LECTURE

24

Chapter 8.4

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COMPONENTS: COMBINED LOADING

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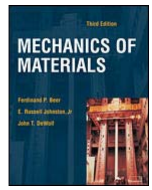

by

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ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

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Thin-Walled Pressure Vessels

- Stresses in Thin-Walled Vessels
  - The thin-walled pressure vessels provide an important application of plane-stress analysis.
  - This their walls offer little resistance to bending, it may be assumed that the internal forces exerted on a given portion of the wall are tangent to the surface of the vessel, as shown in Fig. 32.

Thin-Walled Pressure Vessels

- Stresses in Thin-Walled Vessels

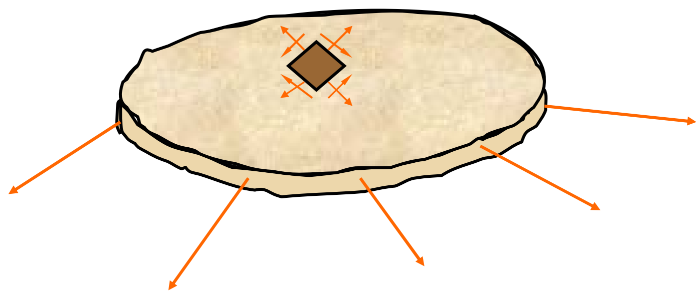


Figure 32. Internal Forces are Tangent

Thin-Walled Pressure Vessels

- Stresses in Thin-Walled Vessels
  - The resulting stresses on an element of the wall will thus be contained in a plane tangent to the surface of the vessel.
  - Two types of thin-walled vessels are investigated:
    - **Spherical Pressure Vessels**
    - **Cylindrical Pressure Vessels**

Thin-Walled Pressure Vessels

- Stresses in Thin-Walled Vessels
  – The stress in thin-walled vessel varies from a maximum value at the inside surface to a minimum value at the outside surface of the vessel.
  – It can be shown that if the ratio of the wall thickness to inner radius of the vessel is less than 0.1, the maximum normal stress is less than 5% greater than the average stress.

Thin-Walled Pressure Vessels

- Definition

  "A pressure vessel is defined as thin-walled when the ratio of the wall thickness to the radius of the vessel is so small that the distribution of normal stress on a plane perpendicular to the surface of the vessel is essentially uniform throughout the thickness of the vessel."

Thin-Walled Pressure Vessels

- General Types of Vessels
  - The following types of vessels can be analyzed as thin-walled elements:
    - Boilers
    - Gas Storage Tanks
    - Pipelines
    - Metal Tires
    - Hoops

Thin-Walled Pressure Vessels

- General Types of Vessels
  - The following types of vessels can be treated as thick-walled elements:
    - Gun Barrels
    - Certain High-pressure Vessels in Chemical Processing Industry
    - Cylinders and Piping for Heavy Hydraulic Pressure

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - A typical thin-walled spherical vessel used for gas storage is shown in Fig. 33.
  - If the weights of the gas and vessel are negligible (in most cases), symmetry of loading and geometry requires that stresses on sections that pass through the center of the sphere be equal.

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels

Figure 33

Thin-Walled Pressure Vessels

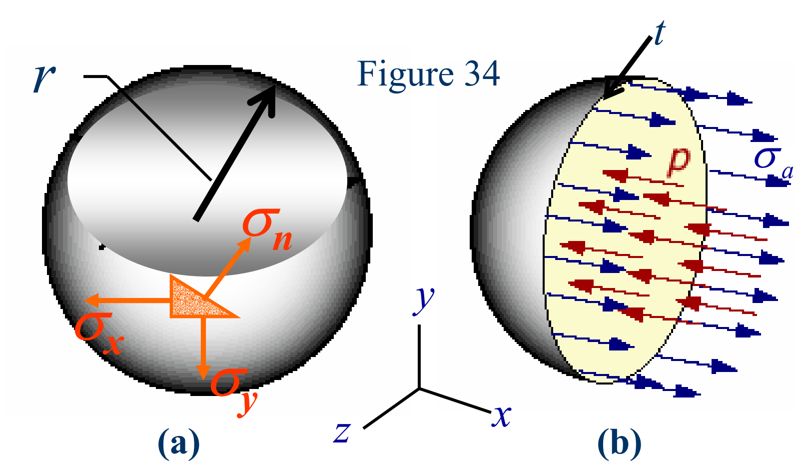
- Spherical Pressure Vessels
  - Consider the element shown in Fig. 34a.
  - The stresses σ_x, σ_y, and σ_n are related by the following equation:

$$\sigma_x = \sigma_y = \sigma_n \qquad (40)$$

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels

Figure 34



Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - Shearing stresses on any of these planes are not present because there are no loads to induce them.
  - The normal stress component in a sphere is known as a *meridional* or *axial* stress and is commonly denoted as

$$\sigma_m \quad \text{or} \quad \sigma_a$$

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - Derivation of Axial or Meridional Stress in Spherical Vessel
    - Consider the thin-walled spherical pressure vessel with radius *r* and thickness *t*, shown in Fig. 34b.
    - The free-body diagram of that figure can be used to compute the stresses

$$\sigma_x = \sigma_y = \sigma_n = \sigma_a \qquad (41)$$

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - Derivation of Axial or Meridional Stress in Spherical Vessel

    in terms of the pressure *p*, and the inside radius *r* and thickness *t* of the spherical vessel.

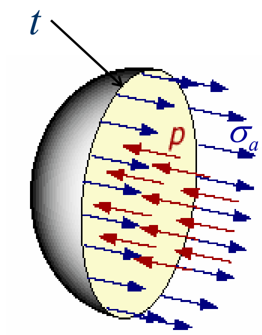
    - The force *R* is the resultant of the internal forces that act on the cross-sectional area of the sphere that exposed by passing a plane through the center of the sphere.

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - Derivation of Axial or Meridional Stress in Spherical Vessel
    - The force *P* is the resultant of the fluid forces acting on the fluid remaining within the hemisphere.
    - Since the vessel is under static equilibrium, it must satisfy Newton's first law of motion. In other words, the stress around the wall must have a net resultant to balance the internal pressure across the cross-section

Thin-Walled Pressure Vessels

- Spherical Pressure Vessels
  - Derivation of Axial or Meridional Stress in Spherical Vessel



$$\sum F = 0;\ R - P = 0$$

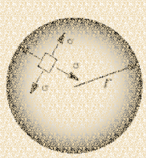
$$R = P$$

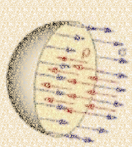
$$\sigma_a (t)(2\pi r) = p(\pi r^2)$$

$$\Rightarrow \sigma_a = \frac{pr}{2t} \qquad (42)$$

Thin-Walled Pressure Vessels

- Stress on Spherical Pressure Vessels





$$\sigma_a = \frac{pr}{2t} \qquad (42)$$

p = pressure of gas or fluid
r = inside radius of sphere
t = thickness of thin-walled sphere

Thin-Walled Pressure Vessels

- Example 12

A steel pressure vessel of spherical shape has the following specifications:

- inside radius r of 36 inches
- thickness t of 3/16"
- allowable yield stress σ_y of 50 ksi
- modulus of elasticity E of 29,000 ksi
- Poisson's ratio ν of 0.25

a) What is the maximum pressure p carried by the tank before yielding occurs?

b) If p = 100 psi, what is the new outer radius of the tank?

Thin-Walled Pressure Vessels

- Example 12 (cont'd)

(a) Normal in-plane stresses are given by Eq. 42. Rewrite the equation to solve for the maximum p

$$\sigma_a = \frac{pr}{2t} \quad \Rightarrow \quad \rho = \frac{2t\sigma_a}{r}$$

$$p = \frac{2\left(\frac{3}{16}\right)50}{36} = 0.521\,\text{ksi} = 521\,\text{psi}$$

Thin-Walled Pressure Vessels

- Example 12 (cont'd)

(b) First find the normal in-plane stress in the shell:

$$\sigma_a = \frac{pr}{2t} = \frac{100(36)}{2\left(\frac{3}{16}\right)} = 9{,}600 \text{ psi}$$

Now apply Hooke's law for plane stress:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) = \frac{1-\nu}{E}\sigma_a = \frac{1-\nu}{E}\frac{pr}{2t}$$

$$\varepsilon_x = \frac{1-0.25}{29\times10^6}(9{,}600) = 0.000248$$

The circumference, and therefore the radius, of the sphere will increase by 1 + e, so

$$\text{New } r_{\text{outer}} = \left(36\frac{3}{16}\right)(1.000248) = 36.196 \text{ in.}$$

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  – A typical thin-walled spherical vessel used for liquefied gas storage is shown in Fig. 35.
  – Normal stresses, as shown in Fig. 36a, are easy to evaluate by using appropriate free-body diagram.
  – Again, The normal stress component on a transverse plane is known as a *meridional* or *axial* stress and is commonly denoted as

$$\sigma_m, \text{ or } \sigma_a.$$

Thin-Walled Pressure Vessels

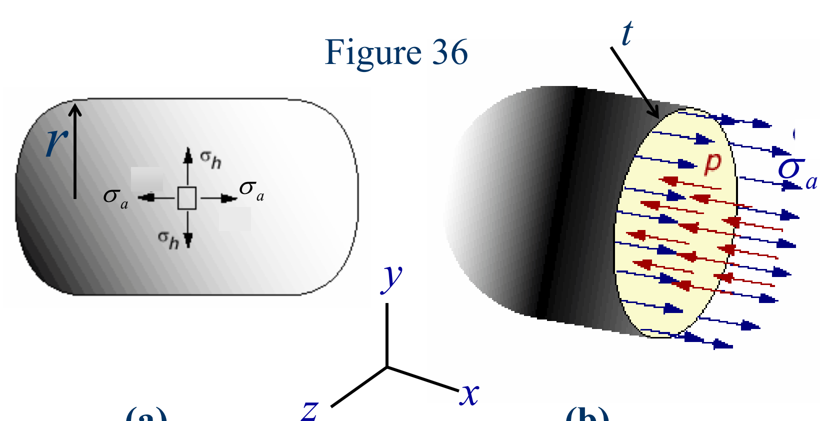
- Cylindrical Pressure Vessels

Figure 35

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels

Figure 36



(a) (b)

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  - The normal stress component on a longitudinal plane is known as *hoop*, *tangential, or circumferential stress*, and commonly denoted as σ_h, σ_t, or σ_c.
  - Again, there are no shearing stresses on transverse or longitudinal planes.
  - Stress determination in this case will be the same as in the case of spherical shape.

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  - Derivation of Normal Stress σ_a
    - To determine the longitudinal stress σ_a, we make a cut across the cylinder similar to analyzing the spherical pressure vessel. The free body, illustrated on the left (Fig. 36a), is in static equilibrium. This implies that the stress around the wall must have a resultant to balance the internal pressure across the cross-section.

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  - Derivation of Normal Stress σ_a
    - Applying statics (Newton's first law of motion, we have

$$\sum F_x = 0,\ R - P = 0 \Rightarrow R = P$$

Or

$$\sigma_a (t)(2\pi r) = p\left(\pi r^2\right)$$

Or

$$\sigma_a = \frac{pr}{2t} \qquad (43)$$

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  - Derivation of hoop or tangential stress σ_h
    - To determine the hoop stress σ_h, we make a cut along the longitudinal axis and construct a small slice as illustrated Fig. 37.
    - The free body is in static equilibrium. According to Statics (Newton's first law of motion), the hoop stress yields,

$$2(\sigma_h)(t)(dx) = p(2r)(dx)$$

$$\text{Therefore, } \sigma_h = \frac{pr}{t} \qquad (44)$$

Thin-Walled Pressure Vessels

- Cylindrical Pressure Vessels
  - Derivation of hoop or tangential stress σ_h

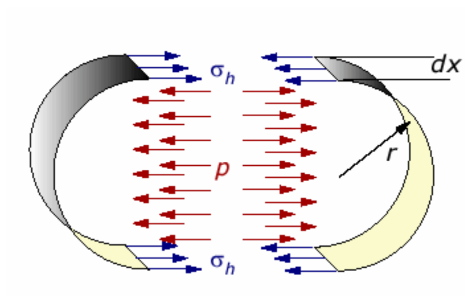


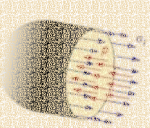
Figure 37

$$2(\sigma_h)(t)(dx) = p(2r)(dx)$$

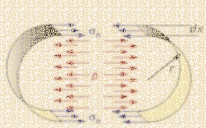
$$\text{Therefore, } \sigma_h = \frac{pr}{t}$$

Thin-Walled Pressure Vessels

- Stress on Cylindrical Pressure Vessels



$$\sigma_a = \frac{pr}{2t} \qquad (45)$$



$$\sigma_h = \frac{pr}{t} \qquad (46)$$

p = pressure of gas or fluid
r = inside radius of sphere
t = thickness of thin-walled sphere

Thin-Walled Pressure Vessels

- Example 1

A steel pipe with inside diameter of 12 in. will be used to transmit steam under a pressure of 1000 psi. If the hoop stress in the pipe must be limited to 10 ksi because of a longitudinal weld in the pipe, determine the maximum satisfactory thickness for the pipe.

Thin-Walled Pressure Vessels

- Example 1 (cont'd)

For the cylinder, the hoop stress is given given by Eq. 46 as

$$\sigma_h = \frac{pr}{t}$$

Therefore,

$$t = \frac{pr}{\sigma_h} = \frac{1000\left(\frac{12}{2}\right)}{10{,}000} = \boxed{0.6\text{ in.}}$$

Thin-Walled Pressure Vessels

- Remarks of Thin-Walled Pressure Vessels
  1. The above formulas are good for thin-walled pressure vessels. Generally, a pressure vessel is considered to be "thin-walled" if its radius *r* is larger than 5 times its wall thickness *t* ($r > 5 \cdot t$).

Thin-Walled Pressure Vessels

- Remarks of Thin-Walled Pressure Vessels
  2. When a pressure vessel is subjected to external pressure, the above formulas are still valid. However, the stresses are now negative since the wall is now in compression instead of tension.

Thin-Walled Pressure Vessels

- Remarks of Thin-Walled Pressure Vessels
  3. The hoop stress is twice as much as the longitudinal stress for the cylindrical pressure vessel. This is why an overcooked hotdog usually cracks along the longitudinal direction first (i.e. its skin fails from hoop stress, generated by internal steam pressure).