LECTURE

25

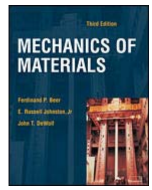
Chapter 8.4

The McGraw-Hill Companies

Third Edition

COMPONENTS: COMBINED LOADING

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

Dr. Ibrahim A. Assakkaf

SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

Combined Axial, Torsional, and Flexural Loads

- Introduction
  – In many industrial cases, structural or machine members are subjected to combinations of three general types of loads:
    - Axial
    - Torsional
    - Flexural

Combined Axial, Torsional, and Flexural Loads

- Introduction
  - Axial Loading

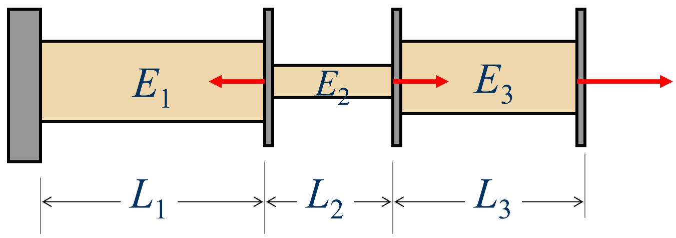


Figure 38. A Structural Member under Axial Loads

Combined Axial, Torsional, and Flexural Loads

- Introduction
  - Torsional Loading

Cylindrical members

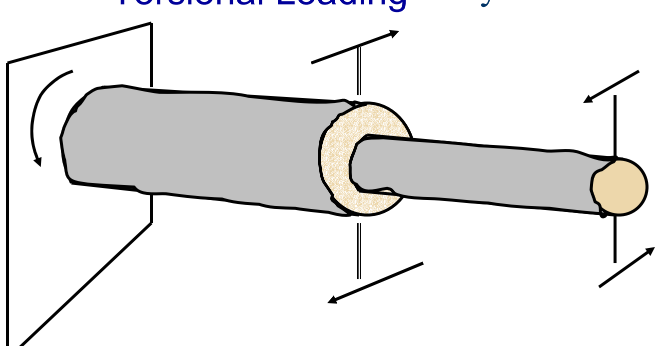


Figure 39. A Structural Member under Torsional Loads

Combined Axial, Torsional, and Flexural Loads

- Introduction
  - Flexural Loading

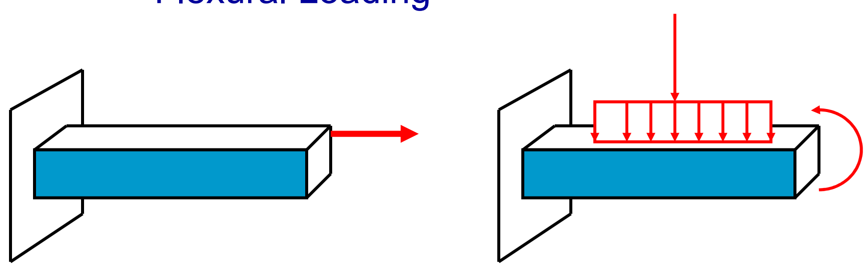


Figure 40. Beams under Combinations of Loads

Combined Axial, Torsional, and Flexural Loads

- Introduction
  - When a flexural load is combined with torsional and axial loads, it is often difficult to locate the points where most severe stresses (maximum) occur.
  - Example of structural member that is subjected to combined axial, torsional, and flexural loadings is shown in Figure 41

Combined Axial, Torsional, and Flexural Loads

- Introduction

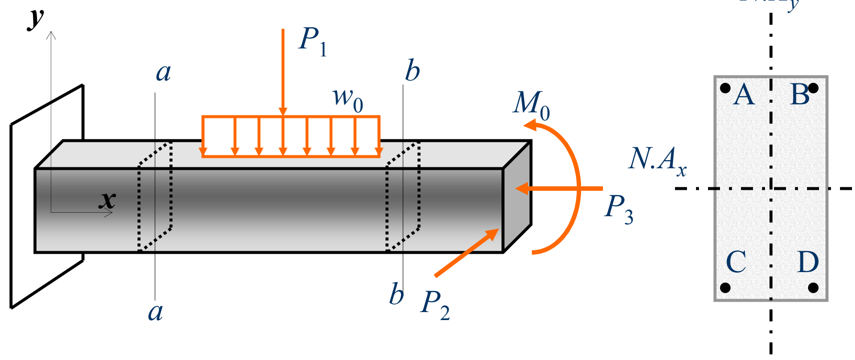


Figure 41. Beam under a Combination of Loads

Combined Axial, Torsional, and Flexural Loads

- Maximum Stress in a Member due to Combined Loads
  - When a machine or a structural element is subjected to combinations of loads, it is usually difficult to locate the point or points that gives (give) the absolute largest or smallest stress (tension or compression)
  - For example, the maximum absolute normal stress in the cross section of the beam of Fig. 41 that is subjected to the

Combined Axial, Torsional, and Flexural Loads

- Maximum Stress in a Member due to Combined Loads
  - combinations of loads shown, could be located at any point lying in the cross section.
  - Furthermore, the location of the critical section along the beam length that provides the critical stress needs to be identified.

Combined Axial, Torsional, and Flexural Loads

- Maximum Stress in a Member due to Combined Loads
  - Depending on the shear and bending moment variations (shear and moment diagrams) along the beam, the critical cross section of the beam of Fig. 41 could be Section *a-a*, Section *b-b*, or any other section located a distance *x* from the origin.

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses
  1. The longitudinal and transverse shearing stresses in a beam are maximum where *Q* is maximum; usually at the centroidal axis of the section where *V* is maximum.

$$\tau = \frac{Q_{\max} V_{\max}}{It} \qquad (47)$$

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses
  2. The flexural stress is maximum at the greatest distance (fiber) from the centroidal axis on the section where *M* is maximum

$$\sigma = \frac{M_{\max} y_{\max}}{I} \qquad (48)$$

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses

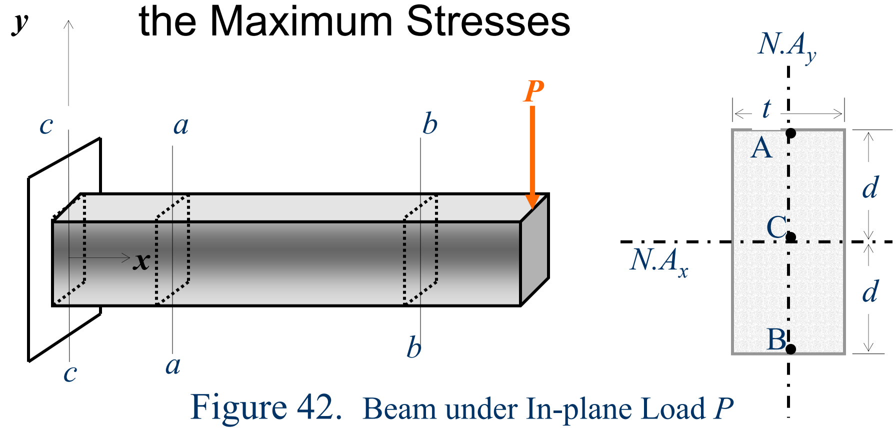


Figure 42. Beam under In-plane Load *P*

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses
  - Example A
    - The maximum *V* for the cantilever beam of Fig. 42 is at the fixed support. Therefore, the maximum shearing stress is represented by point C on the cross section at the support (Section *c*-*c*).

$$\tau = \frac{Q_{\max} V_{\max}}{It}, \quad Q_{\max} = \frac{td^2}{2}, \quad I = \frac{2td^3}{3}$$

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses
  - Example B
    - The maximum *M* for the cantilever beam of Fig. 42 is at the fixed support. Therefore, the maximum flexural normal stress is represented by points A or B on the cross section at the support (Section *c*-*c*).

$$\sigma = \frac{M_{\max} y_{\max}}{I}, \quad y_{\max} = d, \quad I = \frac{2td^3}{3}$$

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses
  3. The torsional shearing stress is maximum at the surface of a shaft at the section where *T* is maximum

$$\tau = \frac{T_{\max} \rho_{\max}}{J} = \frac{T_{\max} c}{J} \qquad (49)$$

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses

4. With reference to remarks 1, 2, and 3, it is possible to locate one or more possible point of high stress. Even so, it may be necessary to compute the various stresses at more than one point of a member before locating the most critically stressed point.

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses

5. The superposition method can be used to combine stresses on any given plane at any specific point of a loaded member provided that the stresses are below the proportional limit of the material.

Combined Axial, Torsional, and Flexural Loads

- Some Helpful Remarks for Identifying the Maximum Stresses

  6. After the stresses on a pair of mutually perpendicular planes at a specific point are determined, plane-stress transformation equations and Mohr's circle can be used to compute the principal stresses as well as the maximum shearing stress at the point.

Combined Axial, Torsional, and Flexural Loads

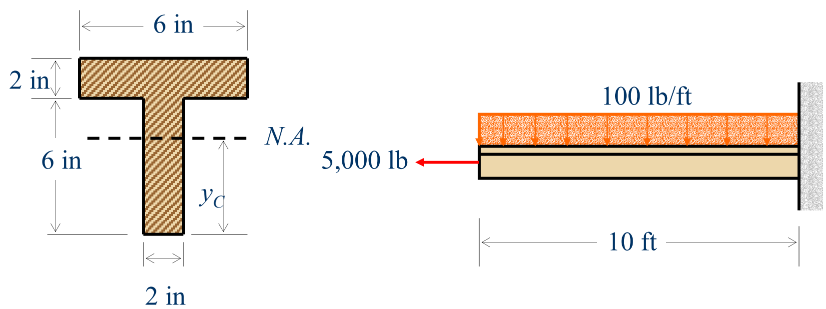
- Example 2

  The dimensions of a T-shaped beam and the loads on it are shown in Figs. 43a and 43b. The horizontal 5000-lb tensile force acts through the centroid of the cross-sectional area. Determine the maximum principal normal stress and the maximum shearing stress in the beam, assuming elastic behavior.

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)

Figure 43

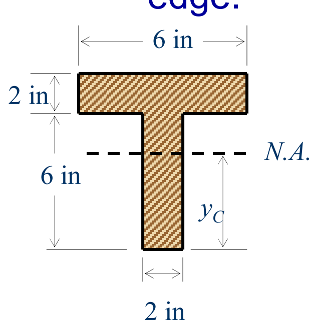


(a) (b)

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)

First, we need to locate the neutral axis. Let's make our reference from the bottom edge.



$$y_C = \frac{(3)(2\times 6)+(6+1)(2\times 6)}{2\times 6+2\times 6} = 5.0\text{ in}$$

$$y_{\text{ten}} = 8-5 = 3.0\text{ in} \qquad y_{\text{com}} = 5.0\text{ in} = y_{\max}$$

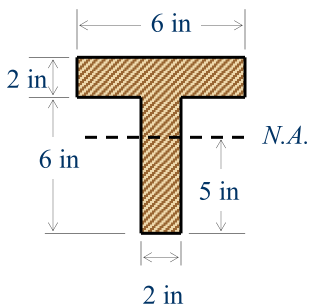
$$\text{Max. Stress} = \frac{M_r y_{\max}}{I_x}$$

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)

Next find the moment of inertia about the neutral axis:

$$I_x = \frac{2(5)^3}{3} + \frac{6(3)^3}{3} - \left[\frac{4(1)^3}{3}\right] = 136 \text{ in}^4$$



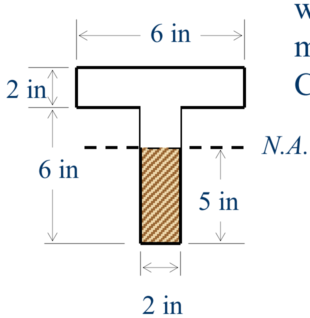
Both the maximum shearing force V and maximum bending moment M occur at the fixed support of the beam

$$V_{\max} = 10(100) = -1000 \text{ lb}$$
$$M_{\max} = 10(100)(5) = -5{,}000 \text{ ft} \cdot \text{lb}$$

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)

Maximum shearing stress:
The maximum value of Q occurs at the neutral axis. Since in this cross section the width t is minimum at the neutral axis, the maximum shearing stress will occur there. Choosing the area below neutral axis, we have



$$Q = \frac{5}{2}(2)(5) = 25 \text{ in}^3$$

$$\tau_{\max} = \frac{V_{\max} Q_{\max}}{It} = \frac{1000(25)}{136(2)} = 91.9 \text{ psi}$$

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)
  - The longitudinal normal stress on the uppermost fibers is

$$\sigma_{\text{up}} = \frac{P}{A} + \frac{M_{\max} y_{\text{up}}}{I_x} = \frac{5000}{24} + \frac{(5000 \times 12)(3)}{136}$$

$$= 208.3 + 1323.5 = 1{,}531.8 \text{ psi (T)}$$

  - While the normal stress on the lowest fibers is

$$\sigma_{\text{up}} = \frac{P}{A} - \frac{M_{\max} y_{\text{up}}}{I_x} = \frac{5000}{24} - \frac{(5000 \times 12)(5)}{136}$$

$$= 208.3 - 2{,}205.9 = -1{,}998 \text{ psi (C)}$$

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)
  - At any point between either outer surface and centroid, there will be both a normal stress ans a shear stress. However, a comparison of their orders of magnitude suggests that the normal stresses at the outside far exceed the transverse shear stresses at the centroid. Hence we can conclude that the critical fibers will be those at the outside surface, where the maximum principal normal stresses will be.

Combined Axial, Torsional, and Flexural Loads

- Example 2 (cont'd)

Therefore, the principal normal stresses are

$$\sigma_{\max} = 1531.8 \text{ psi (T)} \quad \text{and} \quad \sigma_{\max} = \boxed{1998 \text{ psi (C)}}$$

And the maximum shearing stress will occur at the lower fiber, that is

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{1998 - 0}{2} = \boxed{999 \text{ psi}}$$

and will act 45^0 to the longitudinal axis of beam.

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry
  - When the line of action of the axial load *P* passes through the centriod of the cross section, it can be assumed that the distribution of normal stress is uniform throughout the section.
  - Such a loading is said to be *centric,* as shown in Fig 44.

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

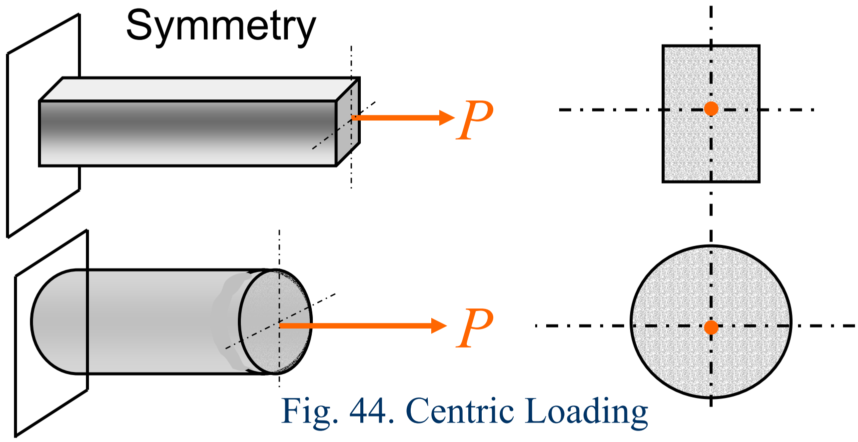


Fig. 44. Centric Loading

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry
  - When the line of action of the concentrated load *P* dose not pass through the centroid of the cross section, the distribution of normal stress is no longer uniform.
  - Such loading is said to eccentric, as shown in Fig 45.

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

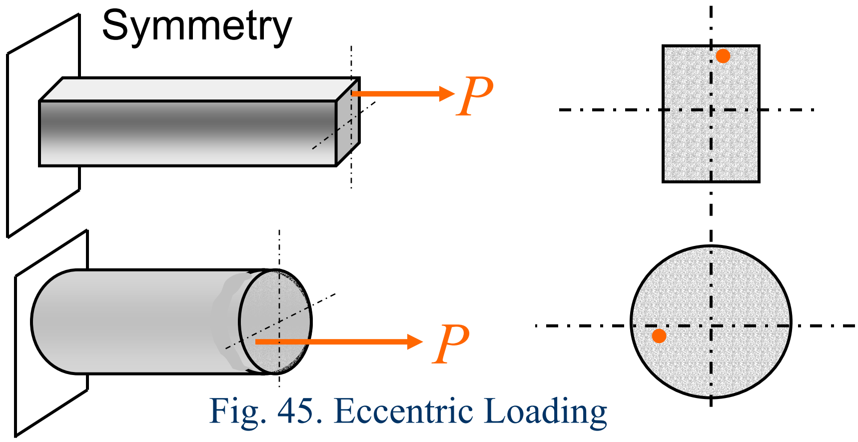


Fig. 45. Eccentric Loading

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry
  - Consider the member of Fig. 46a that subjected to an axial load *P*.
  - The internal force acting on a given cross section may then be represented by a force *F* applied at the centroid y_C of the section and a couple *M* acting in the plane of symmetry of the member (Fig. 46.b).

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

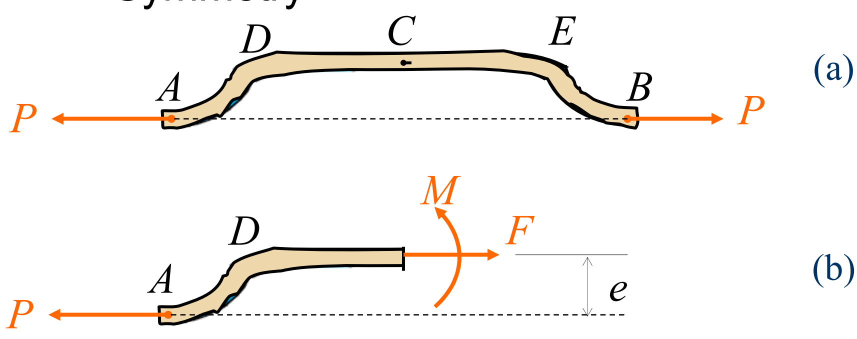


Fig. 46. Eccentric Loading

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry
  - The conditions of equilibrium of the free body *AC* require that the force *F* be equal and opposite to *P* and that the moment of the couple *M* be equal and opposite to the moment of *P* about *C*.
  - Denoting by *e* the distance from the centroid y_C to line of action *AB* of the forces

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

  we have

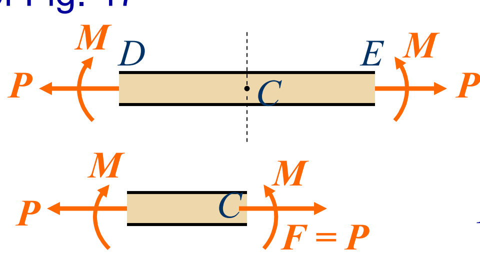
$$F = P \qquad \text{and} \qquad M = Pe \qquad (50)$$

  – The internal forces in the section would have been represented by he same force and couple if the straight portion *DE* of member *AB* had been detached from *AB* and subjected simultaneously to the

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

  centric loads *P* and the bending couple *M* of Fig. 47



$$M = Pe$$

Figure 47

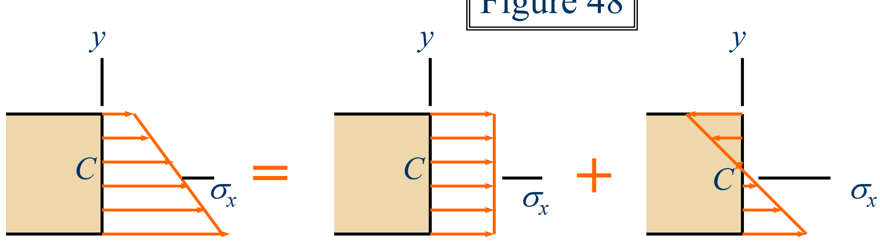
Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry
  - Thus, the stress distribution due to the original eccentric loading may be obtained by superposing the uniform stress distribution corresponding to the centric load *P* and the linear distribution corresponding to the bending moment *M,* as shown in Fig. 48.

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in a Plane of Symmetry

Figure 48



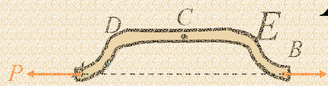
$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$

Combined Axial, Torsional, and Flexural Loads

- Eccentric Axial Loading in A Plane of Symmetry

  The stress due to eccentric loading on a beam cross section is given by

$$\sigma_x = \frac{P}{A} \pm \frac{My}{I} \qquad (51)$$



Combined Axial, Torsional, and Flexural Loads

- Equivalent Force System for Eccentric Loading
  - When there are eccentric loadings, that is loadings that their line of action does not pass through the centroid of the cross section, it is necessary to provide an equivalent-force system, consisting of *P* and *M* for use in Eq. 51.
  - Example of such system is shown in Fig 49.

Combined Axial, Torsional, and Flexural Loads

- Equivalent Force System for Eccentric Loading

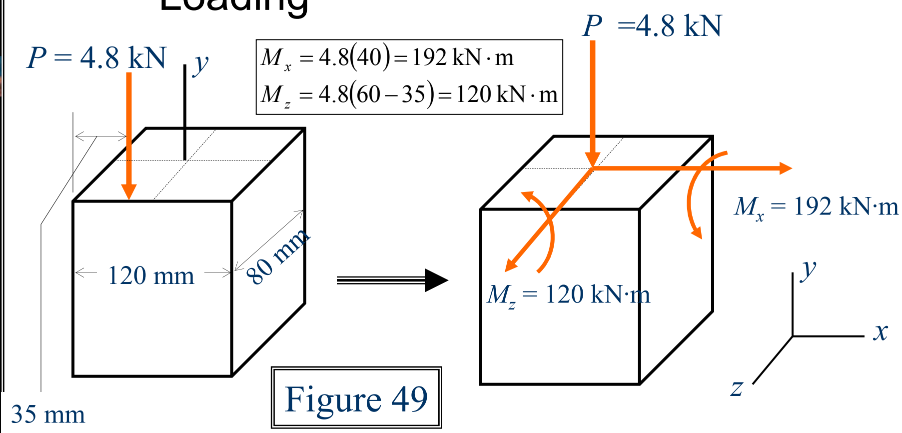


Figure 49

Combined Axial, Torsional, and Flexural Loads

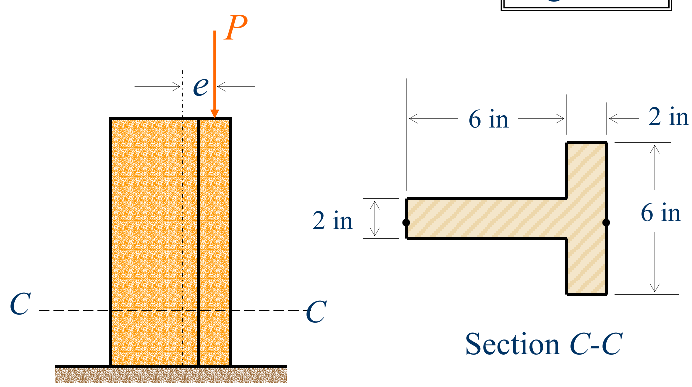
- Example 3

  The T-section shown in Fig. 50 is used as a short post to support a compressive load *P* of 150 kip. The load is applied on centerline of the stem at a distance *e* = 2 in. from the centroid of the cross section. Determine the normal stresses at points *A* and *B* on a transverse plane *C-C* near the base of the post.

Combined Axial, Torsional, and Flexural Loads

- Example 3 (cont'd)

Figure 50



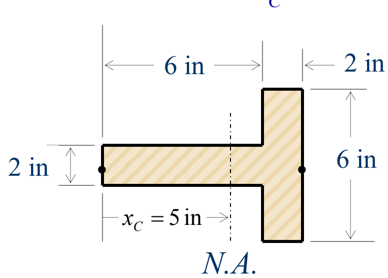
Combined Axial, Torsional, and Flexural Loads

- Example 3 (cont'd)

Computing the cross-sectional properties:

$$\text{Area} = A = 2[6\times 2] = 24\ \text{in}^2$$

$$x_C = \frac{3(6\times 2)+(6+1)(6\times 2)}{24} = 5\ \text{in. from point } A$$



$$I_y = \frac{2(5)^3}{3} + \frac{6(3)^3}{3} - \frac{4(1)^3}{3} = 136\ \text{in}^4$$

Combined Axial, Torsional, and Flexural Loads

- Example 3 (cont'd)

Equivalent force system:

$$P = 150 \text{ kip acts through centroid}$$
$$M = Pe = (150)(2) \times 12 = 3{,}600 \text{ kip} \cdot \text{in}$$

Computations of normal stresses:

$$\sigma_A = -\frac{P}{A} + \frac{My}{I_x} = -\frac{150}{24} + \frac{300(5)}{136} = \boxed{4.78 \text{ ksi (T)}}$$
$$\sigma_B = -\frac{P}{A} - \frac{My}{I_x} = -\frac{150}{24} - \frac{300(3)}{136} = \boxed{-12.87 \text{ ksi (C)}}$$

Combined Axial, Torsional, and Flexural Loads

- Example 4

The beam shown in Fig. 51 has a 3 × 8-in rectangular cross section and is loaded in a plane of symmetry. Determine and show on a sketch the principal and maximum shearing stresses at point *A.*

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

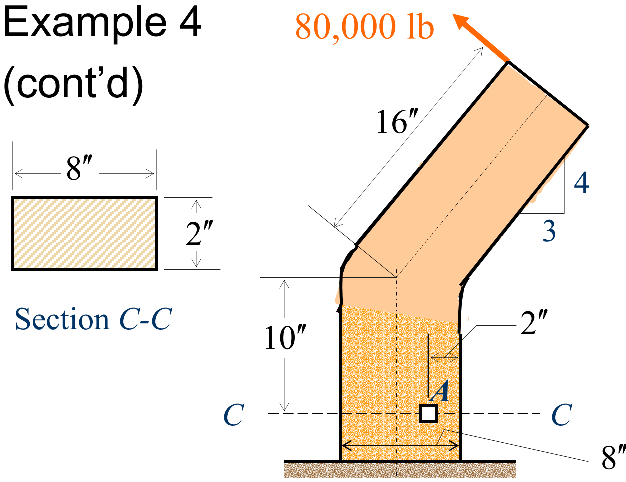
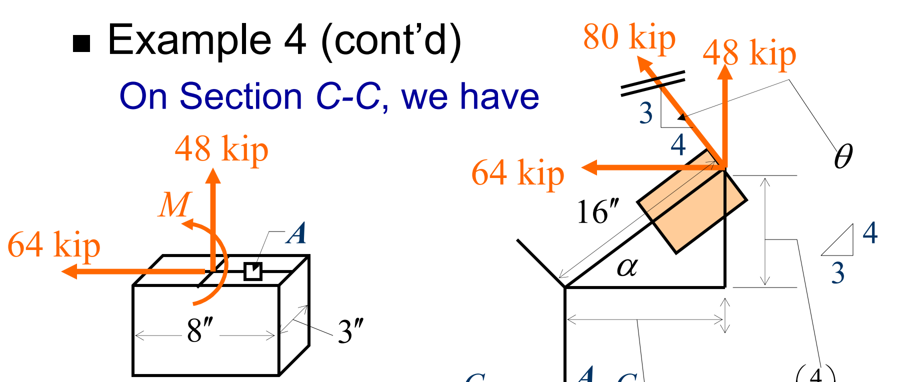


Figure 51

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

  On Section *C-C*, we have



$$P_x = 80{,}000\cos\theta = 80{,}000\left(\frac{4}{5}\right) = 64\text{ kip}$$

$$P_x = 80{,}000\sin\theta = 80{,}000\left(\frac{3}{5}\right) = 48\text{ kip}$$

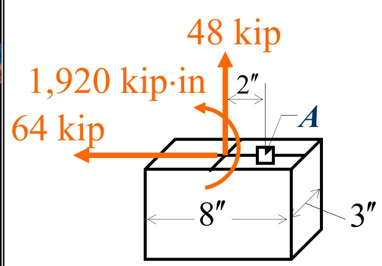
$$16c\sin\alpha = 16\left(\frac{4}{5}\right) = 12.8''$$

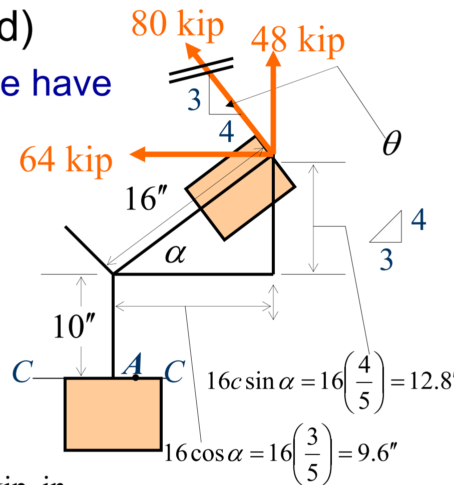
$$16\cos\alpha = 16\left(\frac{3}{5}\right) = 9.6''$$

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

On Section *C-C*, we have





$$M = 64(12.8 + 10) + 48(9.6) = 1{,}920 \text{ kip} \cdot \text{in}$$

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

On Section *C-C* through point *A*, we have

$$P = 48 \text{ kip} \qquad A = 3(8) = 24 \text{ in}^2$$

$$V = 64 \text{ kip} \qquad Q = 2(3)(3) = 18 \text{ in}^3$$

$$M = 1920 \text{ kip} \cdot \text{in} \qquad I = \frac{3(8)^3}{12} = 128 \text{ in}^4$$

Therefore,

$$\sigma_y = \frac{P}{A} + \frac{My}{I} = \frac{48}{24} + \frac{1920(2)}{128} = 32.0 \text{ ksi (T)}$$

$$\tau = \frac{VQ}{It} = \frac{64(18)}{128(3)} = 3.0 \text{ ksi}$$

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

Principal Stresses:

$$\sigma_{p1,p2} = \frac{0+32}{2} \pm \sqrt{\left(\frac{0-32}{2}\right)^2 + (-3)^2}$$

$$\sigma_{p1} = 16 + 16.279 = 32.3 \text{ ksi (T)}$$

$$\sigma_{p2} = 16 - 16.279 = -0.279 \text{ ksi (C)}$$

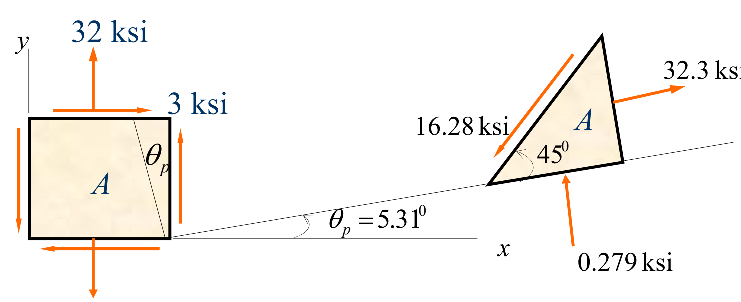
$$\sigma_{p3} = 0 \qquad \tau_{max} = \tau_p = 16.28 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2(-3)}{0-32.0} = +5.31^0$$

Combined Axial, Torsional, and Flexural Loads

- Example 4 (cont'd)

Sketch of the principal and maximum shearing stresses at point *A:*



Combined Axial, Torsional, and Flexural Loads

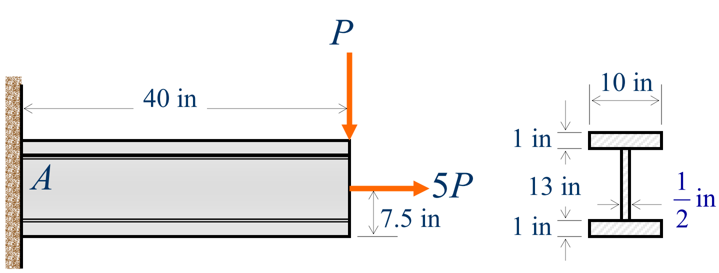
- Example 5

The cantilever beam shown in Fig. 52a has the cross section shown in Fig.52b. If the allowable stresses are 10,000 psi shear and 16,000 psi tension and compression at point *A* (just below the flange), determine the maximum allowable load *P*.

Combined Axial, Torsional, and Flexural Loads

- Example 5 (cont'd)

Figure 52



(a) (b)

Combined Axial, Torsional, and Flexural Loads

- Example 5 (cont'd)

On a section through point *A*, we have

$$N_A = 5P \qquad A = 2(10)(1) = 20\ \text{in}^2$$
$$V_A = P \qquad Q = 10(1)(7) = 70\ \text{in}^3$$
$$M_A = 40\text{P} \qquad I = \frac{10(15)^3}{12} - \frac{9.5(13)^3}{12} = 1073\ \text{in}^4$$

Therefore,

$$\sigma_A = \frac{N}{A} + \frac{My}{I} = \frac{5P}{26.5} + \frac{(40P)(6.5)}{1073} = 0.4310\text{P}$$
$$\tau_A = \frac{VQ}{It} = \frac{P(70)}{1073(0.5)} = 0.1305\text{P}$$

Combined Axial, Torsional, and Flexural Loads

- Example 5 (cont'd)

Principal Stresses and Max Shear Stress

$$\sigma_{\max} = \sigma_{p1} = \frac{0.4310P}{2} + \sqrt{\left(\frac{0.4310P}{2}\right)^2 + (0.1305P)^2}$$
$$= 0.2155P + 0.2519P = 0.4674P < 16{,}000\ \text{psi} \Rightarrow P < 34{,}232\ \text{lb}$$
$$\tau_{\max} = \tau_p = 0.2519P < 10{,}000\ \text{psi}$$

Therefore,

$$P_{\max} = 34{,}232\ \text{lb} \cong 34.2\ \text{kip}$$

Combined Axial, Torsional, and Flexural Loads

- Example 6

  A 100 mm-diameter steel rod is loaded and supported as shown in Fig. 53. Determine the principal stresses and maximum shearing stress

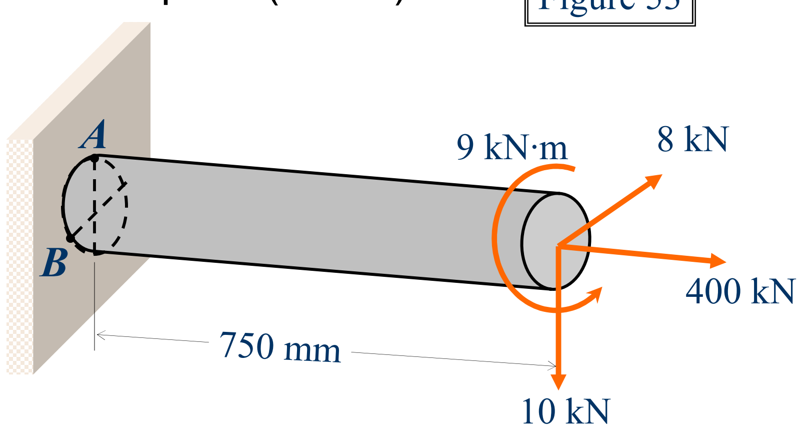
  (a) At point *A* on a section adjacent to the support.

  (b) At point *B* on a section adjacent to the support.

Combined Axial, Torsional, and Flexural Loads

- Example 6 (cont'd)

Figure 53



Combined Axial, Torsional, and Flexural Loads

- Example 6 (cont'd)

On a section adjacent to the support,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(100)^2 = 7854\ \text{mm}^2 \qquad Q_{NA} = \frac{1}{12}(100)^3 = 83.33\times10^3\ \text{mm}^3$$

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(100)^4 = 4.909\times10^6\ \text{mm}^4 \qquad M_A = 10\times10^3(0.750) = 7500\ \text{kN}\cdot\text{m}$$

$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}(100)^4 = 9.817\times10^6\ \text{mm}^4 \qquad M_B = 8\times10^3(0.750) = 6000\ \text{kN}\cdot\text{m}$$

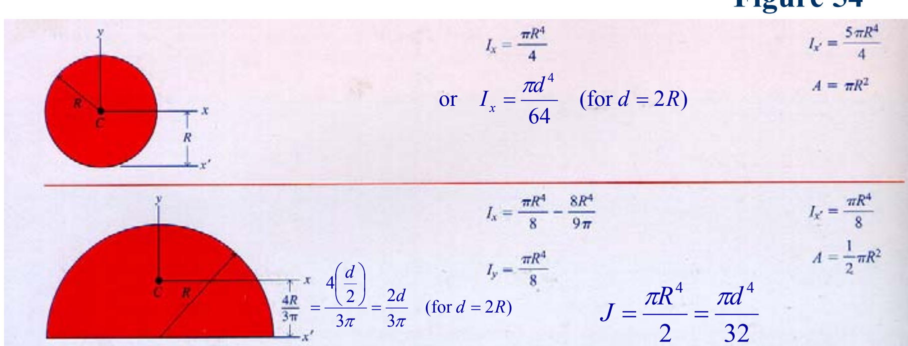
Note for Q_{NA} :

$$Q_{NA} = \frac{\text{area}}{2}\times\text{distance from NA to Centroid} = \frac{\pi d^2}{8}\left(\frac{2d}{3\pi}\right) = \frac{d^3}{12}$$

Combined Axial, Torsional, and Flexural Loads

- Commonly Used Second Moments of Plane Areas (For Example 6)

Figure 54



Combined Axial, Torsional, and Flexural Loads

- Example 6 (cont'd)

(a) For point *A*:

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{400\times10^3}{7854\times10^{-6}} + \frac{7500(0.05)}{4.909\times10^{-6}} = 127.32\times10^6 \text{ N/m}^2$$

$$\tau_{xy} = \frac{VQ}{It} - \frac{Tc}{J} = \frac{8\times10^3(83.33\times10^{-6})}{4.909\times10^{-6}(0.1)} - \frac{9\times10^3(0.05)}{9.817\times10^{-6}} = -44.48\times10^6 \text{ N/m}^2$$

$$\sigma_{p1,p2} = \frac{127.32}{2} \pm \sqrt{\left(\frac{127.32}{2}\right)^2 + (-44.48)^2}$$

$$\sigma_{p1} = 63.66 + 77.66 = \boxed{141.3 \text{ MPa (T)}} \qquad \sigma_{p2} = 63.66 - 77.66 = \boxed{-14 \text{ MPa (C)}}$$

$$\sigma_{p3} = 0 \qquad \tau_{max} = \tau_p = \boxed{77.7 \text{ MPa}}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2(-44.48)}{127.32-0} = \boxed{-17.47^0}$$

Combined Axial, Torsional, and Flexural Loads

- Example 6 (cont'd)

(b) For point *B*:

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{400\times10^3}{7854\times10^{-6}} + \frac{6000(0.05)}{4.909\times10^{-6}} = 112.04\times10^6 \text{ N/m}^2$$

$$\tau_{xy} = -\frac{VQ}{It} - \frac{Tc}{J} = \frac{10\times10^3(83.33\times10^{-6})}{4.909\times10^{-6}(0.1)} - \frac{9\times10^3(0.05)}{9.817\times10^{-6}} = -47.54\times10^6 \text{ N/m}^2$$

$$\sigma_{p1,p2} = \frac{112.04}{2} \pm \sqrt{\left(\frac{112.04}{2}\right)^2 + (-47.54)^2}$$

$$\sigma_{p1} = 56.02 + 73.47 = \boxed{129.5 \text{ MPa (T)}} \qquad \sigma_{p2} = 56.02 - 73.47 = \boxed{-17.45 \text{ MPa (C)}}$$

$$\sigma_{p3} = 0 \qquad \tau_{max} = \tau_p = \boxed{73.5 \text{ MPa}}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2(-47.54)}{112.04-0} = \boxed{-20.2^0}$$

Combined Axial, Torsional, and Flexural Loads

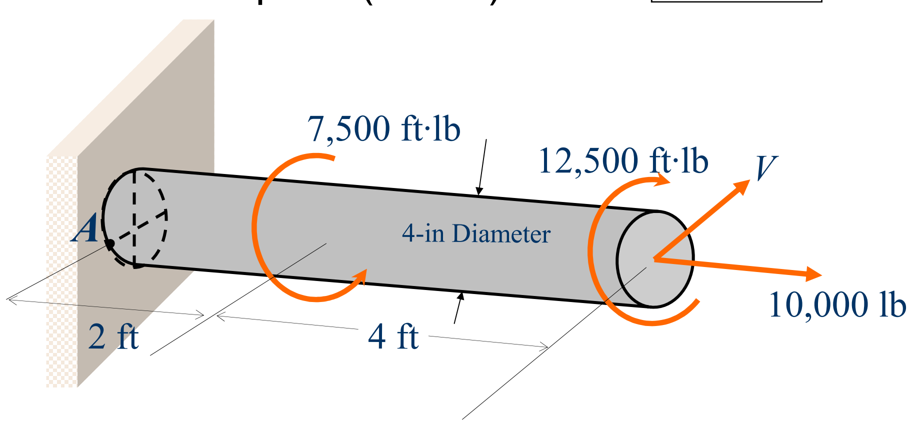
- Example 7

  A 4-in-diameter solid circular steel shaft is loaded and supported as shown in Fig. 55. If the maximum normal and shearing stresses at point *A* must be limited to 7500 psi tension and 5000 psi, respectively, determine the maximum permissible value for the the transverse load *V*.

Combined Axial, Torsional, and Flexural Loads

- Example 7 (cont'd)

Figure 55



Combined Axial, Torsional, and Flexural Loads

- Example 7 (cont'd)

On a section adjacent to the support,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(4)^2 = 12.566\ \text{in}^2 \qquad T = 12{,}500 - 7{,}500 = 5000\ \text{ft}\cdot\text{lb}$$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(4)^4 = 12.566\ \text{in}^4 \qquad M = V(6) = 6V\ \text{ft}\cdot\text{lb}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(4)^4 = 25.133\ \text{in}^4 \qquad P = 10{,}000\ \text{lb}$$

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{10{,}000}{12.566} + \frac{6V \times 12(2)}{12.566} = 795.8 + 11.459V$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{5000 \times 12(2)}{25.133} = 4775\ \text{psi}$$

Combined Axial, Torsional, and Flexural Loads

- Example 7 (cont'd)

Principal Stresses and Max Shearing Stress

$$\sigma_{p1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (4775)^2} < 7{,}500\ \text{psi} \Rightarrow \sigma_x < 4{,}460\ \text{psi}$$

$$\tau_{max} = \tau_p = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (4775)^2} < 5{,}000\ \text{psi} \Rightarrow \sigma_x < 2{,}966\ \text{psi}$$

Therefore,

$$\sigma_x = 795.8 + 11.459V < 2{,}966$$

and

$$V_{max} = \frac{2{,}966 - 795.8}{11.459} = \boxed{189.4\ \text{lb}}$$