure 19.2. For many ships, the maximum value of the horizontal moment M_y is much smaller than the vertical moment M_z, typically 19% or less (5).

The vertical bending moment varies along the length of the ship. It can take values from zero at the ends to a maximum at or near the midlength of the ship. This maximum value of the vertical moment for hull girder is the single most important load effect in the analysis and design of ship structures. Hull girder bending can be caused by either hogging or sagging depending on the curvature due waves as shown in Figure 19.2. The hull girder analysis and design assumes that the hull girder satisfies simple beam theory that implies the following assumptions (5):

- Plane cross sections remain plane,
- The beam is essentially prismatic,

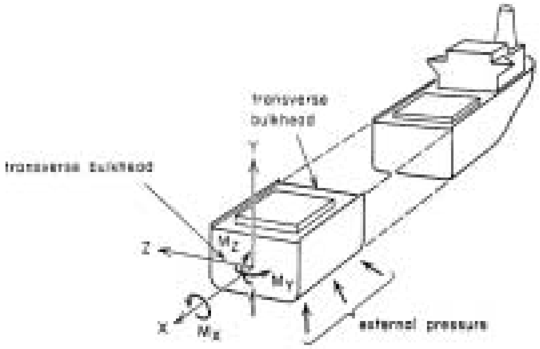


Figure 19.1 Hull Girder Model of a Ship (5)

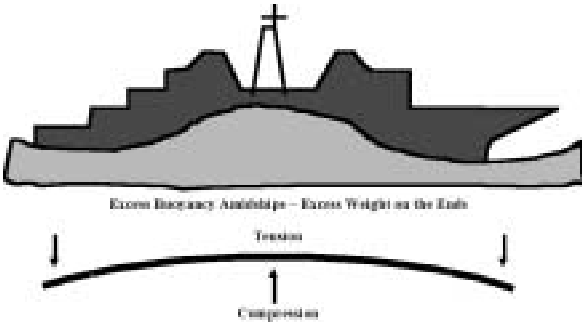


Figure 19.2A Hogging Condition of a Ship Due to Sea Waves

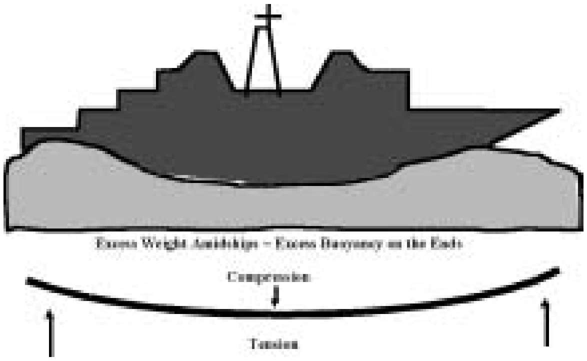


Figure 19.2B Sagging Condition of a Ship Due to Sea Waves

- Other modes of response to the loads do not affect hull girder bending and may treated separately, and
- The material is homogeneous and elastic.

19.2.2 Ship Steel Panels

The structural components that make up the hull girder are the panels or plate elements. Ship panels, in general, can be divided into three distinct categories, 1) unstiffened, 2) stiffened, and 3) gross panels or grillages (Figures 19.3 and 19.4).

These panels (or called plates) are very important components in ship and offshore structures, and, therefore, they should be designed for a set of failure modes that govern their strength.

They form the backbone of most ship's structure, and they are by far the most commonly used element in a ship. They can be found in bottom structures, decks, side shell, and superstructures. The modes of failure, which govern the strength of these panels, can be classified to produce two distinct limit states, strength and serviceability limit states. Strength limit states are based on safety consideration or ultimate load-carrying capacity of a panel and they include plastic strengths, buckling, and permanent deformation. Serviceability limit states, on the other hand, refer to the performance of a panel under normal service loads and are concerned with the uses of unstiffened and stiffened plates, and gross panels. They include such terms as excessive deflections and first yield. Also, strength limit states require the definition of the lifetime extreme loads and their combinations, whereas serviceability limit states require annual-extreme loads and their combinations.

The primary purpose of a panel is to absorb out of plane (or lateral) loads and distribute those loads to the ship's primary structure. It also serves to carry part of the longitudinal bending stress because of the orientation of the stiffeners. The amount of in-plane compression or tension experienced depends primarily on the location of the panel within the ship. Deck panels tend to experience large in-plane compression and small lateral pressures, while bottom panels can be exposed to large in-plane tension and compression with a significant amount of lateral pressures.

The main type of framing system found in ships nowadays is a longitudinal one, which has stiffeners running in two orthogonal directions (Figure 19.3). Deck and bottom structures panels are reinforced mainly in the longitudinal direction with widely spaced heavier transverse stiffeners. The main purpose of the transverse stiffeners is to provide resistance to the loads induced on bottom and side shell by water pressure (6). The types of stiffeners used in the longitudinal direction are the T-beams, angles, bulbs, and flat bars, while the transverse stiffeners are typically T-beam sections. This type of structural configuration is commonly called gross stiffened panel or grillage (6). Besides their use in ship structures, these gross stiffened panels are also widely used in land-based structures such as box and plate girders.

The overall collapse of a gross panel involves global deflection of both longitudinal and transverse stiffeners. However, except for lightly stiffened panels found in superstructures, this type of failure rarely occurs because most ship structures are designed to prevent the overall mode of collapse (7,8). In most cases local plate buckling is the weakest failure mode. Global failure of a stiffened panel can be partially controlled by careful design of strength of the plate elements (unstiffened panels) between stiffeners. The most common mode of failure of the whole panel

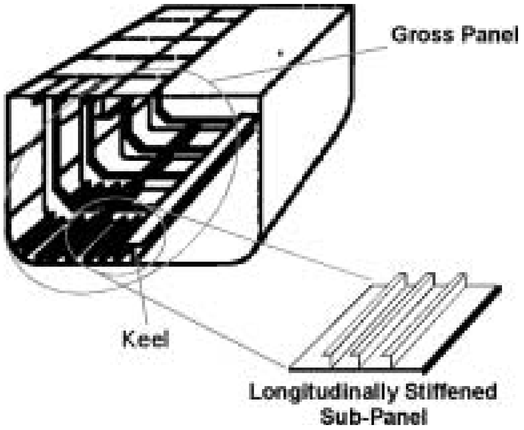


Figure 19.3 Portion of the Hull Girder Showing the Gross Panel and a Longitudinally Stiffened Subpanel (5)

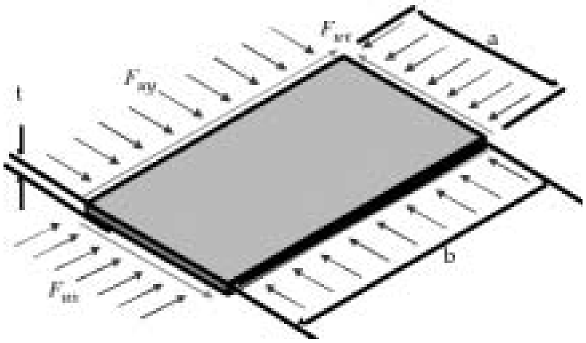


Figure 19.4 Unstiffened Panel Subjected to In-Plane Stresses

involves the collapse of the longitudinally stiffened subpanel. Choosing the size of the transverse stiffeners so that they provide sufficient flexural rigidity to enforce nodes at the location of the transverse stiffeners can prevent the collapse of longitudinally stiffened subpanel. If the transverse stiffeners act as nodes, then the collapse of the stiffened panel is controlled by the strength of the longitudinally stiffened subpanel.

A typical longitudinal stiffened subpanel, as shown in Figure 19.3, is bounded on each end by a transverse structure, which has significantly greater stiffness in the plane of the lateral load. The sides of the panel are defined by the presence of a large structural member that has greater stiffness in bending and much greater stiffness in axial loading. Structural members such as keels, bottom girders, longitudinal bulkheads, deck girders, etc., can act as the side boundaries of the panel. When the panel is located to be in a position to experience large in-plane compression, the boundary conditions for the ends are taken as simply supported. The boundary conditions along the sides also can be considered simply supported.

In ship structures, there are three primary types of load effects that can influence the strength of a plate-stiffener panel (negative bending moment, positive bending moment, and in-plane compression or tension). Negative bending loads are the lateral loads due to lateral pressure. They cause the plate to be in tension and the stiffener flange in compression. Positive bending loads are those loads that put the plating in compression and the stiffener flange in tension. The third type of loading is the uniform in-plane compression. This type of loading arises from the hull girder bending, and will be considered positive when the panel is in compression. The three types of loading can act individually or in combination with one another.

To evaluate the strength of a stiffened or gross panel element it is necessary to review various strength prediction models and to study their applicability and limitations for different loading conditions acting on the element. Although stiffened plate strength has been studied for many years, several advanced strength models have been developed during the last few decades. These advanced models take into account the effects of initial distortion; weld induced residual stresses, and various parameters concerning strength prediction. Some of these models are empirical in nature but they are highly representative of real world scenario because they were developed on the bases of experimental data. An exact stiffened panel-strength prediction can only be achieved by a method of analysis, either numerical or experimental, in which all the characteristics of the panel and the loading variables are presented and are properly accounted for in the method.

19.3 RELIABILITY, RISK, SAFETY, AND PERFORMANCE

Reliability of a system can be defined as its ability to fulfill its design functions for a specified time period. This ability is commonly measured using probabilities. Reliability is, therefore, the occurrence probability of the complementary event to failure resulting into

$$\text{Reliability} = 1 - \text{Failure Probability} \qquad [2]$$

Based on this definition, reliability is one of the components of risk. The concept of risk is used to assess and evaluate uncertainties associated with an event. Risk can be defined as the potential of losses as a result of a system failure, and can be measured as a pair of the probability of occurrence of an event, and the outcomes or consequences associated with the event's occurrence. This pairing can be represented by the following equation:

$$\text{Risk} \equiv \left[\left(p_1, c_1\right), \left(p_2, c_2\right), \ldots, \left(p_x, c_x\right)\right] \qquad [3]$$

In this equation p_x is the occurrence probability of event x, and c_x is the occurrence consequences or outcomes of the event. Risk is commonly evaluated as the product of likelihood of occurrence and the impact of an accident:

$$\begin{aligned} &\text{RISK}\left(\frac{\text{Consequence}}{\text{Time}}\right) = \\ &\text{LIKELIHOOD}\left(\frac{\text{Event}}{\text{Time}}\right) \times \text{IMPACT}\left(\frac{\text{Consequence}}{\text{Event}}\right) \end{aligned} \qquad [4]$$

In equation 4, the likelihood can also be expressed as a probability. A plot of occurrence probabilities that can be annual and consequences is called the Farmer curve (9).

The risk assessment process answers three questions including,

1. what can go wrong,
2. what is the likelihood that it will go wrong, and
3. what are the consequences if it does go wrong?

In order to perform risk assessment several methods have been created including: Preliminary Hazard Analysis (PrHA), HAZOP, Failure Modes and Effects Analysis (FMEA), Failure Modes Effects, and Criticality Analysis (FMECA), Fault Tree Analysis (FTA), and Event Tree Analysis (ETA). Each of these methods of risk assessment is suitable in certain stages of the system life cycle. The characteristics of these methods are shown in Table 19.I. In-depth description of risk management, methods for reliability and consequence analysis and assessment are described in references 10 and 11.

Safety can be defined as the judgment of risk accept-

TABLE 19.I Risk Assessment Methods (9)

Safety/Review Audit	Identify equipment conditions or operating procedures that could lead to a casualty or result in property damage or environmental impacts.
Checklist	Ensure that organizations are complying with standard practices.
What-If	Identify hazards, hazardous situations, or specific accident events that could result in undesirable consequences.
Hazard and Operability Study (HAZOP)	Identify system deviations and their causes that can lead to undesirable consequences. Determine recommended actions to reduce the frequency and/or consequences of the deviations.
Failure Modes and Effects Analysis (FMEA)	Identifies the components (equipment) failure modes and the impacts on the surrounding components and the system.
Failure Modes Effects, and Criticality Analysis (FMECA)	Identifies the components (equipment) failure modes and the impacts on the surrounding components and the system, and criticality of failures.
Fault Tree Analysis (FTA)	Identify combinations of equipment failures and human errors that can result in an accident.
Event Tree Analysis (ETA)	Identify various sequences of events, both failures and successes that can lead to an accident.
Preliminary Hazard Analysis (PrHA)	Identify and prioritize hazards leading to undesirable consequences early in the life of a system. Determine recommended actions to reduce the frequency and/or consequences of prioritized hazards.
Consequence Assessment and Cause Consequence Diagrams	Assess consequences and scenarios leading to them.

ability for the system making it a component of risk management.

After performing risk and safety analysis, system improvement in terms of risk can be achieved by one or more of the following cases:

- consequence reduction in magnitude or uncertainty,
- failure-probability reduction in magnitude or uncertainty, and
- reexamination of acceptable risk.

It is common in engineering that attention is given to failure-probability reduction in magnitude or uncertainty because it offers more system variables that can be controlled by analysts than the other two cases. As a result, it is common to perform reliability-based design of systems. However, the other two cases should be examined for possible solution since they might offer some innovative system improvement options.

The performance of a systems can be defined by a set of requirements stated in terms of tests and measurements of how well the system serves various or intended functions. Reliability and risk measures can be considered as performance measures.

19.3.1 Measures and Assessment of Reliability and Risk

Traditionally, the reliability of engineering systems has been achieved through the use of factors of safety (FS) in the so-called working stress (or allowable stress design, ASD) formats. The safety factor, whose value provides a quantitative measure of reliability or safety, differs from one design specification to another and from one type of structure (that is, beam, column, plate, etc.) to another. It reflects the degree of reliability and risk associated with that particular component. For example, this value can range from 2 to 4 for land-based structural systems, and from 3 to 5 or even 6 in geotechnical engineering applications, depending on the type of structural system or component under consideration.

This measure of reliability or safety was intended to reflect the probability of failure of the system and the risk associated with it.

The traditional approach is difficult to quantify and lacks the logical basis for addressing uncertainties. Therefore, the level of reliability or safety cannot be evaluated quantitatively. Also, for new systems in which there is no prior basis for calibration, the assurance of performance can be a very difficult task.

In reliability-based design and analysis approaches, the measure of reliability or safety is accomplished through the use of reliability (safety) index β. In this respect, the role of β is to reflect the reliability level used in the analysis. In practical structural analysis, β can be computed using structural reliability theory and knowledge of the first and second moments statistical characteristics (that is, mean and *COV*) for both the strength and load variables. Sometimes in more rigorous analyses, the distribution types of these variables are needed. Also, a definition of a performance (or criterion) function is required. For two variables and linear performance function, the reliability index β can be defined as the shortest distance from the origin to the failure line as shown in Figure 19.5. Mathematically, it can be expressed as

$$\beta = \frac{\mu_R - \mu_L}{\sqrt{\sigma_R^2 + \sigma_L^2}} \qquad [5]$$

where

μ_R = mean value of strength R
μ_L = mean value of the load effect L
σ_R = standard deviation of strength R
σ_R = standard deviation of the load effect L

The reliability index according to this definition is commonly referred to as the Hasofer-and-Lind index (12).

A distinction should be made between the reliability index β and target reliability index β_0. Target reliability index values are used by the classification societies to set the standards for code provisions to meet the design requirements of various structural components (or systems). These values can vary depending on the type of structural component being analyzed and the risk associated with its design. On the other hand, computed reliability index values are used to check the adequacy and performances of existing structures. In this approach, the computed value of the safety or reliability index is compared with the target reliability index.

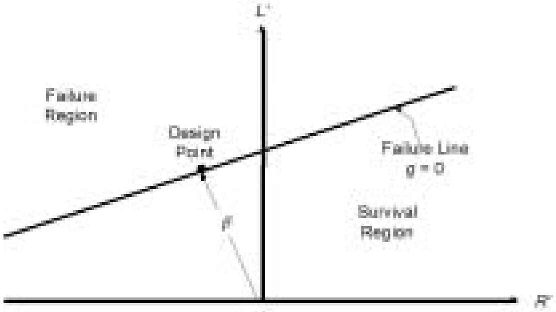


Figure 19.5 Performance Space in Reduced Coordinates

If, for example, the computed value of the reliability index β is greater than the target reliability index β_0, then the structural component under study is adequate to withstand the prescribed load effect.

Table 19.II and III provide examples target reliability levels used in the industry, while Table 19.IV gives target reliability index values for ship structural components.

19.3.2 Selection of Target Reliability Levels

As was alluded to earlier, target reliability levels, β_0s, are used by the classification societies to set the standards for code provisions to meet the intended design requirements of various structural components (or systems).

These target levels can vary depending on the type of structural component being analyzed and the risk associated with its design. Reliability-based design guidelines and rules for ship structures require establishing these target levels for the design and analyses of the structural components. The selected reliability level determines the proba-

TABLE 19.II Target Reliability Levels (13)

Structural Type	*Target Reliability Level (β_0)*
Metal structures for buildings (dead, live, and snow loads)	3
Metal structures for buildings (dead, live, and wind loads)	2.5
Metal structures for buildings (dead, live, and snow, and earthquake loads)	1.75
Metal connections for buildings (dead, live, and snow loads)	4 to 4.5
Reinforced concrete for buildings (dead, live, and snow loads)	
ductile failure	3
brittle failure	3.5

ED: Unfortunately, Greek symbols do not print in italic on press.

TABLE 19.III Target Reliability Levels Used by Ellingwood and Galambos (14)

Member, Limit State	*Target Reliability Level* (β_0)
Structural Steel	
Tension member, yield	3.0
Beams in flexure	3.0
Column, intermediate slenderness	3.5
Reinforced Concrete	
Beam in flexure	3.0
Beam in shear	3.0
Tied column, compressive failure	3.5
Masonry, unreinforced	
Wall in compression, uninspected	5.0
Wall in compression, uninspected	7.5

TABLE 19.IV Recommended Target Safety Indices Relative to Service Life of Ships (13)

	Tanker β_0	*Cruiser* β_0
Hull girder collapse	4	5
Hull girder initial yield	4.5	5.5
Unstiffened panel	3	3.5
Stiffened panel	3.5	4
Fatigue		
Category 1 (Not Serious)	2.0	2.5
Category 2 (Serious)	2.5	3.0
Category 3 (Very Serious)	3.0	3.5

bility of failure of the ship structural component being analyzed. The following three methods can be used to select a target reliability value, 1) agreeing upon a reasonable value in cases of novel structures without prior history, 2) calibrating reliability levels implied in currently used successful design codes, and 3) choosing target reliability level that minimizes total expected costs over the service life of the structure for dealing with design for which failures result in only economic losses and consequences.

Since the development herein is limited to ship structural components that are not novel structures, the first method is excluded. The modes of failure for ship structural components have serious consequences such as the entire loss of the ship, loss of lives, and environmental damages (water pollution in case of tankers or chemical carriers). Accordingly, the second method seems to be the proper one to be adopted for selecting target reliability levels since there are a lot of data available from currently used design codes that resulted in safe structures with adequate reliability.

19.4 RELIABILITY-BASED STRUCTURAL DESIGN APPROACHES

The reliability-based design of any structural system requires the consideration of the following three components 1) loads, 2) structural strength, and 3) methods of reliability analysis.

These three components can be presented in the form of several blocks for each to show their logical sequence and interaction. The reliability-based design procedure also requires the probabilistic characteristics of the strength and load basic random variables as well as defining performance functions that correspond to limit states for significant failure modes. There are two primary approaches for reliability-based design (9), 1) direct reliability-based design, and 2) load and resistance factor design (LRFD).

The direct reliability-based design approach can include both Level 2 and/or Level 3 reliability methods. Level 2 reliability methods are based on the moments (mean and variance) of random variables and sometimes with a linear approximation of nonlinear limit states, whereas, Level 3 reliability methods use the complete probabilistic characteristics of the random variables. In some cases, Level 3 reliability analysis is not possible because of lack of complete information on the full probabilistic characteristics of the random variables. Also, computational difficulty in Level 3 methods sometimes discourages their uses. The LRFD approach is called a Level 1 reliability method. Level 1 reliability methods utilize partial safety factors (PSF) that are reliability based; but the methods do not require explicit use of the probabilistic description of the variables.

The many advantages and benefits of using reliability-based design methods include the following:

- they provide the means for the management of uncertainty in loading, strength, and degradation mechanisms,
- they provide consistency in reliability,
- they result in efficient and possibly economical use of materials,
- they provide compatibility and reliability consistency across materials, such as, steel grades, aluminum and composites,

- they allow for future changes as a result of gained information in prediction models, and material and load characterization,
- they provide directional cosines and sensitivity factors that can be used for defining future research and development needs,
- they allow for performing time-dependent reliability analysis that can form the bases for life expectancy assessment, life extension, and development of inspection and maintenance strategies,
- they are consistent with other industries, AISC, ASHTO, ACI, API, ASME, …, etc, and
- they allow for performing system reliability analysis.

19.4.1 Fundamentals of Reliability-Based Design

The design of any structural system or element must provide for adequate safety and proper functioning of that system or element regardless of what philosophy of design is used. The structural systems or elements must have adequate strength to permit proper functioning during their intended service life. For example, the performance of a ship hull girder as presented in the chapter is defined by a set of requirements stated in terms of tests and measurements of how well the hull girder serves various or intended functions over its service life. Reliability and risk measures can be considered as performance measures, specified as target reliability levels (or target reliability indices, β_0s). The selected reliability levels of a particular structural element reflect the probability of failure of that element. These levels can be set based on implied levels in the currently used design practice with some calibration, or based on cost benefit analysis.

For ship structures, the reliability-based design approaches for a system start with the definition of a mission and an environment for a ship. Then, the general dimensions and arrangements, structural member sizes, scantlings, and details need to be assumed. The weight of the structure can then be estimated to ensure its conformance to a specified limit. Using an assumed operational-sea profile, the analysis of the ship produces a stochastic still water and wave-induced responses. The resulting responses can be adjusted using modeling uncertainty estimates that are based on any available results of full-scale or large-scale testing.

The reliability-based design procedure also requires defining performance functions that correspond to limit states for significant failure modes. In general, the problem can be considered as one of supply and demand. Failure of a structural element occurs when the supply (that is, strength of the element) is less than the demand (that is, loading on the element). On the other hand, the reliability of this element is achieved when the supply is greater than the demand.

19.4.1.1 Reliability of structural components

The reliability of a structural component constitutes the basis for performing system reliability of larger structure. In general, a component can fail in one of several failure modes. The treatment of multiple failure modes requires modeling the component behavior as a system. In addition, the system can be defined as a collection or an assemblage of several components that serves some function or purpose (15). A multi-component system can fail in several failure modes. Once the reliability or probability of failures for all of the components that make up the whole systems is evaluated, system reliability can be performed on the overall system. The theory of system reliability is beyond the scope of this chapter. Numerous excellent books and references have been written for the subject, and the reader is encouraged to read references (1,9,15,29,31).

The reliability of a structural component can be defined as the probability that the component meets some specified demands. For example, the reliability of a structural component such as a beam can be defined as the probability that structural strength of the beam (that is, ultimate moment capacity) exceeds the applied load (that is, moment due to the total combined loads). The first step in evaluating the reliability or probability of failure of a structural component is to decide on specific performance function g and the relevant load and resistance variables. The generalized form of the performance function can be expressed as

$$g = R - L \qquad [6]$$

or

$$g = f(X_1, X_2, \ldots X_n) \qquad [7]$$

where

g = the performance function
$X_1, X_2, \ldots, X_n$ = n basic random variables for R and L
$f(.)$ = a function that gives the relationship between R and L and the basic random variables.

The failure in this case is defined in the region where *g* is less than zero (see Figure 19.6) or R is less than L, that is

$$g = < 0.0 \text{ or } R < L \qquad [8]$$

whereas the reliability is defined in the region where *g* is greater than zero (Figure 19.6) or R is greater than L, that is

$$g = > 0.0 \text{ or } R > L \qquad [9]$$

The limit state is defined when $g = 0$.

Due to the variability in both strength and loads, there is always a probability of failure that can be defined as

$$P_f = P(g < 0.0) = P(R < L) \qquad [10]$$

The reliability of a structural component can be defined as the probability that the component meets some specified demands for a specified time frame.

Mathematically, it can be given by the following expression:

$$R_c = P(g > 0.0) = P(R > L) \qquad [11]$$

where P_f = probability of the system or component and R_c = reliability of the component. According to probability theory, since failure and non-failure (or success) constitute two complementary events, therefore,

$$P_f = 1 - R_c \qquad [12]$$

For the general case, where the basic random variables can be correlated, the probability of failure for the component can be determined by solving the following integral:

$$P_f = \int\limits_{\text{over } g\le 0} \cdots \int f_{\underline{X}}\left(x_1, x_2, \cdots x_n\right) dx_1 dx_2 \cdots dx_n \qquad [13]$$

where $f_{\underline{X}}$ is the joint probability density function (PDF) of the random vector $\underline{X} = [X_1, X_2, \ldots, X_n]$; and the integration is performed over the region where g = f(.) < 0. The computation of P_f by Equation 13 is called the *full distributional approach* and can be considered the fundamental equation of reliability analysis (29). In general, the determination of the probability of failure by evaluating the integral of Equation 13 can be a difficult task. In practice, the joint probability density function $f_{\underline{X}}$ is hard to obtain. Even, if the PDF is obtainable, evaluation of the integral of Equation 13 requires numerical methods. In practice, there are alternative methods for evaluating the above-mentioned integral through the use of analytical approximation procedures such as the First-Order Reliability Method (FORM), which is the focus of our discussion in the next section.

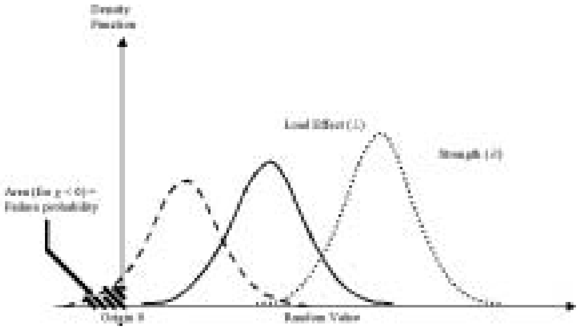


Figure 19.6 Frequency Distribution of Strength *R* and Load *L*

19.4.1.2 First-Order reliability method

The First-Order Reliability Method (FORM) is a convenient tool to assess the reliability of a ship structural element. It also provides a means for calculating the partial safety factors ϕ and γ_i that appear in the LRFD design formula of Equation 1 for a specified target reliability level β_0. The simplicity of the first-order reliability method stems from the fact that this method, beside the requirement that the distribution types must be known, requires only the first and second moments; namely the mean values and the standard deviations of the relevant random variables. Knowledge of the joint probability density function (PDF) of the design basic variables is not needed as in the case of the direct integration method for calculating the reliability index β. Even if the joint PDF of the basic random variables is known, the computation of β by the direct integration method as given by equation 13 can be a very difficult task.

The development of FORM over the years resulted in many variations of the method. These variations (29) include such methods as the first-order second moment (FOSM) and the advanced first-order second moment (AFOSM). Both of these methods use the information on first and second moments of the random variables, namely, the mean and standard deviation (or the coefficient of variation, *COV*) of a random variable. However, the FOSM method ignores the distribution types of the random variables, while AFOSM takes these distributions into account. Clearly, the AFOSM method as the name implies produces more accurate results than FOSM. Nevertheless, FOSM can be used in many situations of preliminary design or analysis stages of a structural component, where the strength and load variables are assumed to follow a normal distribution and the performance function is linear. In these cases, the results of the two methods are essentially the same.

The importance of FORM is that it can be used in structural analysis to compute the reliability index β, and also to determine the partial safety factors (PSF's) in the development of various design codes. The reliability index was defined earlier as shortest distance from the origin to the failure line as shown in Figure 19.5. For normal distributions of the strength and load variables, and linear performance function, β can be computed using Equation 5. The important relationship between the reliability index β and the probability of failure P_f is given by

$$P_f = 1 - \Phi(\beta) \qquad [14]$$

where $\Phi(.)$ = cumulative probability distribution function of the standard normal distribution. It is to be noted that equation 14 assumes all the random variables in the limit state equation to have normal probability distribution and the performance function is linear. However, in practice, it is common to deal with nonlinear performance functions with a relatively small level on linearity. If this is the case, then the error in estimating the probability of failure P_f is very small, and thus for all practical purposes, equation 14 can be used to evaluate P_f with sufficient accuracy (3).

The nominal values of partial safety factors (PSFs) according to the linear performance function given by Equations 6 and 7, and for normal distributions of the strength and load variables can be calculated using the following two expressions as suggested by Halder and Mahadevan (16):

For single load case:

$$\phi = \frac{1 - \varepsilon\beta\delta_R}{1 - S_R\delta_R} \qquad [15]$$

$$\gamma_L = \frac{1 + \varepsilon\beta\delta_L}{1 + S_L\delta_L} \qquad [16]$$

where

$$\varepsilon = \frac{\sqrt{\sigma_R^2 + \sigma_L^2}}{\sigma_R + \sigma_L} \qquad [17]$$

and in which, σ_R = standard deviation of strength R, σ_L = standard deviation of the load effect L, δ_R = coefficient of variation (COV) of the strength R, δ_L = COV of the load effect L, and S_R and S_L are parameters used by some classification societies and the industry to approximate the nominal values of the strength and the load effect, respectively. Typical values for S_R and S_L range from 1 to 3.

For multiple load case:

The nominal reduction factor ϕ of strength can still be computed from Equation 15. However, the nominal load factors γ_is for the ith load effect become (22)

$$\phi = \frac{1 - \varepsilon\beta\delta_R}{1 - S_R\delta_R} \qquad [18]$$

where

$$\phi = \frac{1 - \varepsilon\beta\delta_R}{1 - S_R\delta_R} \qquad [19]$$

and in which, $\sigma_{L_1}^2, \sigma_{L_2}^2, \cdots, \sigma_{L_{n1}}^2$ = standard deviations of the load effects ($L_1, L_2, \ldots, L_n$) and δ_{L_i} = *COV* of the load effect L_i, and S_{L_i} = parameter used to approximate the nominal value of load effect L_i.

In general, the nominal value of the strength is less than the corresponding mean value, and the nominal value of the load effect is larger than its mean value. For example, if both S_R and S_L equal to 2, the nominal value of R would be 2 standard deviations below the mean, and the nominal value for L would be 2 standard deviations above its mean value. If S_R and S_L have zero values, then Equations 15 and 16 essentially result into the mean values of the partial safety factors $\overline{\phi}$ and $\overline{\gamma}_L$, respectively. The nominal values of partial safety factors can be used in LRFD design format of the type

$$\phi R_n \geq \gamma_1 L_1 + \gamma_2 L_2 + \ldots \gamma_n L_n \qquad [20]$$

For purposes of design, this relationship needs to be satisfied.

It is to be noted that Equations 15 and 16 apply only for linear performance function with two variables (strength and one load effect) having normal distributions, while Equation 18 applies for multiple linear case. For a general case of nonlinear function with multiple random variables having different distribution types (that is, lognormal, Type I, etc.), an advanced version of FORM should be used. Detailed algorithms of advanced FORM version as well as procedures for calculating and calibrating the partial safety factors using FORM can be found in Appendix A. It is to be noted that the version of FORM given in the appendix is the advanced first-order second moment (AFOSM). This version of FORM applies for a general case of nonlinear performance function and for any distribution type of the random variables.

EXAMPLE 19.1

Given:

A tension member in a truss has an ultimate strength T with a mean value of 623 kN and standard deviation of 53 kN. The tension load L applied to the member has a mean value of 400 kN kips and standard deviation of 111 kN. If normal distributions are assumed for T and L, what is the reliability index for this member? What is its failure probability?

Solution:

The following parameters are given:

$$\mu_T = 53 \text{ kN}$$
$$\mu_L = 111 \text{ kN}$$
$$\sigma_T = 623 \text{ kN}$$
$$\sigma_L = 400 \text{ kN}$$

Using Equation 5, therefore,

$$\beta = \frac{623 - 400}{\sqrt{(53)^2 + (111)^2}} = 1.81$$

The probability of failure according to Equation 14 is

$$P_f = 1 - \Phi(1.81) = 1 - 0.9649 = 0.035$$

Note: Φ(1.81) can be obtained from Tables that provide values for the cumulative distribution function of standard normal.

EXAMPLE 19.2

Given:

The fully plastic flexural capacity of a beam section can be estimated as F_y Z, where F_y = yield strength of the material (steel) of the beam and Z = plastic section modulus. If the simply supported beam shown in Figure 19.7 is subjected to mean values of distributed dead and live loads: w_D and w_L, respectively; and if Z and L are assumed to be constant, develop the nominal and mean partial safety factors for this beam and the corresponding LRFD-based design formula for a target reliability index of 3. Assume that the nominal values are one standard deviation below the mean for the strength, and one standard deviations above the corresponding mean values for both the dead and live loads. The probabilistic characteristics of the basic random variables are as provided in Table 19.V.

Solution:

For this analysis, the following linear performance function is considered:

$$g = M_R - M_D - M_L$$

The plastic moment capacity of the beam M_p can be considered the mean moment capacity, thus

$$g = ZF_y - M_D - M_L$$

$$\begin{aligned} M_R &= M_P = ZF_y \\ &= (4588 \times 10^{-6})(248 \times 10^{3}) \\ &= 1137.8 \text{ kN} - \text{m} \end{aligned}$$

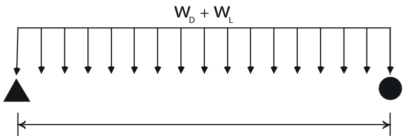


Figure 19.7 Beam Design for Example 19.2

$$\sigma_R = 1137.8\left(\frac{12.4}{248}\right) = 56.9 \text{ kN} - \text{m}$$

$$\delta_R = \frac{12.4}{248} = 0.05$$

$$\delta_D = \frac{\sigma_D}{\mu_D} = \frac{0.044}{0.315} = 0.14$$

$$\delta_L = \frac{\sigma_L}{\mu_L} = \frac{0.16}{0.438} = 0.36$$

For simply supported beam, the applied maximum moments at its mid-span can be computed as follows:

$$M_D = \frac{w_D L^2}{8} = \frac{0.315(915)^2}{8(100)} = 329.7 \text{ kN} - \text{m}$$

$$M_L = \frac{w_L L^2}{8} = \frac{0.438(915)^2}{8(100)} = 458.4 \text{ kN} - \text{m}$$

Denoting the total moment due to applied dead and live loads as M, its mean, standard deviation, and COV can be estimated:

$$\begin{aligned} \mu_M &= 329.7 + 458.4 = 788.1 \text{ kN} - \text{m} \\ \mu_{M_D} &= 329.7(0.14) = 46.16 \text{ kN} - \text{m} \\ \mu_{M_L} &= 458.4(0.36) = 165.02 \text{ kN} - \text{m} \end{aligned}$$

Therefore,

$$\sigma_M = \sqrt{(46.16)^2 + (165.02)^2} = 171.4 \text{ kN} - \text{m}$$

$$\delta_M = \frac{171.4}{788.1} = 0.22$$

Using Equations 17 and 19, the parameters ε and ε_n are calculated as follows:

$$\varepsilon = \frac{\sqrt{(56.9)^2 + (171.4)^2}}{56.9 + 171.4} = 0.79$$

TABLE 19.V Probabilistic Characteristics of Random Variables for the Beam Problem

Variable	μ	σ	*Distribution*
F_y	248 MPa	12.4 MPa	Normal
Z	4588 cm^3	n/a	n/a
L	915 cm	n/a	n/a
w_D	0.315 kN/cm	0.044 kN/cm	Normal
w_L	0.438 kN/cm	0.16 kN/cm	Normal

$$\varepsilon_n = \frac{\sqrt{(46.16.24)^2 + (165.02)^2}}{46.16 + 165.02} = 0.81$$

According to Equations 15 and 18, and noting that $S_R = S_D = S_L = 1$ for both the strength and load effects, the nominal partial safety factors (PSF's) are obtained as follows:

$$\phi = \frac{1 - 0.79(3)(0.05)}{1 - (1)(0.05)} = 0.93$$

$$\gamma_D = \frac{1 + 0.79(0.81)(3)(0.14)}{1 + (1)(0.14)} = 1.11$$

$$\gamma_L = \frac{1 + 0.79(0.81)(3)(0.36)}{1 + (1)(0.36)} = 1.24$$

Thus, the LRFD-based design formula is given by

$$0.93R \geq 1.11D + 1.24L$$

The mean values of the partial safety factors can be found using Equations 15 and 18, with $S_R = S_D = S_L = 0$. The results are:

$$\bar{\phi} = 0.88$$
$$\bar{\gamma}_D = 1.27$$
$$\bar{\gamma}_L = 1.69$$

EXAMPLE 19.3

Given:
Develop the mean values of partial safety factors for the simply supported beam of Example 19.2 using the probabilistic characteristics for the random variables as provided in Table 19.VI.

Solution:
In this example, we note that the distribution types of the random variables are no longer normal. We have a mixture of distributions for these variables. Therefore, the simplified methods of this section cannot apply directly even though the performance function is the same, that is

$$g = ZF_y - M_D - M_L$$

To compute the mean values of the partial safety factors, the general procedure of FORM, as outlined in Appendix A, should be utilized. The results are as follows:

$$\bar{\phi} = 0.97$$
$$\bar{\gamma}_D = 1.05$$
$$\bar{\gamma}_L = 2.63$$

TABLE 19.VI Probabilistic Characteristics of Random Variables for Example 19.3

Variable	μ	σ	*Distribution*
F_y	248 MPa	12.4 MPa	Lognormal
Z	4588 cm^3	n/a	n/a
L	915 cm	n/a	n/a
w_D	0.315 kN/cm	0.044 kN/cm	Normal
w_L	0.438 kN/cm	0.16 kN/cm	Type I

19.4.2 Direct Reliability-Based Design

The direct reliability-based design method uses all available information about the basic variables, including correlation, and does not simplify the limit state in any manner. It requires performing spectral analysis and extreme analysis of the loads. In addition, linear or nonlinear structural analysis can be used to develop a stress frequency distribution. Then, stochastic load combinations can be performed. Linear or nonlinear structural analysis can then be used to obtain deformation and stress values. Serviceability and strength failure modes need to be considered at different levels of the ship, that is, hull girder, grillage, panel, plate and detail. The appropriate loads, strength variables, and failure definitions need to be selected for each failure mode. Using reliability assessment methods such as FORM, reliability indices βs for all modes at all levels need to be computed and compared with target reliability indices β'_0s. Equation 14 gives the relationship between the reliability index β and the probability of failure.

19.4.3 Load and Resistance Factor Design

The second approach (LRFD) of reliability-based design consists of the requirement that a factored (reduced) strength of a structural component is larger than a linear combination of factored (magnified) load effects as given by the following general format:

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_{ni} \qquad [21]$$

where ϕ = strength factor, R_n = nominal (or design) strength, γ_i = load factor for the *i*th load component out of *n* components, and L_{ni} = nominal (or design) value for the ith load component out of m components.

In this approach, load effects are increased, and strength is reduced, by multiplying the corresponding characteristic (nominal) values with factors, which are called strength (resistance) and load factors, respectively, or partial safety factors (PSFs). The characteristic or nominal value of some quantity is the value that is used in current design practice, and it is usually equal to a certain percentile of the probability distribution of that quantity. The load and strength factors are different for each type of load and strength. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with strength, the lower the corresponding strength factor. These factors are determined probabilistically so that they correspond to a prescribed level of reliability or safety. It is also common to consider two classes of performance function that correspond to strength and serviceability requirements.

The difference between the allowable stress design (ASD) and the LRFD format is that the latter uses different safety factors for each type of load and strength. This allows for taking into consideration uncertainties in load and strength, and to scale their characteristic values accordingly in the design equation. ASD (or called working stress) formats cannot do that because they use only one safety factor as seen by the following general design format:

$$\frac{R}{FS} \geq \sum_{i=1}^{m} L_i \qquad [22]$$

where R = strength or resistance, L_i = load effect, and FS = factor of safety. In this design format, all loads are assumed to have average variability. The entire variability of the strength and the loads is placed on the strength side of the equation. The factor of safety FS accounts for this entire variability.

In the LRFD design format, ship designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods (that is, ADS) without explicitly performing probabilistic analysis. The LRFD format as described in this chapter is concerned mainly with the structural design of ship hull components under combinations of different effects of environmental loads acting on a ship. As was noted earlier, these loads are considered primary loads acting on the hull girder of a ship, and in most cases they control the design of various structural elements. They include load effects due to still water, waves, and dynamic vertical bending moments on the hull girder (see Figure 19.1). Other load effects such as horizontal bending moments, static (dead), live, cargo, and their combinations with the primary environmental loads can also be incorporated in an LRFD design format. The intention herein is to provide naval architects and ship designers with sample reliability-based LRFD methods for their use in both early and final design stages and for checking the adequacy of the scantlings of all structural members contributing to the longitudinal and transverse strength of ships. Equation 21 gives the general form of the LRFD format used in this chapter.

EXAMPLE 19.4

Given:

Suppose that the simply supported beam of Figure 19.7 has a rectangular cross sectional area as shown in Figure 19.8 below. If this beam is subjected to nominal dead (including beam weight) and live uniform loads of intensity 0.5 and 0.76 kN per centimeter (kN/cm), respectively, design the web depth d_w using, the LRFD design format developed in Example 19.2, and the ASD (working stress design) given by Equation 22 with a factor of safety equals to 2.

Assume that the length L of the beam is 5.5 m, and the yield strength of the steel is 248 MPa.

Solution:

LRFD Design According to LRFD design philosophy, the ultimate capacity of the beam is the fully plastic flexural capacity $F_y Z$.

Assume that the plastic neutral axis is at the base of the flange, therefore,

$$38.1(d_w) = 254(50.8) = 12\,903 \text{ mm}^2$$

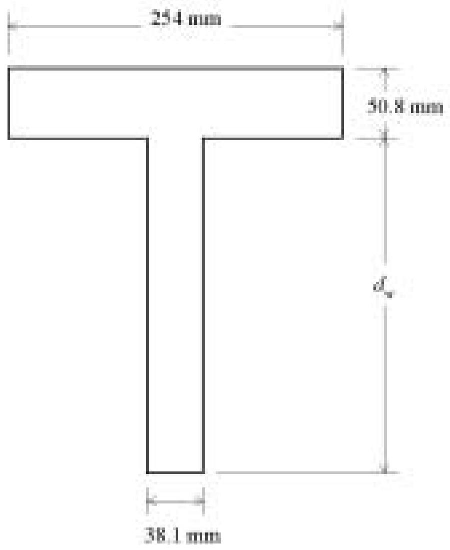


Figure 19.8 Cross Section of Simply Supported Beam for *Example 19.4*

or

$$d_w = 338.7 \text{ mm}$$

The section modulus can be computed as follows:

$$\begin{aligned} Z &= 254(50.8)(25.4) + 38.1(338.7)\left(\frac{338.7}{2}\right) \\ &= 2.51 \times 10^6 \ \text{mm}^3 \end{aligned}$$

$$M_n = F_y Z = 248 \left(2.51 \times 10^{-3}\right) = 623 \ \text{kN} - \text{m}$$

The maximum moment for a simply supported beam is located at the mid span of the beam. Therefore, the maximum moments due to the dead and live loads are calculated as follows:

$$M_D = \frac{w_D L^2}{8} = \frac{0.5(5.5)^2}{8} \times 100 = 189 \ \text{kN} - \text{m}$$

$$M_L = \frac{w_L L^2}{8} = \frac{0.76(5.5)^2}{8} \times 100 = 287 \ \text{kN} - \text{m}$$

Based on the partial safety factors of the design equation of Example 19.2, the reduced strength is

$$0.93M_n = 0.93(623) = 579.4\text{kN} - \text{m}$$

and the amplified load is

$$\begin{aligned} 1.11M_D + 1.24M_L &= 1.11(189) + 1.24\ (287) \\ &= 566\text{KN} - \text{m} \end{aligned}$$

$$\therefore\ (0.93M_n = 579) > 566 \ \textbf{acceptable}$$

Therefore,

$$\text{Select } d_w = 338.7.5 \text{ mm}$$

ASD Design In this design approach, the moment capacity of the beam is based on elastic strength of the beam. The elastic moment capacity of the beam is given by

$$M_y = F_y S$$

where *S* = elastic section modulus. In order to find *S*, we have to perform elastic calculations:

Assume that d_w = 340 mm., therefore,

$$\text{Area} = (254)(50.4) + (38.1)(340) = 25,756 \text{ mm}^2$$

$$\bar{y} = \frac{38.1(340)\left(\frac{340}{2}\right) + 254(50.8)(365.4)}{25,756} = 268.6 \ \text{mm}$$

from tip of web.

$$\begin{aligned} I &= \frac{38.1(268.6)^3}{3} + \frac{38.1(71.4)^3}{3} + \frac{254(50.8)^3}{12} \\ &\quad + (254)(50.8)(96.8)^2 = 374.4 \times 10^6 \ \text{mm}^4 \end{aligned}$$

$$S = \frac{I}{c} = \frac{374.4 \times 10^6}{268.6} = 1.394 \times 10^6 \ \text{mm}^3$$

$$M_y = F_y S = 248 \left(1.394 \times 10^{-3}\right) = 345.7 \ \text{kN} - \text{m}$$

According to ASD design format of Equation 22,

$$\frac{M_y}{FS} \geq (M_D + M_L = 476 \ \text{kN} - \text{m})$$

$$\frac{M_y}{FS} = \frac{345.7}{2} = 172.9 \ \text{kN} - \text{m}$$

$$172.9 < 476 \ \textbf{unacceptable}$$

Try now d_w = 619 mm, hence

$$\text{Area} = (254)(50.8) + (38.1)(619) = 36,525 \text{ mm}^2$$

$$\begin{aligned} \bar{y} &= \frac{38.1(620)\left(\frac{620}{2}\right) + 254(50.8)(645.4)}{36,525} \\ &= 428.5 \ \text{mm} \end{aligned}$$

from tip of web.

$$\begin{aligned} I &= \frac{38.1(428.5)^3}{3} + \frac{38.1(191.5)^3}{3} \\ &\quad + \frac{254(50.8)^3}{12} + (254)(50.8)(216.9)^2 \\ &= 1.698 \times 10^9 \text{ mm}^4 \end{aligned}$$

$$S = \frac{I}{c} = \frac{1.698 \times 10^9}{428.5} = 3.963 \times 10^6 \ \text{mm}^3$$

$$M_y = F_y S = 248 \left(3.963 \times 10^{-3}\right) = 982.7 \ \text{kN} - \text{m}$$

According to the ASD design format of Equation 22,

$$\frac{M_y}{FS} \geq (M_D + M_L = 476 \ \text{kN} - \text{m})$$

$$\frac{M_y}{FS} = \frac{982.7}{2} = 491.4 \ \text{kN} - \text{m}$$

$$491.4 > 476 \ \textbf{acceptable}$$

Therefore,

$$\text{Select } d_w = 619 \text{ mm.}$$

19.5 LRFD-BASED DESIGN CRITERIA FOR SHIP STRUCTURES

The design of ship structural elements is controlled by the relevant agencies and classifications societies that set up the rules and specifications. Even if ship structural design is not controlled by these specifications, the designer will probably refer to them as a guide. Ship design specifications, which are developed over the years by various organizations and classifications societies, present the best opinion of those organizations as to what represents good practice. The main objective of ship structural design is to insure safety, functional, and performance requirements of the components and the overall system of a ship. Traditionally, the so-called deterministic methods such as the allowable stress design, ASD, (also called working stress design, WSD) have been the primary methods for ship design and analyses. Because it is difficult in these methods to quantify and address uncertainties in a rational manner, and also to provide consistent levels of reliability among various structural components, there has been an increased interest in reliability-based design and analyses for ship structures. As was mentioned earlier, numerous efforts have been made to implement the theory or at least develop the basis for the analyses of some aspects of the design. This chapter is part of these efforts to provide the reader with sample reliability-based load and resistance factor design (LRFD) guidelines for surface ships.

Like any other design methods, reliability-based LRFD approach requires identifying the loads and their combinations, selecting a strength model, and the associated modes of failure of the structural component being analyzed or designed. This section provides, for demonstration purposes, the needed ingredients for the design and analysis of ship structural components through the use of partial safety factors in reliability-based LRFD formats similar to equation 21. One of the advantages of the LRFD is that it does not require performing probabilistic analysis. Ship designers can use the load and resistance factors (or called partial safety factors) in the limit-state equations to account for the uncertainties that might be considered properly by deterministic methods without explicitly performing reliability analyses.

19.5.1 Design Criteria and Modes of Failure

Ship structural steel elements, like any other structural elements found in land-based structures, can fail in different modes of failure depending on the type of the element and the type of loading exerted on the that element. Failure can occur when a member or component of a structure ceases to perform the function it was designed for. Fracture is a common and important type of failure, however every failure is not due to fracture. Some failures can occur before inelastic behavior or permanent deformation of the structural component is reached. For example, it is possible for a structural component to cease to perform its function due to excessive elastic deformation. Therefore, it should be realized that failure of a member or component must be defined with reference to the function of the member or component, and not necessarily to its degree of fracture (18). Some of the more common modes of failures are summarized in Table 19.VII. A well-written design code for ship structures, whether it adopts the traditional deterministic approach for design or reliability-based LRFD format, must consider all of these failure modes in its provisions. However, it is recognized that no matter how the code or the specification are written, it is impossible to cover every possible case.

As a result, the ultimate responsibility for the design of a safe structure lies with the structural engineer.

To insure public safety and proper functioning of the structural components, modern reliability-based LRFD codes such as of the AISC (4), AASHTO (19), and API (20) usually incorporate some of these failures modes in their provisions. As was mentioned earlier, the load and resistance factor design, or LRFD, is based on a limit states phi-

TABLE 19.VII Modes of Failures for a Structural Component (18)

Type of Failure	*Description*
Fracture	For brittle material, failure by fracture is usually sudden and complete in nature and likely to be initiated with crack in or near an area of high stress concentration. For ductile material such as steel, failure usually occurs as a result of excessive inelastic behavior (or called collapse mechanism), which leads to very large deformation long before fracture.
General Yielding	This type of failure applies to ductile material. When an element fails by general yielding, it loses its ability to support the load.
Buckling	Buckling is considered as structural stability problem. This type is the cause of failure for many structural elements that are long and cylindrical in nature. Failure by buckling can occur when a member or structure becomes unstable.
Fatigue	This type of failure is referred to as fatigue failure. It is a fracture type of failure that can be caused by repeated loading on the element or structural detail of high stress concentration, and for thousands or millions of load cycles. Usually this type failure is initiated by a crack within the element.

losophy. The limit state describes the condition at which the structural system (element) or some part of the system ceases to perform its intended function. These limit states can be classified into two categories,

1. strength limit states, and
2. serviceability limit states.

Strength limit states are based on safety consideration or ultimate load-carrying capacity of a structure and they include plastic strengths, buckling, and permanent deformation. Serviceability limit states, on the other hand, refer to the performance of a structure under normal service loads and they are concerned with the uses and functioning of the structure. They include such terms as excessive deflections, first yield, slipping, vibration, and cracking (6). Also, strength limit states require the definition of the lifetime extreme loads and their combinations, whereas serviceability limit states require annual-extreme loads and their combinations.

The LRFD specifications usually focus on very specific requirements pertaining to strength limit states and allows the engineer or designer some freedom or judgment on serviceability issues. This, off course, does not mean that the serviceability limit state is not significant; rather the life and safety of the public are considered to be the most important items (6). The modes of failure for ship structural components have serious consequences such as the entire loss of ship, loss of lives, and environmental damages (that is, water pollution in case of tankers of chemical carriers). Accordingly, only strength limit states that take into account the ultimate capacity of ship structural element are considered in this chapter for demonstration purposes. In fact, most of the strength models for ship structural elements as provided in the subsequent sections are based on the ultimate strength capacity of the member, and therefore, strength limit states are used.

19.5.2 Design Loads and Load Combinations

Load determination in a random sea environment, in which a ship operates, can be a challenge to ship designers. Adequate load determination is crucial to any ship structural design effort, and must be given a great deal of considerations. When using any design code, the structural designer should be aware of any simplifying assumptions made in load calculations in order to permit recognition of those instances in which these simple models do not apply. Because of the large variety of loads that may act on a single structural member, it is sometimes important to define the conditions under which these loads occur and the frequency of their occurrences.

Loads of ship structures are categorized into two primary types (9),

1. loads due to a natural environment, and
2. loads due to a man-made environment.

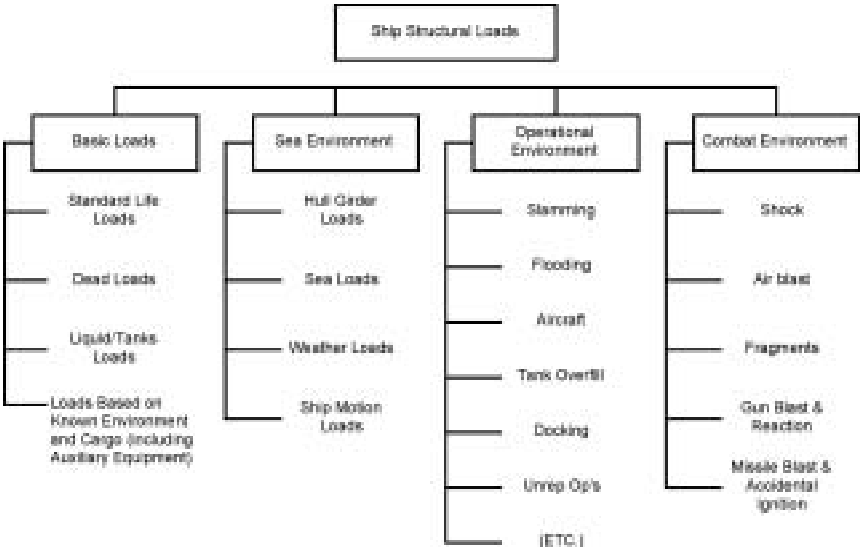


Figure 19.9 Hull Structural Load Categories

The main groups of loads for ship structures and their categories are shown in Figures 19.9. These loads are further subdivided into four main types,

1. basic loads,
2. loads due to the sea environment,
3. operational, environmental, and rare loads, and
4. loads due to combat environment.

The basic and sea-environment loads can be considered in load combinations; whereas operational and combat loads are beyond the scope of the LRFD methods presented in this chapter, and should be treated individually.

Basic or gravity loads are applied to all ship structural elements regardless of environmental influences and operational conditions. These loads include, for example, dead and live loads, liquid loads in tanks, and equipment loads. Live standard loads represent cargo, personnel, and minor equipment. Table 19.VIII provides an example distribution, intensities, and the applications of this type of load.

Liquid/Tank loads are the loads that are due to the hydrostatic force caused by the head of liquid inside tanks (such as ballast, fuel, cargo, and fresh water).

The loads acting on the ship's hull girder can be categorized into three main types

1. stillwater loads,
2. wave loads, and
3. dynamic loads.

The load effect of concern herein is the vertical bending moment exerted on the ship hull girder.

TABLE 19.VIII Example Standard Live Load Distribution (17,22)

Type of Compartment	*Live Loading (kPa)*
Living and control space, offices and passages, main deck and above	3.6
Living spaces below main deck	4.8
Offices and control spaces below main deck	7.2
Shop spaces	9.6
Storeroom/Magazines	14.4[a]
Weather portions of main deck and O1 level	12.0[b]

a. Or stowage weight, whichever is greater.

b. Or maximum vehicle operating load (including helicopter operational loads), whichever is greater.

Stillwater loads can be predicted and evaluated with a proper consideration of variability in weight distribution along the ship length, variability in its cargo loading conditions, and buoyancy. Both wave loads and dynamic loads are related and affected by many factors such as ship characteristics, speed, heading of ship at sea, and sea state (waves heights). Waves height is a random variable that requires statistical and extreme analyses of ship response data collected over a period of time in order to estimate maximum wave-induced and dynamic bending moments that the ship might encounter during its life. The statistical representation of sea waves allows the use of statistical models to predict the maximum wave loads in ship's life.

Procedures for computing design wave loads for a ship's hull girder based on spectral analysis can be found in numerous references pertaining to ship structures such as Hughes (5), Sikora et al (23), and Ayyub et al. (9).

19.5.2.1 Design loads

The design load effects that are of concern in this chapter and used for developing reliability-based design ship structural elements are those load effects resulting from ship hull girder vertical bending and their combinations. As indicated earlier, the loads acting on the ship's hull girder can be categorized into three main types: still water loads, wave loads, and dynamic loads.

The calm water or still water loading should be investigated in design processes although it rarely governs the design of a ship on its own. The ship is balanced on the draft load waterline with the longitudinal center of gravity aligned with the longitudinal center of buoyancy in the same vertical plan. Then, the hull girder loads are developed based on the differences between the weights and the buoyancy distributions along the ship's length. The net load generates shear and bending moments on the hull girders. The resulting values from this procedure are to be considered the design (nominal) values in the LRFD format for the still water shear forces and bending moments on the hull girder.

Wave-induced bending moment is treated as a random variable dependent on ship's principal characteristics, environmental influences, and operational conditions. Spectral and extreme analyses can be used to determine the extreme values and the load spectra of this load type during the design life of the ship. The outcome of this analysis can be in the form of vertical or horizontal longitudinal bending moments or stresses on the hull girder. Computer programs have been developed to perform these calculations for different ships based on their types, sizes, and operational conditions (23).

Spectral and extreme analyses can be used to determine the design value of the dynamic and combined wave-induced and dynamic bending moments on a ship hull girder during its design life (23).

19.5.2.2 Load combinations and ratios

Reliability-based LRFD formats for ship structural elements presented in this chapter is based on two load combinations that are associated with correlation factors as presented in the subsequent sections (24).

The load effect on a ship hull girder or any structural element such as unstiffened or stiffened panel due to combinations of still water and vertical wave-induced bending moments is given by

$$f_c = f_{SW} + k_{WD}f_{WD} \qquad [23]$$

where f_{SW} = stress due to still water bending moment, f_{WD} = stress due to wave-induced bending moment, f_c = unfactored combined stress, k_W = correlation factor for wave-induced bending moment and can be set equal to one (24).

The load effect on ship structural element due to combinations of still water, vertical wave-induced and dynamic bending moments is given by

$$f_c = f_{SW} + k_W(f_W + k_Df_D) \qquad [24]$$

where f_W = stress due to waves bending moment, f_D = stress due to dynamic bending moment, and k_D = correlation factor between wave-induced and dynamic bending moments. The correlation factor k_D is given by the following two cases of hogging and sagging conditions (7, 22,24):

Hogging Condition:

$$k_D = \text{Exp}\left[\frac{53080}{\left(158\,\text{LBP}^{-0.2} + 14.2\,\text{LBP}^{0.3}\right)\text{LBP}}\right] \qquad [25]$$

TABLE 19.IX Correlation Coefficient of Whipping Bending Moment (k_D) for *LBP* between 90 and 305 m (7, 24)

Length of Ship, LBP (meters)	$k_{D(sag)}$	$k_{D(hog)}$
27.9	0.578	0.254
37.2	0.672	0.369
46.5	0.734	0.461
55.8	0.778	0.533
65.0	0.810	0.591
74.4	0.835	0.637
83.6	0.854	0.675
92.9	0.870	0.706

Sagging Condition:

$$k_D = \text{Exp}\left[\frac{21200}{\left(158\,\text{LBP}^{-0.2} + 14.2\,\text{LBP}^{0.3}\right)\text{LBP}}\right] \qquad [26]$$

where LBP = length between perpendiculars for a ship in feet. Values of k_D for LBP ranging from 90 to 305 m can be obtained either from Table 19.IX or from the graphical chart provided in Figure 19.10.

19.5.3 Limit States and Design Strength

The design of ship structural component for all stations along the length of a ship should meet one of the following conditions; the selection of the appropriate equation depends on the availability of information as required by these two limit state equations:

Limit State I

$$\phi R_u \geq \gamma_{SW}f_{SW} + \gamma_{WD}k_{WD}f_{WD} \qquad [27]$$

Limit State II

$$\phi R_u \geq \gamma_{SW}f_{SW} + k_W(\gamma_W f_W + \gamma_D k_D f_D) \qquad [28]$$

where R_u = ultimate strength capacity of ship structural component (that is, force, stress, moment, etc.), ϕ = strength reduction factors for ultimate strength capacity of the structural component being analyzed, γ_{SW} = load factor for the load due to still water bending moment, f_{SW} = load effect due to still water bending moment, k_{WD} = combined wave-induced and dynamic bending moment factor, γ_{WD} = load

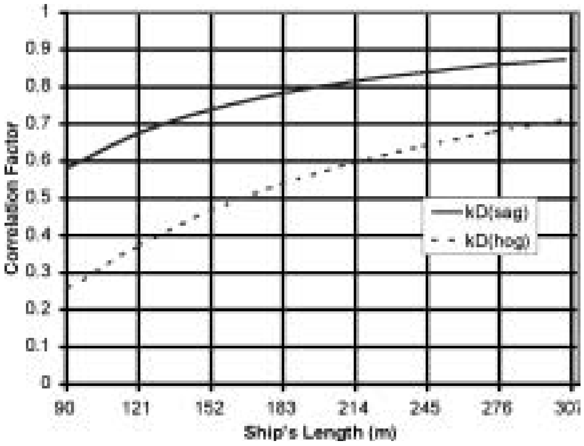


Figure 19.10 Correlation Coefficient of Whipping Bending Moment (k_D) for 90 < LBP < 305 m (7, 24)

factor for the stress due combined wave-induced and dynamic bending moment, f_{WD} = load effect due to combined wave-induced and dynamic bending moments, k_W = load combination factor, can be taken as 1.0, γ_W = load factor for the load effect due waves bending moment, f_W = load effect due to waves bending moment, k_D = load combination factor, can be taken as 0.7 or obtained from Figure 19.10 and Table 19.IX, γ_{WD} = load factor for the load effect due to dynamic bending moment, and f_D = load effect due to dynamic bending moment.

For cases of unstiffened panels where the limit state is formulated to take into account various combinations of uniaxial, biaxial, edge shear, and lateral pressure load effects, the design of these panels for all stations along the length of a ship should meet one of the following conditions:

$$\left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right)^2 - \eta_b \left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)\left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right) \le 1 \qquad [29]$$

$$\left(\frac{f_{2x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{2y}}{\phi_{R_{uy}} R_{uy}}\right)^2 - \eta_b \left(\frac{f_{2x}}{\phi_{R_{ux}} R_{ux}}\right)\left(\frac{f_{2y}}{\phi_{R_{uy}} R_{uy}}\right) \le 1 \qquad [30]$$

$$\left(\frac{f_{1x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{1y}}{\phi_{R_{uy}} R_{uy}}\right)^2 + \left(\frac{f_{1\tau}}{\phi_{R_{u\tau}} R_{u\tau}}\right)^2 \le 1 \qquad [31]$$

$$\left(\frac{f_{2x}}{\phi_{R_{ux}} R_{ux}}\right)^2 + \left(\frac{f_{2y}}{\phi_{R_{uy}} R_{uy}}\right)^2 + \left(\frac{f_{2\tau}}{\phi_{R_{u\tau}} R_{u\tau}}\right)^2 \le 1 \qquad [32]$$

where R_{ux}, and R_{uy} = ultimate strength capacity of a plate that depends on the loading conditions (that is, uniaxial stress, edge shear, etc.) for the unstiffened plate element, and $\phi_{R_{ux}}$ and $\phi_{R_{uy}}$ = strength reduction factors correspond to the ultimate strength capacity R_{ux} and R_{uy}, respectively, $\phi_{R_{u\tau}}$ = strength reduction factor for plates in shear, $R_{u\tau}$ = ultimate load capacity of plate in shear, f_{1x} = magnification of the applied stress in the x-direction for limit state I, f_{2x} = magnification of the applied stress in the x-direction for limit state II, f_{1y} = magnification of the applied stress in the y-direction for limit state I, f_{2y} = magnification of the applied stress in the y-direction for limit state II, $f_{1\tau}$ = magnification of the applied stress in the τ-direction for limit state I, $f_{2\tau}$ = magnification of the applied stress in the τ-direction for limit state II, and

$$\eta_b = \begin{cases} 0.25 & \alpha \ge 3.0 \\ 0.25 - \left(\dfrac{\alpha - 3}{2}\right)\left[3.2\,e^{-0.35B} - 2.25\right] & 1.0 < \alpha < 3.0 \\ 3.2\,e^{-0.35B} - 2 & \alpha = 1.0 \end{cases} \qquad [33]$$

in which α = aspect ratio of plate (a/b), and B = plate slenderness ratio. The magnified stresses f_{1x} , f_{2x}, f_{1y}, f_{2y}, $f_{1\tau}$, and $f_{2\tau}$ can be determined according to the following equations:

$$f_{1x} = \gamma_{SW} f_{SWx} + k_{WD}\gamma_{WD} f_{WDx} \qquad [34]$$

$$f_{2x} = \gamma_{SW} f_{SWx} + k_W(\gamma_W f_{Wx} + k_D\gamma_D f_{Dx}) \qquad [35]$$

$$f_{1y} = \gamma_{SW} f_{SWy} + k_{WD}\gamma_{WD} f_{WDy} \qquad [36]$$

$$f_{2y} = \gamma_{SW} f_{SWy} + k_W(\gamma_W f_{Wy} + k_D\gamma_D f_{Dy}) \qquad [37]$$

$$f_{1\tau} = \gamma_{SW} f_{SW\tau} + k_{WD}\gamma_{WD} f_{WD\tau} \qquad [38]$$

$$f_{2\tau} = \gamma_{SW} f_{SW\tau} + k_W(\gamma_W f_{W\tau} + k_D\gamma_D f_{D\tau}) \qquad [39]$$

The nominal (that is, design) values of the strength and load components should satisfy these formats in order to achieve specified target reliability levels. The nominal strength for various structural components of a ship can be determined as described in the subsequent sections. It is to be noted that these strength models are provided herein in a concise manner without the detailed background of their bases. The interested reader should consult (9,20,22,26).

19.5.3.1 Design strength for unstiffened panels

An unstiffened panel of ship structures is basically a plate element as shown in Figure 19.4. The design strength of unstiffened panels (plates) can be computed using formulas that correspond appropriately to their loading conditions. This section provides a summary of these formulas. They must be used appropriately based on the loading conditions of the plate between stiffeners. Both serviceability and strength limit states are provided herein although only the strength limit states were considered in the paper for computing strength reduction factors.

Uniaxial compression: The ultimate strength f_u of plates under uniaxial compression stress can be computed from one of the following two cases (27,28):

For a/b > 1.0:

$$f_u = \begin{cases} F_y\sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \ge 3.5 \\ F_y\left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & \text{if } 1.0 \le B < 3.5 \\ F_y & \text{if } B < 1.0 \end{cases} \qquad [40]$$

For a/b < 1.0:

$$f_u = F_y\left[\alpha C_u + 0.08(1-\alpha)\left(1 + \frac{1}{B^2}\right)\right]^2 \le F_y \qquad [41]$$

where

F_y = yield strength (stress) of plate
a = length or span of plate
b = distance between longitudinal stiffeners,

$B = \frac{b}{t}\sqrt{\frac{F_y}{E}}$, plate slenderness ratio
$\alpha = a / b$, aspect ratio of plate
t = thickness of the plate
E= the modulus of elasticity
ν = Poisson's ratio

and

$$C_u = \begin{cases} \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & \text{if } 1.0 \leq B < 3.5 \\ 1.0 & \text{if } B < 1.0 \end{cases} \qquad [42]$$

Edge shear The ultimate strength $f_{u\tau}$ of plates under pure edge shear stress can be computed as:

$$F_{u\tau} = F_{cr\tau} + F_{P\tau} \qquad [43]$$

where $F_{cr\tau}$ = critical or buckling stress and $F_{P\tau}$ = post-buckling strength using tension field action. The buckling strength can be computed based on one of the following three conditions that correspond to shear yield, inelastic buckling, and elastic buckling (25):

$$F_{cr\tau} = \begin{cases} F_{y\tau} & \text{if } B \leq m_1 \\ \sqrt{k_\tau \dfrac{\pi^2 F_y F_{pr}}{12(1-\nu^2)B^2}} & \text{if } m_1 < B \leq m_2 \\ k_\tau \dfrac{\pi^2 F_y}{12(1-\nu^2)B^2} & \text{if } B > m_2 \end{cases} \qquad [44]$$

where $F_{y\tau}$ = yield stress in shear, F_{pr} = proportional limit in shear which can be taken as $0.8F_{y\tau}$, and

$$m_1 = \frac{\sqrt{k_\tau \dfrac{\pi^2 F_y F_{pr}}{12\left(1-\nu^2\right)}}}{F_{y\tau}} \qquad [45]$$

$$m_2 = \sqrt{k_\tau \frac{\pi^2 F_y}{12\left(1-\nu^2\right)F_{pr}}} \qquad [46]$$

The buckling coefficient k_τ can be obtained from Figure 2 or from the following two expressions depending on whether the plate under pure shear is simply supported or clamped, respectively:

For $\alpha \geq 1.0$:

$$k_\tau = \begin{cases} 5.35 + \dfrac{4.0}{\alpha^2} & \text{for simple supports} \\ 8.98 + \dfrac{5.6}{\alpha^2} & \text{for clamped supports} \end{cases} \qquad [47]$$

For $\alpha \leq 1.0$:

$$k_\tau = \begin{cases} 4.0 + \dfrac{5.35}{\alpha^2} & \text{for simple supports} \\ 5.6 + \dfrac{8.98}{\alpha^2} & \text{for clamped supports} \end{cases} \qquad [48]$$

The yield stress in shear ($F_{y\tau}$) is given by

$$F_{y\tau} = \frac{F_y}{\sqrt{3}} \qquad [49]$$

where F_y = yield stress of plate. The post-buckling shear strength $F_{P\tau}$ is given by

$$F_{P\tau} = \frac{F_y - \sqrt{3}F_{cr\tau}}{2\sqrt{1+\alpha^2}} \qquad [50]$$

where α is the aspect ratio of plate (a/b). If the aspect ratio α exceeds 3.0, tension field action is not permitted. In this case, the ultimate shear strength of a plate shall be based on elastic and inelastic buckling theory such that:

$$F_{u\tau} = F_{cr\tau} \qquad [51]$$

where $F_{cr\tau}$ can be computed from equation 44.

Lateral pressure: The ultimate strength f_{up} of plates under lateral pressure is given as (12):

$$f_{up} = \frac{2222 F_y^2}{EB^2}\left[\left(\frac{\dfrac{w_u}{b}}{\left[0.004 + 0.02\,tanh\left(\dfrac{B}{60}\sqrt{\dfrac{E}{F_y}}\right)\right]}\right)^{\frac{1}{3}} + 1\right] \qquad [52]$$

where F_y = yield strength (stress) of plate, b = distance between longitudinal stiffeners, or plate width, B = slenderness ratio of plate, α = aspect ratio of plate, a = length or span of plate, t= thickness of the plate, E = the modulus of elasticity, and w_u = specified permanent set. Values for the ratio of the permanent set to plate width (w_u/b) or the permanent set to plate thickness (w_u/t) varies with both the ma-

TABLE 19.X Ranges of the Ratio w_u/b (17)

Aluminum or Steel Type	*Yield Strength F_y (MPa)*	*Top Side*	*Lower Shell/Tank*	*Flooding/Damage Control*
AL5086	193.0	0.000	0.000	0.009
AL5456	227.5	0.000	0.001	0.032
$\mathrm{M_S}$	234.4	0.000	0.009	0.128
HTS	324.0	0.000	0.006	0.098
HY80	552.0	0.000	0.001	0.021
HY100	690.0	0.000	0.000	0.019

TABLE 19.XI Ranges of the Ratio w_u/t (17)

Aluminum or Steel Type	*Yield Strength F_y (ksi)*	*Top Side*	*Lower Shell/Tank*	*Flooding/ Damage Control*
AL5086	193.0	0.000	0.005	0.821
AL5456	227.5	0.000	0.066	2.792
MS	234.4	0.002	0.801	11.282
HTS	324.0	0.001	0.553	8.658
HY80	552.0	0.000	0.114	1.822
HY100	690.0	0.000	0.037	1.692

terial type and the location of a plate within the ship. When using Equation. 52, these values can be obtained from Tables 19.X and XI, respectively.

Biaxial compression: The ultimate strength f_{ux} and f_{uy} of plates under biaxial compression stresses should meet the requirement of following interaction equation (12,29):

$$\left(\frac{f_x}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 - \eta_b\left(\frac{f_x}{f_{ux}}\right)\left(\frac{f_y}{f_{uy}}\right) \le 1 \qquad [53]$$

where η_b as defined by Equation 33, $\alpha = a/b$, the aspect ratio of plate, f_x = the applied stress in the x-direction, f_y = the applied stress in the y-direction, f_{ux} = the ultimate strength of a plate under compressive normal stress in the x-direction acting alone, and f_{ux} = the ultimate strength of a plate under compressive normal stress in the y-direction acting alone.

The ultimate stresses f_{ux} and f_{uy} can be computed from equations 40 and 41, respectively. It should be noted that when using equations 40 and 41 for calculating both f_{ux} and f_{uy}, the length of plate a, is assumed to coincide with the *x*-direction and the aspect ratio α is greater than unity. If, however, α is less than unity, then f_{ux} and f_{uy} should be interchanged in equations 40 and 41.

Biaxial compression and edge shear: The ultimate strength f_{ux}, f_{uy}, and $f_{u\tau}$ of plates under biaxial compression and edge shear stresses should meet the requirement of following interaction equation as adopted by the API (20) and the DnV (30):

$$\left(\frac{f_x}{f_{ux}}\right)^2 + \left(\frac{f_y}{f_{uy}}\right)^2 + \left(\frac{f_\tau}{f_{u\tau}}\right)^2 \le 1 \qquad [54]$$

where f_x = the applied stress in the *x*-direction, f_y = the applied stress in the *y*-direction, f_τ = the applied shear stress, f_{ux} = the ultimate strength of a plate under compressive normal stress in the *x*-direction acting alone, f_{ux} = the ultimate strength of a plate under compressive normal stress in the *y*-direction acting alone, and $f_{u\tau}$ = the ultimate shear stress when the plate is subjected to pure edge shear. The ultimate stresses f_{ux}, f_{uy}, $f_{u\tau}$ can be computed from equations 40, 41, and 43, respectively.

Other load combinations with lateral pressure: The loading conditions for unstiffened plates that are covered in this chapter are the combined in-plane and lateral pressure loads. Lateral pressure in combination with the other cases of loading presented in the previous sections can lead to a number of loading conditions that can have an effect on the overall strength of plates. In such situations, the designer should consider the following cases:

- lateral pressure and uniaxial compression,
- lateral pressure and biaxial compression,
- lateral pressure, uniaxial compression and edge shear,
- lateral pressure, biaxial compression and edge shear, and
- lateral pressure and edge shear.

The effect of lateral pressure on the ultimate strength of plates subjected to in-plane loads is so complex that there are no simple models (formulas) available to predict the strength of plates under these types of loading. However, there are design charts available for some of these load combinations. For example, large deflection solutions for case 4 (lateral pressure, biaxial compression, and edge shear) exits, but the results cannot be put in the form of a simple formula as those given in the previous sections. Researchers demonstrated that the lateral pressure has negligible effect on both the uniaxial and biaxial compressive strength of plates when *b*/*t* is less than 50. However, for values of the ratio *b*/*t* greater than 50, the lateral pressure can have a negative impact on the biaxial strength (case 2). Also, they pointed out that a clear understanding of the influence of pressure on strength of plates subjected to in-plane loads is lacking and that additional testing and research on the subject deemed to be appropriate to clarify some of the aspects involved. Therefore, it is recommended to treat lateral pressure as an uncoupled load from other in-plane loads, and to design for them individually and separately.

Figure 19.3. The design strength of stiffened and gross panels can be computed using formulas that correspond appropriately to their loading conditions. In this section, a summary of selected strength models that are deemed suitable for LRFD design formats is presented. These strength models are for longitudinally stiffened panels subjected to uniaxial stress and combined uniaxial stress with lateral pressure. Three strength models for stiffened panels that are deemed appropriate for reliability-based LRFD format are those of Herzog (31), Hughes (5), and Adamchak (32). Herzog's model can be applied for stiffened panel under axial stress loading, while both Hughes and Adamchak models are suitable for predicting the ultimate strength of stiffened panel when it is subjected to combined axial stress and lateral pressure. A formula for performing reliability (safety) checking on the design of gross panel, which is based on the transverse and longitudinal stiffness of stiffeners, is also provided. These strength models are presented herein in a concise manner, and they were evaluated in terms of their applicability, limitations, and biases with regard to ship structures. A complete review of the models used by different classification agencies such as the AISC (4), ASSHTO (19), and the API (20) is provided in (17,22).

Axial compression: Based on reevaluation of 215 tests by various researchers and on empirical formulation, Herzog (31) developed a simple model (formula) for the ultimate strength of stiffened panels that are subjected to uniaxial compression without lateral loads. The ultimate strength F_u of a longitudinally stiffened plate is given by the following empirical formula (31):

$$F_u = \begin{cases} m\bar{F}_y\left[0.5 + 0.5\left(1 - \dfrac{ka}{r\pi}\sqrt{\dfrac{\bar{F}_y}{E}}\right)\right] & \text{for } \dfrac{b}{t} \le 45 \\ mc_1\bar{F}_y\left[0.5 + 0.5\left(1 - \dfrac{ka}{r\pi}\sqrt{\dfrac{\bar{F}_y}{E}}\right)\right] & \text{for } \dfrac{b}{t} > 45 \end{cases} \qquad [55]$$

where

$\bar{F}_y = \dfrac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p}$, mean yield strength for the entire plate-stiffener cross section
F_{yp} = yield strength of plating
F_{ys} = yield strength of stiffener
E = modulus of elasticity of stiffened panel
A_p = bt, cross sectional area of plating
$A_s = t_f f_w + t_w d_w$, cross sectional area of stiffener
$A = A_s + A_p$, cross sectional area of plate-stiffener
t_f = stiffener flange thickness
f_w = stiffener flange width or breadth
t_w = stiffener web thickness
d_w = stiffener web depth
a = length or span of longitudinally stiffened panel
b = distance between longitudinal stiffeners
t = plate thickness
I = moment of inertia of the entire cross section
$r = \sqrt{\dfrac{I}{A}}$, radius of gyration of entire cross section
m = corrective factor accounts for initial deformation and residual stresses
k = buckling coefficient depends on the panel end constraints
$c_1 = 1 - 0.007\left(\dfrac{b}{t} - 45\right)$

Values for *m* and *k* for use in Equation 55 can be obtained from Tables 19.XII and XIII, respectively.

The 215 tests evaluated by Herzog belong to three distinct groups. Group I (75 tests) consisted of small values for imperfection and residual stress, Group II (64 tests) had average values for imperfection and residual stress, while

TABLE 19.XII Recommended *m* Values (31)

Degree of Imperfection and Residual Stress	*m*
No or average imperfection and no residual stress	1.2
Average imperfection and average residual stress	1.0
Average or large imperfection and high value for residual stress	0.8

TABLE 19.XIII Recommended *k* Values (31)

End Condition	*k*
Both ends are simply-supported	1.0
One end is simply-supported and the other is clamped	0.8
Clamped ends	0.65

TABLE 19.XIV Statistics of 215 Tests Conducted on Longitudinally Stiffened Plates in Uniaxial Compression (31)

Group	*Number of Tests*	*Mean Value* (μ)	*Standard Deviation* (σ)	*COV*
I	75	1.033	0.134	0.130
II	64	0.999	0.100	0.100
III	76	0.981	0.162	0.169
All	215	1.004	0.136	0.135

the third group (Group III, 76 tests) consisted of higher values for imperfection and residual stress. The statistical uncertainty (*COV*) associated with Herzog model of Equation 55 is 0.218. The mean value μ, standard deviation σ, and *COV* of the measurement to prediction are given in Table 19.XIV.

Axial compression and lateral pressure: According to Hughes (5), there are three types of loading that must be considered for determining the ultimate strength of longitudinally stiffened panels. These types of loading are:

1. lateral load causing negative bending moment of the plate-stiffener combination (the panel),
2. lateral load causing positive bending moment of the panel, and
3. in-plane compression resulting from hull girder bending.

The sign convention to be used throughout this section is that of Hughes (5). Bending moment in the panel is considered positive when it causes compression in the plating and tension in the stiffener flange, and in-plane loads are positive when in compression (Figure 19.11). The deflection, w_0, due to the lateral load (that is, lateral pressure) M_0 and initial eccentricity, δ_0, is considered positive when they are toward the stiffener as shown in Figure 19.11. In beam-column theory, the expressions for the moment M_0 and the corresponding deflection w_0 are based upon an ideal column, which is assumed to be simply supported.

Disregarding plate failure in tension, there can be three distinct modes of collapse (Figure 19.11) according to Hughes (5), 1) compression failure of the stiffener (Mode I Collapse), 2) compression failure of the plating (Mode II Collapse), and 3) combined failure of stiffener and plating (Mode III Collapse).

The ultimate axial strength (stress) F_u for a longitudinally stiffened panel under a combination of in-plane compression and lateral loads (including initial eccentricities) can be, therefore, defined as the minimum of the collapse (ultimate) values of applied axial stress computed from the expressions for the three types (modes) of failure. Mathematically, it can be given as

$$F_u = \min(F_{a,uI},\ F_{a,uII},\ \text{and } F_{a,uIII}) \tag{56}$$

where $F_{a,uI}$, $F_{a,uII}$, and $F_{a,uIII}$ correspond to the ultimate collapse value of the applied axial stress for Mode I, Mode II, and Mode III, respectively. The mathematical expressions for the collapse stress for each mode of failures are provided in references 5 and 24.

Adamchak (32) developed a model in 1979 to estimate the ultimate strength of conventional surface ship hulls or hull components under longitudinal bending or axial compression. The model itself is very complex for hand calculation and therefore it is not recommended for use in a design code without some computational tools or a computer program. To overcome the computational task for this model, Adamchak developed a computer program (ULTSTR) based on this model to estimate the ductile collapse strength of conventional surface ship hulls under longitudinal bending.

The recent version of the ultimate strength (ULTSTR) program is intended for preliminary design and based on a variety of empirically based strength of material solutions for the most probable ductile failure modes for stiffened and unstiffened plate structures. The probable ductile failure modes include section yielding or rupture, inter-frame

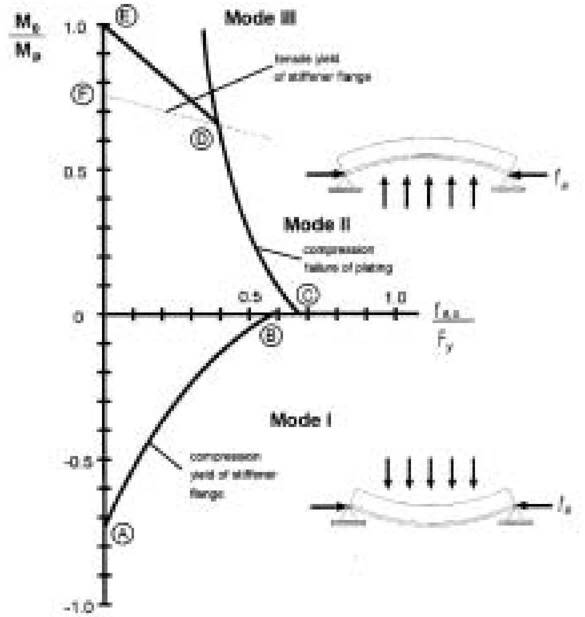
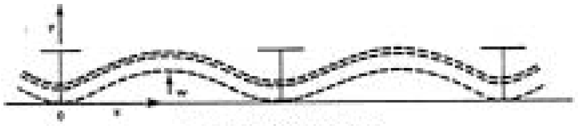
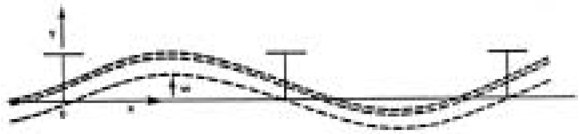


Figure 19.11 Interaction Diagram for Collapse Mechanism of a Stiffened Panel under Lateral and In-plane Loads (5)



TYPE 1 BEAM-COLUMN FAILURE



TYPE 2 BEAM-COLUMN FAILURE

Figure 19.12 Types of Beam-column Failure (2)

Euler beam-column buckling, and inter-frame stiffener tripping (lateral-torsional buckling). The program also accounts for the effects of materials having different yield strength in plating and stiffeners, for initial out-of-plane distortion due to fabrication, and for lateral pressure loading.

The basic theory behind this model (or ULTSTR) originated preliminary in a joint project on ship structural design concepts involving representatives of the Massachusetts Institute of Technology (MIT), the Ship Structural Committee (SSC), and navy practices in general. Longitudinally stiffened panel elements can fail either by material yielding, material rupture (tension only), or by some form of structural stability. The instability failure modes for this model include Euler beam-column buckling and stiffener lateral torsional buckling (tripping). Euler beam-column buckling is actually treated in this model as having two distinct types of failure patterns as shown in Figure 19.12. Type I is characterized by all lateral deformation occurring in the same direction. Although this type of failure is depended on all geometrical and material properties that define the structural element, it is basically yield strength dependent. Type I failure is assumed to occur only when either lateral pressure or initial distortion, or both, are present. On the other hand, Type II failure is modulus (*E*) depended, as far as initial buckling is concerned. This type of failure can be initiated whether or not initial distortion or lateral pressure, or both, are present. Type III failure is a stiffener tripping or lateral-torsional buckling.

Therefore, the ultimate axial strength (stress) for longitudinally stiffened panel under various types of loading (including material fabrication distortion) is the minimum value of the axial compressive stress computed from the expressions for the three types (modes) of failures, that is:

$$F_u = \min(F_{uI},\ F_{uII},\ \text{and } F_{uIII}) \qquad [57]$$

Detailed mathematical expressions for the three modes of failures as implemented in the program ULTSTR can be found in references 17 and 33.

Gross panels and grillages: To perform a a reliability (safety) checking on the design of gross panel, the reduced ratio of the stiffness of the transverse and longitudinal stiffeners should at least equal to the load effect given by the geometrical parameters shown in the second hand term of the following expression:

$$\phi_g \frac{I_y}{I_x} \ge \frac{(n+1)^5}{n\pi^2\left(0.25+\frac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5 \qquad [58]$$

where I_x = moment of inertia of longitudinal plate-stiffener, I_y= moment of inertia of transverse plate-stiffener, a = length or span of the panel between transverse webs, b = distance between longitudinal stiffeners, n = number of longitudinal stiffeners, N = number of longitudinal subpanels in overall (or gross) panel, and ϕ_g = gross panel strength reduction factor. A target reliability level can be selected based on the ship type and usage. Then, the corresponding safety factor can be looked up from Table 19.XXI.

19.5.3.3 Design strength for hull girder

The ultimate bending strength capacity for a section at any station can be estimated using the incremental strain approach by calculating the moment-curvature relationship and as the maximum resisting moment for the section. This approach calculates the moment-curvature relationship and the ultimate bending capacity of a ship's hull girder cross section using strength and geometry information about scantlings of all structural members contributing to the longitudinal strength. The ultimate strength for hull girder can be given as (13)

$$M_u = cF_uZ \qquad [59]$$

where *Z* = section modulus of the hull and *c* = is a buckling knockdown factor. The buckling knockdown factor *c* is equal to the ultimate collapse bending moment of the hull, taking buckling into consideration, divided by the initial yield moment (13).

The ultimate collapse moment can be calculated using a nonlinear finite element program such as ULTSTR or using software based on the Idealized Structural Unit Method (13). Approximate nonlinear buckling analysis may also be used. The initial yield moment is simply equal to the yield strength of the material multiplied by the section modulus of the hull at the compression flange, that is, at deck in sagging condition, or at bottom in hogging condition. The default values for the buckling knockdown factor *c* may be taken as 0.80 for mild steel and 0.60 for high-strength steel.

19.5.3.4 Fatigue strength

Assessment of ship structural capacity for fatigue and fracture was provided in greater detail in Chapter 19. This section summarizes fatigue strength in the context of structural reliability. Reliability-based LRFD design format requires the use of partial safety factors (PSFs) in the limit state equations. The PSFs are both for strength and load variables. They are commonly termed strength reduction and load amplification factors.

The structural detail or joint element of a ship should meet the following performance functions or limit state:

$$\gamma_{S_e} S_e \le \left[\frac{n_i}{\phi_A A \gamma_{k_s}^{b} k_s^{b} \phi_\Delta \Delta_L}\right]^{\frac{1}{b}} \qquad [60]$$

where

$$S_e = \sqrt[m]{\sum_{i=1}^{n_b} f_i S_i^m} \qquad [61]$$

S_e = Miner's equivalent stress range,
ϕ_Δ = reduction safety factor corresponds to fatigue damage ratio Δ_L,
ϕ_A = reduction safety factor corresponds to the intercept of the S-N curve,
γ_{k_s} = amplification safety factor for fatigue stress uncertainty, and
γ_{s_e} = amplification safety factor for Miner's rule equivalent stress range.

It is to be noted that the nominal S_e is the best estimate resulting from spectral analysis. The nominal (that is, design) values of the fatigue variables should satisfy these formats in order to achieve specified target reliability levels.

The probabilistic characteristics and nominal values for the strength and load components were determined based on statistical analysis, recommended values from other specifications, and by professional judgment. These factors are determined using structural reliability methods based on the probabilistic characteristics of the basic random variables for fatigue including statistical and modeling (or prediction) uncertainties. The factors are determined to meet target reliability levels that were selected based on assessing previous designs. This process of developing reliability-based LRFD rules based on implicit reliability levels in current practices is called code calibration.

The LRFD design for fatigue, as given by Equation 61, requires partial safety factors and nominal values. The partial safety factors (PSF's) are provided in Tables 19.XXIII and XXIV according to the following requirements:

- Target reliability levels in the range from 2.0 to 4.0,
- Fatigue strength prediction methods based on Miner's linear cumulative damage theory and on the characteristic *S-N* curve, and
- Selected details of the British standards (BS 5400).

A target reliability level should be selected based on the ship class and usage. Then, the corresponding partial safety factors can be looked up from Tables 19.XXIII and 19.XXIV based on the appropriate detail for joint for selected details. Similar tables can be developed for other details.

19.5.4 LRFD-based Partial Safety Factors for Ship Structural Components

19.5.4.1 Load factors

This section provides load factors for different categories of hull structural members. The factors can be used in the limit state equations for the design of these elements, and also for checking the adequacy of their strength capacity. The load factors are tabulated by load type and load combinations for selected target reliability levels β_0s as shown in Table 19.XVII. The ranges of target levels depend on the type of structural member under investigation. Recommended target reliability levels for various hull structural elements are provided in Table 19.XVIII.

The factors are provided for the load effect of still water *SW*, wave-induced *W*, dynamic *D*, and combined wave-induced and dynamic *WD* bending moments for target reliability levels (β_0) ranging from 3.0 to 6.0. These load factors can be used in the limit states and the load combinations presented in Section 19.5.3. The target reliability, β_0, should be selected based on the ship type and usage. Then, the corresponding load factors can be looked up from Table 19.XV for the load combination of interest.

19.5.4.2 Strength factors

This section gives strength (resistance) factors for different categories of hull structural members. The factors can be used in the limit state equations for the design of these elements, and also for checking the adequacy of their strength capacity. The strength factors can be used in the limited states as provided in Section 19.4.3 for hull girders, unstiffened, stiffened, and gross panels, respectively. Recommended target reliability levels for the design of these various hull structural components are provided in Table 19.XVI.

Tables 19.XVII through 19.XXII provide nominal strength reduction factors for the design of unstiffened, stiffened, and gross panels; and hull girders and fatigue details of ship structures. These factors can be used in the strength limit state equations as provided in Section 19.5.3.

TABLE 19.XV Nominal Load Factors

Target Reliability Index (β_o)	*Load Factors*			
	γ_{SW}	γ_W	γ_D	γ_{WD}
3.0	0.74	1.40	1.10	1.45
3.5	0.74	1.55	1.10	1.50
4.0	0.74	1.70	1.10	1.55
4.5	0.74	1.90	1.10	1.60
5.0	0.74	2.05	1.10	1.63
5.5	0.74	2.30	1.10	1.66
6.0	0.74	2.50	1.10	1.70

TABLE 19.XVI Recommended Target Reliability Levels (B_0) for Reliability-based LRFD Format

Structural System or Element	*Ranges of* β_0
Hull girder collapse	4.0–6.0
Unstiffened panel	3.0–4.0
Stiffened panel	3.5–4.5
Gross panel	2.0–3.0
Fatigue	2.0–4.0

19.6 EXAMPLES: DESIGN AND ANALYSIS

The following examples demonstrate the use of LRFD-based partial safety in the limit state equations for designing and checking the adequacy of structural components of a ship:

EXAMPLE 19.5: UNSTIFFENED PANEL DESIGN

Given:

A 122-cm × 61-cm × *t* unstiffened plate element is to be designed at the bottom deck of a ship to withstand a uniaxial compression stress due to environmental bending moment loads acting on the ship. The stresses due to the environmental loads are estimated to have the following values: 82.7 MPa due to still water bending, 33.1 MPa due to waves bending, and 12.4 MPa due to dynamic bending. If the yield strength of steel is 235 MPa, design the thickness *t* of the plate assuming target level of 3.0.

Solution:

For unstiffened panel under uniaxial compression, the strength is given by Equation 40 as

$$f_u = \begin{cases} F_y \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ F_y \left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & \text{if } 1.0 \leq B < 3.5 \\ F_y & \text{if } B < 1.0 \end{cases}$$

Assume that $t = 6.5$ mm, and the modulus of elasticity for steel is 190 GPa, therefore

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} = \frac{61}{0.65}\sqrt{\frac{235}{200,000}} = 3.22$$

and

TABLE 19.XVII Nominal Strength Factors for Unstiffened Panels

		Strength Factors (ϕ) and Target Reliability Index (β)					
		3.0		3.5		4.0	
Loading Condition	*EQ.*	ϕ	ϕ_τ	ϕ	ϕ_τ	ϕ	ϕ_τ
Uniaxial Compression	27	0.75	N/A	0.70	N/A	0.64	N/A
	28	0.83	N/A	0.79	N/A	0.79	N/A
Edge Shear	27	N/A	0.70	N/A	0.64	N/A	0.59
	28	N/A	0.77	N/A	0.73	N/A	0.68
Lateral Pressure	27	0.39	N/A	0.36	N/A	N/A	0.34
	28	0.47	N/A	0.46	N/A	0.44	N/A
Biaxial Compression	29	0.54	N/A	0.40	N/A	0.29	N/A
	30	0.61	N/A	0.51	N/A	0.42	N/A
Biaxial Compression and Edge Shear	31	0.68	0.70	0.60	0.64	0.53	0.59
	32	0.84	0.77	0.82	0.73	0.80	0.68

TABLE 19.XVIII Nominal Strength Factors for Stiffened Panels

		Strength Factors (ϕ) and Target Reliability Index (β_0)		
Loading Condition	*Limit State Equation*	*3.5*	*4.0*	*4.5*
Axial Compression	1	0.56	0.51	0.46
	2	0.61	0.57	0.54
Axial Compression and Lateral Loads	1	0.61	0.54	0.50
	2	0.66	0.61	0.58

TABLE 19.XIX Nominal Partial Safety Factor for the Stiffness Ratios of Gross Panels

Target Reliability Index (β_0)	*Gross Panel Strength Reduction Factor (ϕ_g)*
2.0	0.82
2.5	0.78
3.0	0.75

TABLE 19.XX Nominal Strength Factors for Hull Girders

Limit State Equation	*Target Reliability Index (β_0)*				
	4.0	4.5	5.0	5.5	6.0
1	0.62	0.58	0.53	0.50	0.46
2	0.70	0.67	0.63	0.62	0.58

TABLE 19.XXI Nominal Partial Safety Factors for Category B of the British Standards (BS 5400)

β_0	ϕ_Δ	ϕ_A	γ_{ks}	γ_S
2.0	0.55	0.60	1.09	1.10
2.5	0.48	0.53	1.11	1.12
3.0	0.42	0.48	1.13	1.15
3.5	0.37	0.43	1.15	1.18
4.0	0.32	0.38	1.17	1.21

TABLE 19.XXII Nominal Partial Safety Factors for Category W of the British Standards (BS 5400)

β_0	ϕ_Δ	ϕ_A	γ_{ks}	γ_S
2.0	0.52	0.57	1.07	1.08
2.5	0.45	0.50	1.09	1.10
3.0	0.39	0.45	1.11	1.12
3.5	0.34	0.40	1.13	1.15
4.0	0.29	0.35	1.14	1.17

$$f_u = F_y\left(\frac{2.25}{B} - \frac{1.25}{B^2}\right)$$

$$= 235\left(\frac{2.25}{3.22} - \frac{1.25}{(3.22)^2}\right) = 135.9\ \text{MPa}$$

The design of the plate should meet the requirement of the reliability-based LRFD format and the partial safety factors as given in Tables 19.XV and XVII for the limit state under consideration and the appropriate partial safety factors for $\beta_0 = 3.0$, that is,

$$\phi f_u = \gamma_{SW}\, f_{SW} + k_W\,(\gamma_W\, f_W + \gamma_D\, k_D\, f_D)$$
$$\phi f_u = 0.83(135.9) = 112.8\ \text{MPa}$$

$$\gamma_{SW}\, f_{SW} + k_W\,(\gamma_W\, f_W + \gamma_D\, k_D\, f_{SW})$$
$$= (1.05)(82.7) + (1)[1.4\,(33.1) + (1.1)(0.7)\,(12.4)]$$
$$= 142.7\ \text{MPa}$$

(ϕf_u = 112.8 ksi) < 142.7 MPa; this is **unacceptable**

Try a value of *t* = 10 mm., therefore

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} = \frac{61}{1.0}\sqrt{\frac{235}{200,000}} = 2.1$$

and

$$f_u = F_y\left(\frac{2.25}{B} - \frac{1.25}{B^2}\right)$$

$$= 235\left(\frac{2.25}{2.1} - \frac{1.25}{(2.1)^2}\right) = 185.2\ \text{MPa}$$

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} = \frac{61}{0.65}\sqrt{\frac{235}{200,000}} = 3.22$$

$$\phi\, f_u = 0.83(185.2) = 153.7\ \text{MPa}$$

$$\gamma_{SW}\, f_{SW} + k_W\,(\gamma_W\, f_W + \gamma_D\, k_D\, f_{SW})$$
$$= (1.05)(82.7) + (1)[1.4\,(33.1) + (1.1)\,(0.7)\,(12.4)]$$
$$= 142.7\ \text{MPa}$$

($\phi\, f_u$ = 185.2 MPa) > 142.7 MPa; this is **acceptable**

Hence, select PL: 122 × 61 × 1 cm

EXAMPLE 19.6: ADEQUACY CHECKING FOR UNSTIFFENED PANEL

Given:

Suppose that the unstiffened plate element of Example 19.5 is to be checked for the effect of lateral pressure. Would this plate be adequate to withstand the lateral pressure generated by the environmental loads?.

Solution:

For unstiffened panel under pure lateral pressure, the strength is given by Equation 52 as

$$f_{up} = \frac{2222\,F_y^2}{EB^2}\left[\left(\frac{\frac{w_u}{b}}{\left[0.004 + 0.02\,tanh\left(\frac{B}{60}\sqrt{\frac{E}{F_y}}\right)\right]}\right)^{\frac{1}{3}} + 1\right]$$

For MS Steel, and Lower Shell/Tank, Table 19.X gives

$$\frac{w_u}{b} = 0.009$$

With *B* = 2.1 as computed in Example 19.5, therefore,

$$f_{up} = \frac{2222\ (235)^2}{200,000(2.1)^2} \times \left[\left(\frac{0.009}{\left[0.004 + 0.02\, tanh\left(\frac{2.1}{60}\sqrt{\frac{200,000}{235}}\right)\right]}\right)^{\frac{1}{3}} + 1\right]$$

$$= 247 \text{ MPa}$$

The design of the plate should meet the requirement of the LRFD method and the partial safety factors as given in Tables 19.XV and XVII for the limit state under consideration and the appropriate partial safety factors for $\beta_0 = 3.0$, that is,

$$\phi f_{up} \geq \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_D)$$

$$\phi f_{up} = 0.47(247) = 116.1 \text{ MPa}$$

$$(1.05)(82.7) + (1)[1.4(33.1) + (1.1)(0.7)(12.4)] = 142.7 \text{ MPa}$$

($\phi f_u = 116.1$ MPa) < 142.7 MPa; this is **unacceptable**

Hence, the plate will not be adequate for lateral pressure. A new plate should be designed.

EXAMPLE 19.7: STIFFENED PANEL DESIGN

Given:

A stiffened panel, pinned at the ends, whose dimensions are shown in Figure 19.13 is to be designed at the bottom deck of a ship to withstand a uniaxial compression stress due to environmental bending moment loads acting on the ship. The stresses due to the environmental loads are estimated to have the following values: 1.035 MPa due to still-water bending, 31.0 MPa due to waves bending, and 15.2 MPa due to dynamic bending. If the yield strength of steel is 235 MPa for the plating and 248 MPa for the stiffener (that is, web & flange), and the dimensions of the panel are as shown in Table 19.XXIII, design the thickness t and length a of the plating assuming a target reliability level of 4.0. Note that the length of the plating is not to exceed 195 cm, and not to be less than 122 cm.

Solution

For stiffened panel under uniaxial compression without lateral pressure, the strength model as given by Equation 19.55 (Herzog) applies.

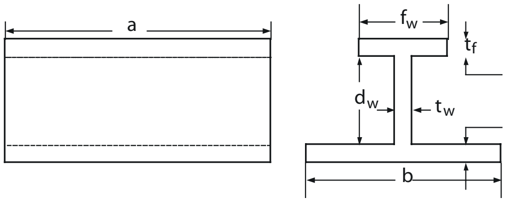


Figure 19.13 Stiffened Panel Design

$$F_u = \begin{cases} m\bar{F}_y\left[0.5 + 0.5\left(1 - \frac{ka}{r\pi}\sqrt{\frac{\bar{F}_y}{E}}\right)\right] & \text{for } \frac{b}{t} \leq 45 \\ m\bar{F}_y\left[0.5 + 0.5\left(1 - \frac{ka}{r\pi}\sqrt{\frac{\bar{F}_y}{E}}\right)\right] & \\ \times\left[1 - 0.007\left(\frac{b}{t} - 45\right)\right] & \text{for } \frac{b}{t} > 45 \end{cases}$$

Assume an initial value for $t = 0.5$ cm, and for $a = 195$ cm, hence

$$A_p = bt = 61(0.5) = 30.5 \text{ cm}^2$$

$$A_s = t_f f_w + t_w d_w = 0.95(4.5) + 0.52(11.5) = 10.26 \text{ cm}^2$$

$$\bar{F}_y = \frac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p} = \frac{248(10.26) + 235(30.5)}{10.26 + 30.5} = 338.3 \text{ MPa}$$

Check the slenderness ratio *b*/*t*:

$$\frac{b}{t} = \frac{61}{0.5} = 122 > 45$$

Therefore, the following equation applies:

$$F_u = m\bar{F}_y\left[0.5 + 0.5\left(1 - \frac{ka}{r\pi}\sqrt{\frac{\bar{F}_y}{E}}\right)\right] \times\left[1 - 0.007\left(\frac{b}{t} - 45\right)\right]$$

The radius of gyration *r* for the cross section can be found when the moment of inertia *I* has been established. To compute *I*, the location of neutral axis must be calculated:

TABLE 19.XXIII Given Dimensions of the Stiffened Panel

Variable	*Value (cm)*
Width of plating, b	61.0
Stiffener web depth, d_w	11.50
Stiffener flange breadth, f_w	4.5
Stiffener web thickness, t_w	0.52
Stiffener flange thickness, t_f	0.95

$$\begin{aligned}\bar{y} &= \frac{1}{10.26+30.5}\left[\frac{0.5}{2}(61)(0.5)\right.\\ &\quad + \left(0.5+\frac{11.5}{2}\right)(11.5)(0.52)\\ &\quad \left.+ \left(0.5+11.5+\frac{0.95}{2}\right)(0.95)(4.5)\right]\\ &= 2.41 \text{ cm from the base of the plating.}\end{aligned}$$

Therefore, $I = 717.2\text{ cm}^4$, and

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{717.2}{10.26+30.5}} = 4.2\,\text{cm}$$

Assuming m and k both equal to one (see Tables 19.XII and XIII), we have

$$\begin{aligned}F_u &= (1)(338.3)\\ &\quad\times\left[0.5+0.5\left(1-\frac{205}{(4.2)\pi}\sqrt{\frac{338.3}{200,000}}\right)\right]\\ &\quad\times\left[1-0.07\left(\frac{61}{0.5}-45\right)\right] = 106.1 \text{ MPa}\end{aligned}$$

In reference to Tables 19.XVII and XX, and for a target reliability index $\beta_0 = 4.0$ as given, the following partial safety factors are obtained for use in the design equation:

$$\phi = 0.57,\ \gamma_{SW} = 1.05,\ \gamma_W = 1.7, \text{ and } \gamma_D = 1.1$$

Therefore,

$$\phi F_u = 0.57(106.1) = 60.5 \text{ MPa}$$

$$\begin{aligned}&\gamma_{SW} f_{SW} + k_W(\gamma_W f_W + \gamma_D k_D f_D)\\ &= (1.05)(1.035) + (1)[1.7(31) + (1.1)(0.7)(15.2)]\\ &= 65.5 \text{ MPa}\end{aligned}$$

($\phi F_u = 60.5$ MPa) < 65.5 MPa; this is **unacceptable**

Now try $t = 0.65$ cm and $a = 195$ cm, hence,

$$A_p = bt = 61\,(0.65) = 39.7\text{ cm}^2$$

$$A_s = t_f f_w + t_w d_w = 10.26\text{ cm}^2$$

$$\begin{aligned}\bar{F}_y &= \frac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p} = \frac{248(10.26) + 235(39.7)}{10.26+39.7}\\ &= 337.7 \text{ MPa}\end{aligned}$$

Check the slenderness ratio b/t:

$$\frac{b}{t} = \frac{61}{0.65} = 94 > 45$$

Therefore, the following equation applies:

$$\begin{aligned}F_u &= m\bar{F}_y\left[0.5+0.5\left(1-\frac{ka}{r\pi}\sqrt{\frac{\bar{F}_y}{E}}\right)\right]\\ &\quad\times\left[1-0.007\left(\frac{b}{t}-45\right)\right]\end{aligned}$$

Again, the radius of gyration r for the cross section can be found when the moment of inertia I is established. To compute I, the location of neutral axis must be calculated:

$$\begin{aligned}\bar{y} &= \frac{1}{10.26+30.5}\times\left[\frac{0.65}{2}(61)(0.5)\right.\\ &\quad + \left(0.65+\frac{11.5}{2}\right)(11.5)(0.52)\\ &\quad \left.+ \left(0.65+11.5+\frac{0.95}{2}\right)(0.95)(4.5)\right]\\ &= 2.1 \text{ cm from the base of the plating.}\end{aligned}$$

Therefore, $I = 758.4\text{ cm}^4$, and

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{758.4}{10.26+39.7}} = 3.9\,\text{cm}$$

Assuming m and k both equal to one (see Tables XII and XIII), we have

$$\begin{aligned}F_u &= (1)(337.7)\\ &\quad\times\left[0.5+0.5\left(1-\frac{205}{(3.9)\pi}\sqrt{\frac{337.7}{200,000}}\right)\right]\\ &\quad\times\left[1-0.007\left(\frac{61}{0.65}-45\right)\right]\\ &= 145.8 \text{ MPa}\end{aligned}$$

$$\phi F_u = 0.57(145.8) = 83.1 \text{ MPa}$$

$= \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_D)$
$= (1.05)(1.035) + (1)[1.7(31) + (1.1)(0.7)(15.2)]$
$= 65.5$ MPa

($\phi F_u = 83.1$ MPa) > 65.5 MPa; this is **acceptable**

Hence, select t = 6.5 mm, and a = 195 cm

EXAMPLE 19.8: ADEQUACY CHECKING FOR GROSS PANEL

Given:

Assume a target reliability level of 2.5, check the adequacy of the following gross panel:

$I_x = 666$ cm^4
$I_y = 1103$ cm^4
$N = 5$
$n = 3$
$a = 152$ cm
$b = 61$ cm

Solution:

For gross panel, the strength is given by Equation 19.58 as

$$\phi_g \frac{I_y}{I_x} \geq \frac{(n+1)^5}{n\pi^2\left(0.25 + \dfrac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5$$

For target reliability index of 2.5, Table 19.XXI gives ϕ_g = 0.78, therefore,

$$\phi_g \frac{I_y}{I_x} = 0.78\frac{1103}{666} = 1.29$$

$$\frac{(n+1)^5}{n\pi^2\left(0.25 + \dfrac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5$$

$$= \frac{(3+1)^5}{(3\pi^2)\left(0.25 + \dfrac{2}{(5)^3}\right)}\left(\frac{61}{152}\right)$$

$$= 1.35$$

Since 1.29 < 1.35, the gross panel will be **inadequate**.

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Chapter 19 Appendix: First-Order Reliability Method

The First-Order Reliability Method (FORM) is a convenient tool to assess the reliability of a ship structural element. It also provides a means for calculating the partial safety factors ϕ and γ_i that appear in Equation 1 for a specified target reliability level β_0. The simplicity of the first-order reliability method stems from the fact that this method, beside the requirement that the distribution types must be known, requires only the first and second moments; namely the mean values and the standard deviations of the respective random variables. Knowledge of the joint probability density function (PDF) of the design basic variables is not needed as in the case of the direct integration method for calculating the reliability index β. Even if the joint PDF of the basic random variables is known, the computation of β by the direct integration method can be a very difficult task.

In design practice, there are usually two types of limit states: the ultimate limit states and the serviceability limit states. Both types can be represented by the following performance function:

$$g(X) = g(X_1, X_2, ..., X_n) \qquad [A1]$$

in which **X** is a vector of basic random variables (X_1, X_2, ..., X_n) for the strengths and the loads. The performance function g(**X**) is sometimes called the limit state function. It relates the random variables for the limit-state of interest. The limit state is defined when g(**X**) = 0, and therefore, failure occurs when g(**X**) < 0 (see Figure 19.A1). The reliability index β is defined as the shortest distance from the origin to the failure surface in the reduced coordinates at the most probable failure point (MPFP) as shown in Figure 19.A1.

As indicated in this chapter, the basic approach for developing reliability-based design guidelines and rules requires the determination of the relative reliability of designs based on current practices. Therefore, reliability assessment of existing structural components of ships such as the hull girder and its structural elements is needed to estimate a representative value of the reliability index β. The first-order-reliability method is very well suited to perform such a reliability assessment. The following are computational steps as described in [3] for determining β using the FORM method:

1. Assume a design point x_i^* and obtain $x_i'^*$ in the reduced coordinate using the following equation:

$$x_i'^* = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}} \qquad [A2]$$

where, $x_i'^* = \alpha_i^*\beta$, μ_{X_i} = mean value of the basic random variable, and σ_{X_i} = standard deviation of the basic random variable. The mean values of the basic random variables can be used as initial values for the design points. The notation x^* and x'^* are used respectively for the design point in the regular coordinates and in the reduced coordinates.

2. Evaluate the equivalent normal distributions for the nonnormal basic random variables at the design point using the following equations:

$$\mu_X^N = x^* - \Phi^{-1}\left(F_X(x^*)\right)\sigma_X^N \qquad [A3]$$

and

$$\sigma_X^N = \frac{\left(\Phi^{-1}\left(F_X(x^*)\right)\right)}{f_X(x^*)} \qquad [A4]$$

where μ_X^N = mean of the equivalent normal distribution, σ_X^N = standard deviation of the equivalent normal distribution, $F_X(x^*)$ = original (non-normal) cumulative distribution function (CDF) of X_i evaluated at the design point, $f_X(x^*)$ = original probability density function (PDF) of X_i evaluated at the design point, $\Phi(\cdot)$ = CDF of the standard normal distribution, and $\phi(\cdot)$ = PDF of the standard normal distribution.

3. Compute the directional cosines at the design point (α_i^*, i = 1,2, ..., *n*) using the following equations:

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial x_i'}\right)_*}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i'}\right)_*^2}} \text{ for } i = 1, 2, \ldots, n \qquad [A4]$$

where

$$\left(\frac{\partial g}{\partial x_i'}\right)_* = \left(\frac{\partial g}{\partial x_i}\right)_* \sigma_{X_i}^N \qquad [A5]$$

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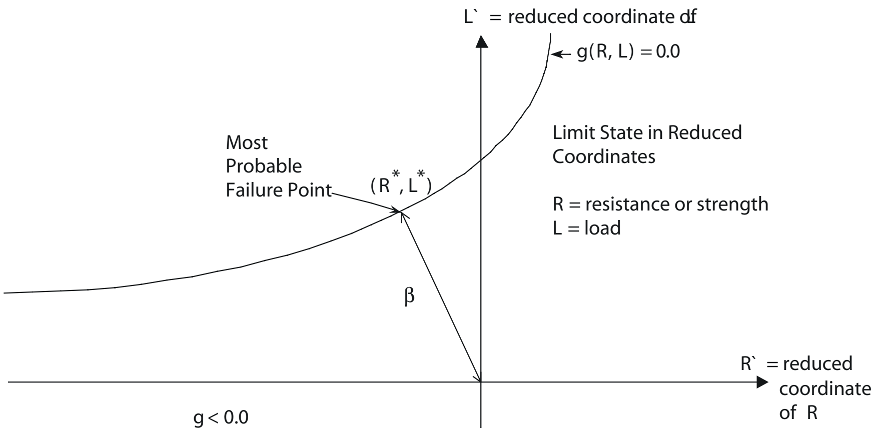

Figure 19.A1 Space of Reduced Random Variables Showing the Reliability Index and the Most Probable Failure Point

4. With α^*_i, $\mu^N_{X_i}$, and $\sigma^N_{X_i}$ now known, the following equation can be solved for the root β:

$$g\left[\left(\mu^N_{X_1} - \alpha^*_{X_1}\sigma^N_{X_1}\beta\right),\ldots,\left(\mu^N_{X_n} - \alpha^*_{X_n}\sigma^N_{X_n}\beta\right)\right] = 0 \quad [A6]$$

5. Using the β obtained from step 4, a new design point can be obtained from the following equation:

$$x^*_i = \mu^N_{X_i} - \alpha^*_i\sigma^N_{X_i}\beta \quad [A7]$$

6. Repeat steps 1 to 5 until a convergence of β is achieved. The reliability index is the shortest distance to the failure surface from the origin in the reduced coordinates as shown in Figure A1.

The important relation between the probability of failure and the reliability (safety) index is given by Equation 14.

A.1 PROCEDURE FOR CALCULATING PARTIAL SAFETY FACTORS (PSF) USING FORM

The first-order reliability method (FORM) can be used to estimate partial safety factors such those found in the design format of Equation 21. At the failure point (R^*, L^*_1, ..., L^*_n), the limit state of Equation 21 can be rewritten as

$$g = R^* - L^*_1 - \ldots - L^*_n = 0 \quad [A8]$$

or, in a general form

$$g(X) = g(x^*_1,\ x^*_2, \ldots,\ x^*_n) = 0 \quad [A9]$$

For given target reliability index β_0, probability distributions and statistics (means and standard deviations) of the load effects, and coefficient of variation of the strength, the mean value of the resistance and the partial safety factors can be determined by the iterative solution of Equations A2 through A7. The mean value of the resistance and the design point can be used to compute the required mean partial design safety factors as follows:

$$\phi = \frac{R^*}{\mu_R} \quad [A10]$$

$$\gamma_i = \frac{L^*_i}{\mu_{L_i}} \quad [A11]$$

The strength factors are generally less than one, whereas the load factors are greater than one.

A.2 DETERMINATION OF A STRENGTH FACTOR FOR A GIVEN SET OF LOAD FACTORS

In developing design code provisions for ship structural components, it is sometimes necessary to follow the current design practice to insure consistent levels of reliability over various types of ship structures. Calibrations of existing design codes is needed to make the new design formats as simple as possible and to put them in a form that is familiar to the users or designers. Moreover, the partial safety factors for the new codes should provide consistent levels of reliability. For a given reliability index β and probability characteristics for the resistance and the load effects, the partial safety factors determined by the FORM approach might be different for different failure modes for the same structural component. Therefore, the calculated partial safety factors (PSFs) need to be adjusted in order to maintain the same values for all loads at different failure modes by the strength factor ϕ for a given set of load factors. The following algorithm can be used to accomplish this objective:

- For a given value of the reliability index β, probability distributions and statistics of the load variables, and the coefficient of variation for the strength, compute the mean strength needed to achieve the target reliability using the first-order reliability method as outlined in the previous sections.
- With the mean value for R computed in step 1, the partial safety factor can be revised for a given set of load factors as follows:

$$\phi^{`} = \frac{\sum_{i=1}^{n} \gamma^{`}_{i} \mu_{L_i}}{\mu_R} \qquad [A12]$$

where = revised strength factor, and μ_R are the mean values of the loads and strength variables, respectively; and, $\gamma^{`}_{i}$ = 1, 2, ..., n, are the given set of load factors.