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LECTURE

22

Chapter 7.4

FAILURE CRITERIA: MOHR'S CIRCLE AND PRINCIPAL STRESSES

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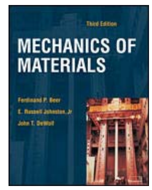

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ENES 220 – Mechanics of Materials

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The Stress Transformation Equations for Plane Stress

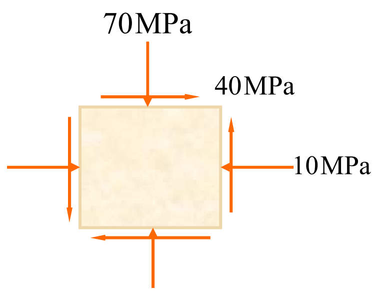
- Example 2

  The stresses shown in Figure 12a act at a point on the free surface of a stressed body. Determine the normal stresses σ_n and σ_t and the shearing stress τ_{nt} at this point if they act on the rotated stress element shown in Figure 12b.

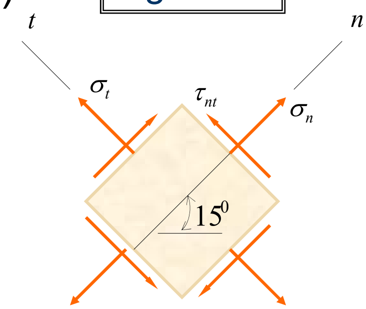
The Stress Transformation Equations for Plane Stress

- Example 2 (cont'd)

Figure 12



(a)



(b)

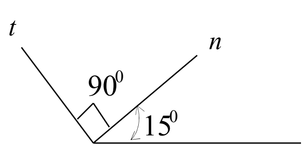
The Stress Transformation Equations for Plane Stress

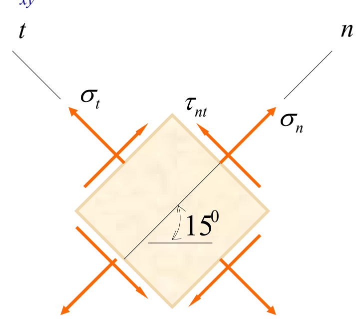
- Example 2 (cont'd)

  The given values are as follows:

  $\sigma_x = -10 \text{ MPa}, \sigma_y = -70 \text{ MPa}, \tau_{xy} = +40 \text{ MPa}$

  $\theta_n = 15^0, \theta_t = 15^0 + 90^0 = 105^0$





The Stress Transformation Equations for Plane Stress

- Example 2 (cont'd)

  Applying Eq. 12 for the given values

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= -10\cos^2(15) - 70\sin^2(15) + 2(40)\sin(15)\cos(15)$$

$$\sigma_n = 5.981\text{ MPa} = \boxed{5.981\text{ MPa (Tension)}}$$

$$\sigma_t = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= -10\cos^2(105) - 70\sin^2(105) + 2(40)\sin(105)\cos(105)$$

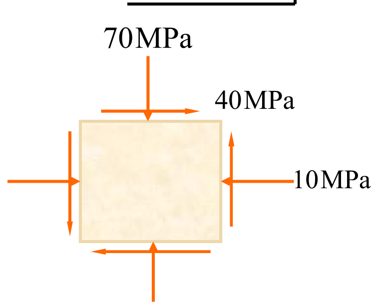
$$\sigma_t = -85.98\text{ MPa} = \boxed{86\text{ MPa (comprssion)}}$$

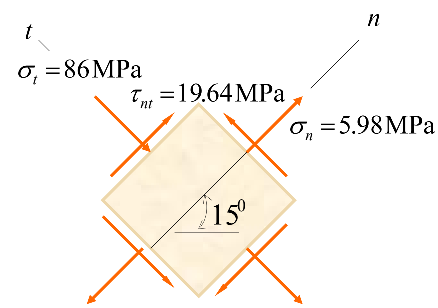
The Stress Transformation Equations for Plane Stress

- Example 2 (cont'd)

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$
$$= -(-10 - (-70))\sin(15)\cos(15) + 40(\cos^2(15) - \sin^2(15))$$

$$\tau_{nt} = \boxed{19.64\text{ MPa}}$$



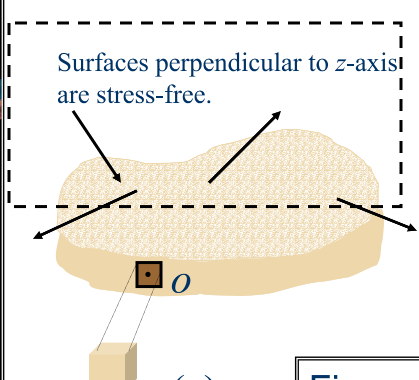


Principal Stresses and Maximum Shearing Stress

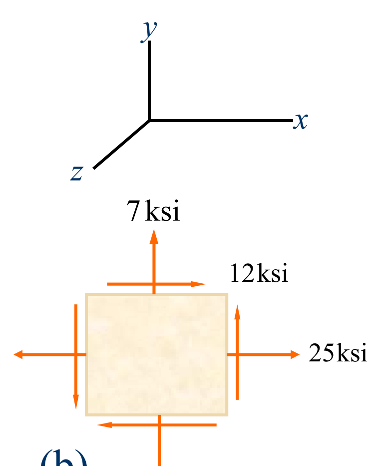
- Principal Stresses
  - The transformation equations (Eq. 12 or 13) provides a means for determining the normal stress σ_n and the shearing stress τ_{nt} on different planes through a point *O* in stressed body.
  - Consider, for example, the state of stress at a point *O* of the free surface of a structure or machine component (Fig. 13).

Principal Stresses and Maximum Shearing Stress

- Principal Stresses



(a)



(b)

Figure 13

Principal Stresses and Maximum Shearing Stress

- Principal Stresses
  - As the element is rotated through an angle θ about an axis perpendicular to the stress-free surface, the normal stress σ_n and the shearing stress τ_{nt} on different planes vary continuously as shown in Figure 14.
  - For design purposes, critical stress at the point are usually the maximum tensile (or compressive) and shearing stresses.

Principal Stresses and Maximum Shearing Stress

- Principal Stresses
  - The principal stresses are the *maximum normal stress* σ_{max} and *minimum normal stress* σ_{min}.
  - In general, these maximum and minimum or principal stresses can be determined by plotting curves similar to those of Fig. 14.
  - But this process is time-consuming, and therefore, general methods are needed.

Principal Stresses and Maximum Shearing Stress

- Variation of Stresses as Functions of θ

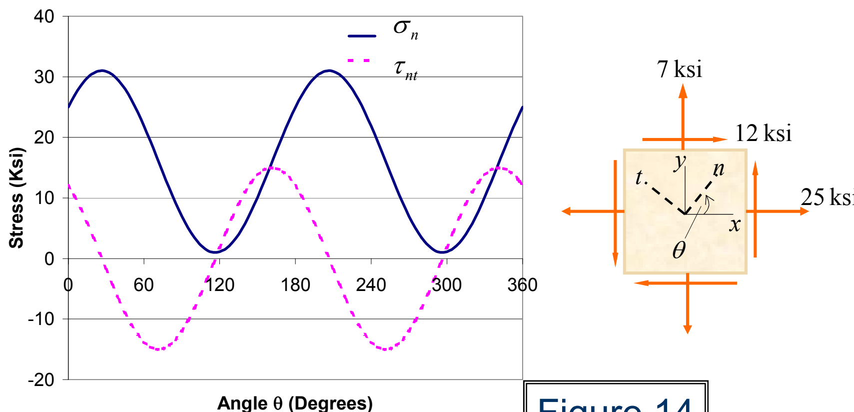


Figure 14

Principal Stresses and Maximum Shearing Stress

- Principal Stresses
  - Principal Stresses for Special Loading Conditions:
    - Bar under axial load

$$\sigma_{max} = \frac{P}{A} \quad \text{and} \quad \tau_{max} = \frac{P}{2A} \qquad (14)$$

    - Shaft under Pure Bending

$$\sigma_{max} = \tau_{max} = \frac{T_{max}c}{J} \qquad (15)$$

Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Axially Loaded Bar

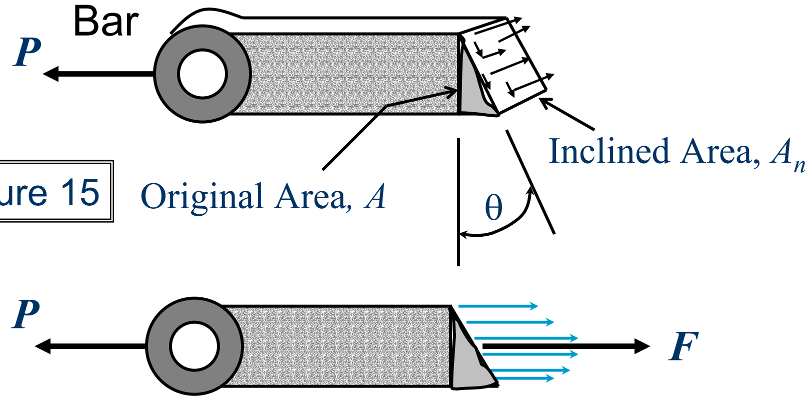
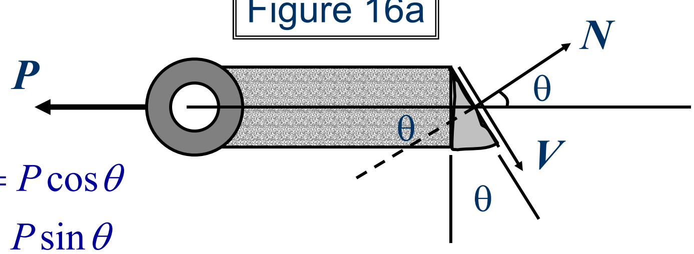


Figure 15

Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Axially Loaded Bar

Figure 16a



$$N = P\cos\theta$$

$$V = P\sin\theta$$

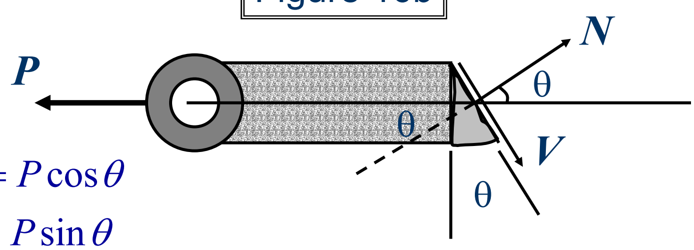
$$A_n = \frac{A}{\cos\theta}$$

$$\sigma_n = \frac{N}{A_n} = \frac{P\cos\theta}{\dfrac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta = \frac{P}{2A}(1+\cos 2\theta)$$

Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Axially Loaded Bar

Figure 16b



$$N = P\cos\theta$$

$$V = P\sin\theta$$

$$A_n = \frac{A}{\cos\theta}$$

$$\tau_n = \frac{V}{A_n} = \frac{P\sin\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta$$

Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Axially Loaded Bar

σ_n is maximum when θ = 0^0 or 180^0

τ_n is maximum when θ = 45^0 or 135^0

– Also

$$\tau_{\max} = \frac{\sigma_{\max}}{2} \qquad (16)$$

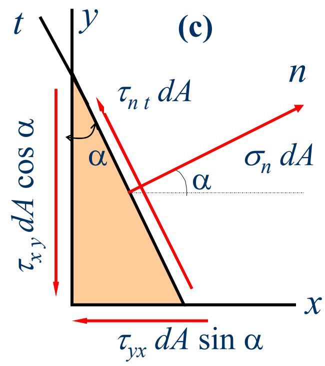
Therefore

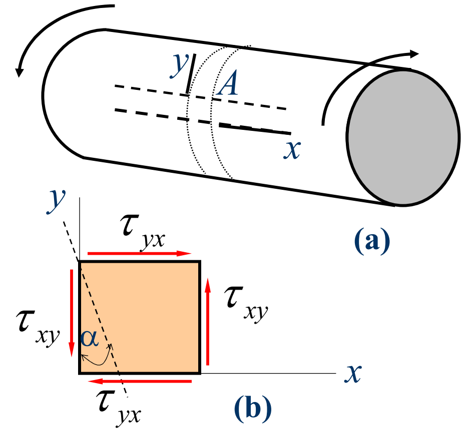
$$\sigma_{\max} = \frac{P}{A} \quad \text{and} \quad \tau_{\max} = \frac{P}{2A} \qquad (17)$$

Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Shaft under Pure Torsion

Fig.17





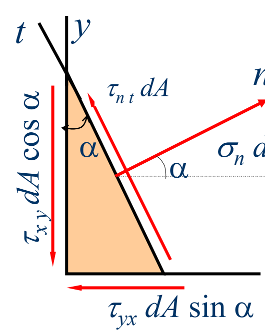
Principal Stresses and Maximum Shearing Stress

- Principal Stresses for Shaft under Pure Torsion

$$\sigma_n = 2\tau_{xy} \sin\alpha \cos\alpha = \tau_{xy} \sin 2\alpha \qquad (18)$$

$$\tau_{nt} = \tau_{xy}\left(\cos^2\alpha - \sin^2\alpha\right) = \tau_{xy} \cos 2\alpha \qquad (19)$$

$$\boxed{\sigma_{\max} = \tau_{\max} = \frac{T_{\max} c}{J}} \qquad (20)$$



Principal Stresses and Maximum Shearing Stress

- Development of Principal Stresses Equations

  Recall Eq 13

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{(13a)}$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{(13b)}$$

Principal Stresses and Maximum Shearing Stress

- Development of Principal Stresses Equations

  Differentiating the first equation with respect to θ and equate the result to zero, gives

$$\frac{d\sigma_n}{d\theta} = \frac{d}{d\theta}\left[\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right]$$

$$= -\left(\sigma_x - \sigma_y\right)\sin 2\theta + 2\tau_{xy}\cos 2\theta \overset{\text{set}}{=} 0$$

or

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \text{or} \quad 2\theta_p = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad (21)$$

Principal Stresses and Maximum Shearing Stress

- Development of Principal Stresses Equations

Substituting the expression for $2\theta_p$ into Eq. 13a, yields

$$\sigma_{p1,\,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (22)$$

Eq. 21 gives the two principal stresses in the *xy*-plane, and the third stress $\sigma_{p3} = \sigma_z = 0$.

Principal Stresses and Maximum Shearing Stress

- Principal Stresses

Principal stresses σ_{max} and σ_{min} can be computed from

$$\sigma_{p1,\,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (22a)$$

where subscript *p* refers to the planes of maximum and minimum values of σ_n.

Principal Stresses and Maximum Shearing Stress

- Location of the Plane of Principal Stresses

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad (22b)$$

Principal Stresses and Maximum Shearing Stress

- Notes on Principal Stresses Equation
  1. Eq. 22 gives the angle θ_p and $\theta_p + 90^0$ between *x*-plane (or *y*-plane) and the mutually perpendicular planes on which the principal stresses act.
  2. When tan $2\theta_p$ is positive, θ_p is positive, and the rotation is *counterclockwise* from the *x*- and *y*-planes to the planes on which the two principal stresses act.

Principal Stresses and Maximum Shearing Stress

- Notes on Principal Stresses Equation
  3. When tan $2\theta_p$ is negative, θ_p is negative, and the rotation is *clockwise*.
  4. The shearing stress is zero on planes experiencing maximum and minimum values of normal stresses.
  5. If one or both of the principal stresses from Eq.22 is negative, the algebraic maximum stress can have a smaller absolute value than the minimum stress.

Principal Stresses and Maximum Shearing Stress

- Development of Maximum Shearing Stress Equation

Recall Eq 13b: $\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$$\frac{d\tau_{nt}}{d\theta} = \frac{d}{d\theta}\left[-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right]$$
$$= -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta \overset{\text{set}}{=} 0$$

or

$$\tan 2\theta_\tau = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \quad \text{or} \quad 2\theta_\tau = \tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \qquad (23)$$

Principal Stresses and Maximum Shearing Stress

- Development of Principal Shearing Stress Equation

  Substituting the expression for $2\theta_\tau$ into Eq. 13b, yields

$$\tau_p = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (24)$$

  Eq. 24 gives the maximum in-plane shearing stress.

Principal Stresses and Maximum Shearing Stress

- Maximum In-Plane Shearing Stress

  Maximum in-plane shearing stress can be computed from

$$\tau_p = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (24a)$$

  where the subscript *p* refers to the plane of maximum in-plane shearing stress τ_p.

Principal Stresses and Maximum Shearing Stress

- Location of the Plane of Maximum Shearing Stress

$$\theta_\tau = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \qquad (24b)$$

Principal Stresses and Maximum Shearing Stress

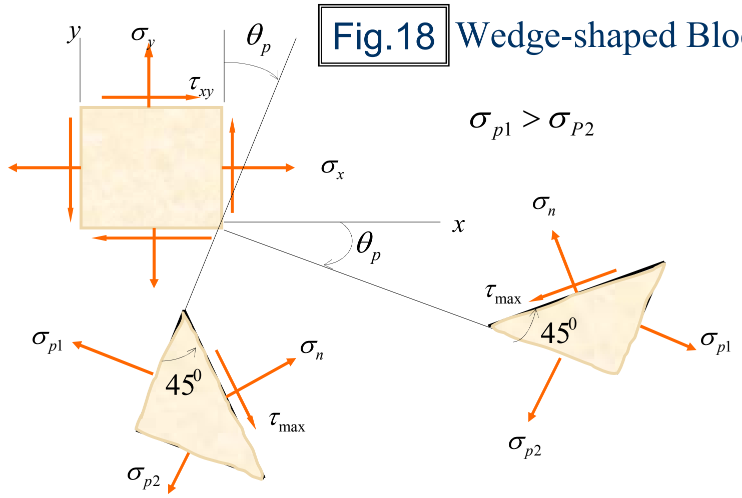
- Notes on Principal Stresses and Maximum In-Plane Shearing Stress Equation
  1. The two angles $2\theta_p$ and $2\theta_\tau$ differ by 90^0, therefore, θ_p and θ_τ are 45^0 apart.
  2. This means that the planes in which the maximum in-plane shearing stress occur are 45^0 from the principal planes.

Principal Stresses and Maximum Shearing Stress

- Notes on Principal Stresses and Maximum In-Plane Shearing Stress Equation
  3. The direction of the maximum shearing stress can be determined by drawing a wedge-shaped block with two sides parallel to the planes having the maximum and minimum principal stresses, and with the third side at an angle of 45^0. The direction of the maximum shearing stress must oppose the larger of the two principal stresses.

Principal Stresses and Maximum Shearing Stress

Fig.18 Wedge-shaped Block



Principal Stresses and Maximum Shearing Stress

- Useful Relationships
  - The maximum value of τ_{nt} is equal to one half the difference between the two in-plane principal stresses, that is

$$\tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} \qquad (25)$$

  - For plane stress, the sum of the normal stresses on any two orthogonal planes through a point in a body is a constant or in invariant.

$$\sigma_{p1} + \sigma_{p2} = \sigma_x + \sigma_y \qquad (26)$$

Principal Stresses and Maximum Shearing Stress

- Useful Relationships
  - When a state of plane exists, one of the principal stresses is zero.
  - If the values of σ_{p1} and σ_{p2} from Eq. 25 have the same sign, then the third principal stress σ_{p3} equals zero, will be either the maximum or minimum normal stresses.
  - Three possibilities:

$$\max = (\sigma_{p1} - \sigma_{p2})/2, \quad \max = (\sigma_{p1} - 0)/2, \quad \max = (0 - \sigma_{p2})/2$$

Principal Stresses and Maximum Shearing Stress

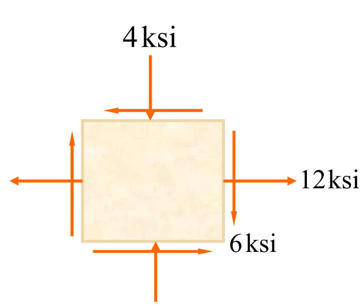
- Example 3

  Normal and shearing stresses on horizontal and vertical planes through a point in a structural member subjected to plane stress are shown in Figure 19. Determine and show on a sketch the principal and maximum shearing stresses.

Principal Stresses and Maximum Shearing Stress

- Example 3 (cont'd)

Fig.19



The given values for use in Eqs. 22 and 24 are:

$$\sigma_x = +12 \text{ ksi}$$
$$\sigma_y = -4 \text{ ksi}$$
$$\tau_{xy} = -6 \text{ ksi}$$

Principal Stresses and Maximum Shearing Stress

- Example 3 (cont'd)

Using Eq. 22a for the given values:

$$\sigma_p = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12 + (-4)}{2} \pm \sqrt{\left(\frac{12 - (-4)}{2}\right)^2 + (-6)^2} = 4 \pm 10$$

Therefore,

$$\sigma_{P1} = 4 + 10 = +14 \text{ ksi} = 14 \text{ ksi (T)}$$

$$\sigma_{p2} = 4 - 10 = -6 \text{ ksi} = 6 \text{ ksi (C)}$$

$$\sigma_{p3} = \sigma_z = 0$$

Principal Stresses and Maximum Shearing Stress

- Example 3 (cont'd)

Since the σ_{p1} and σ_{p2} have opposite sign, the maximum shearing stress is

$$\tau_{\max} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{14 - (-6)}{2} = \frac{20}{2} = +10 \text{ ksi}$$

The location θ_p of the principal stresses is computed from Eq. 22b

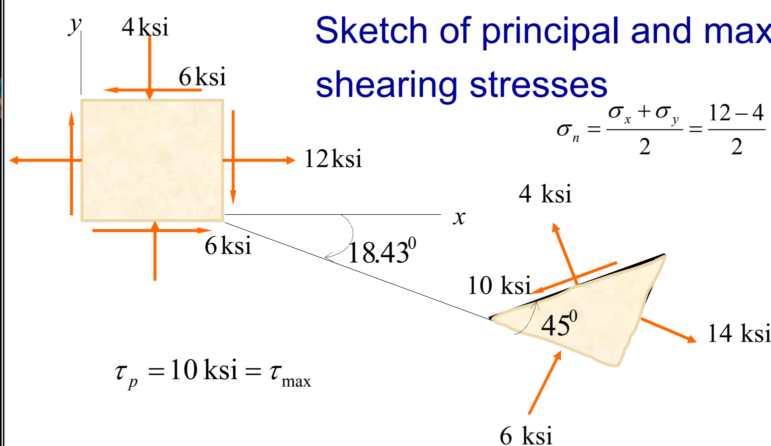
$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2(-6)}{12_x - (-4)_y}\right) = 18.43^0$$

Principal Stresses and Maximum Shearing Stress

- Example 3 (cont'd)

Sketch of principal and max shearing stresses

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{12-4}{2} = 4 \text{ ksi (T)}$$



$$\tau_p = 10 \text{ ksi} = \tau_{max}$$

Principal Stresses and Maximum Shearing Stress

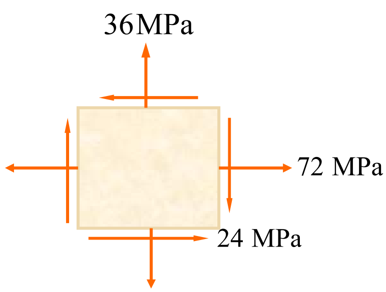
- Example 4

Normal and shearing stresses on horizontal and vertical planes through a point in a structural member subjected to plane stress are shown in Figure 20. Determine and show on a sketch the principal and maximum shearing stresses

Principal Stresses and Maximum Shearing Stress

- Example 4 (cont'd)

Fig.20



The given values for use in Eqs. 22 and 24 are:

$$\sigma_x = +72 \text{ MPa}$$
$$\sigma_y = +36 \text{ MPa}$$
$$\tau_{xy} = -24 \text{ MPa}$$

Principal Stresses and Maximum Shearing Stress

- Example 4 (cont'd)

Using Eq. 22a for the given values:

$$\sigma_p = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{72 + (+36)}{2} \pm \sqrt{\left(\frac{72 - (+36)}{2}\right)^2 + (-24)^2} = 54 \pm 30$$

Therefore,

$$\sigma_{P1} = 54 + 30 = +84 \text{ ksi} = 84 \text{ MPa (T)}$$

$$\sigma_{p2} = 54 - 30 = +24 \text{ ksi} = 24 \text{ MPa (T)}$$

$$\sigma_{p3} = \sigma_z = 0$$

Principal Stresses and Maximum Shearing Stress

- Example 4 (cont'd)

Since the σ_{p1} and σ_{p2} have the same sign, the maximum shearing stress is

$$\tau_{\max} = \frac{\sigma_{p1} - 0}{2} = \frac{84 - 0}{2} = \frac{84}{2} = +42 \text{ MPa}$$

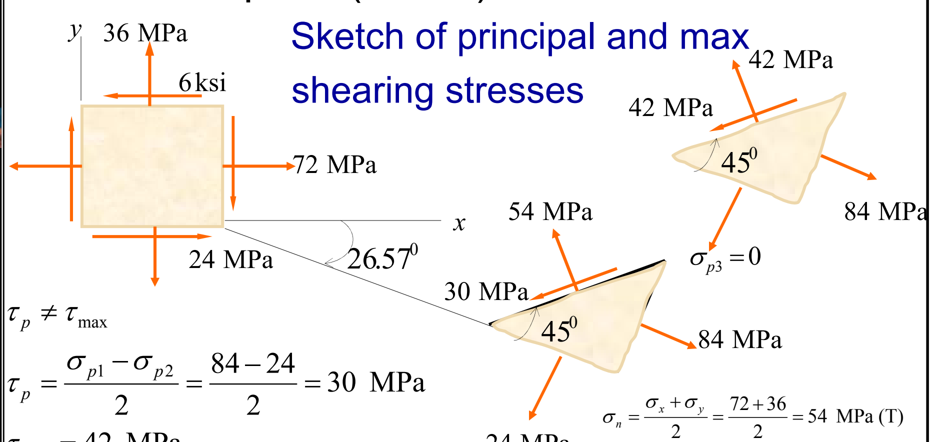
The location θ_p of the principal stresses is computed from Eq. 22b

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-24)}{72 - 36}\right) = -26.57^0$$

Principal Stresses and Maximum Shearing Stress

- Example 4 (cont'd)

Sketch of principal and max shearing stresses



$\tau_p \neq \tau_{\max}$

$$\tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{84 - 24}{2} = 30 \text{ MPa}$$

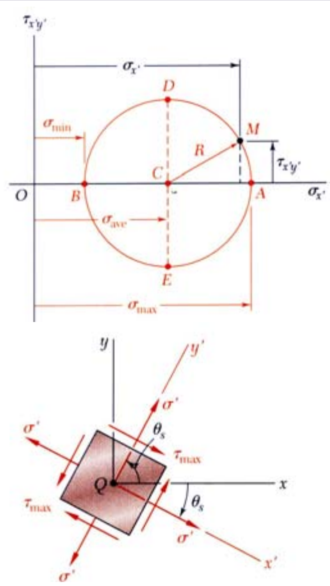
$$\tau_{\max} = 42 \text{ MPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{72 + 36}{2} = 54 \text{ MPa (T)}$$

Mohr's Circle for Plane Stress

- Introduction
  - Mohr's circle is a pictorial or graphical interpretation of the transformation equations for plane stress.
  - The process involves the construction of a circle in such a manner that the coordinates of each point on the circle represent the normal and shearing stresses on one plane through the stressed

Maximum Shearing Stress



Maximum shearing stress occurs for $\sigma_{x'} = \sigma_{ave}$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90^o and offset from θ_p by 45^o

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Mohr's Circle for Plane Stress

- Introduction

  Point, and the angular position of the radius to the point gives the orientation of the plane.

  – The proof that normal and shearing components of stress on arbitrary plane through a point can be represented as points on a circle follows from Eqs. 13a and 13b.

Mohr's Circle for Plane Stress

- Plane Stresses using Mohr's Circle
  - Recall Eqs. 13a and 13b,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (13a)$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (13a)$$

  - Squaring both equations, adding, and simplifying gives

$$\left(\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{nt}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad (27)$$

Mohr's Circle for Plane Stress

- Plane Stresses using Mohr's Circle
  - The previous equation is indeed an equation of a circle in terms of the variable σ_n and τ_{nt}. The circle is centered on the the σ axis at a distance $(\sigma_x - \sigma_y)/2$ from the τ axis, and the radius of the circle is given by

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (28)$$

Mohr's Circle for Plane Stress

- Plane Stresses using Mohr's Circle
  - Normal stresses are plotted as horizontal coordinates, with tensile stresses (positive) plotted to the right of the origin and compressive stresses (negative) plotted to the left.
  - Shearing stresses are plotted as vertical coordinates, with those tending to produce a clockwise rotation of the stress element

Mohr's Circle for Plane Stress

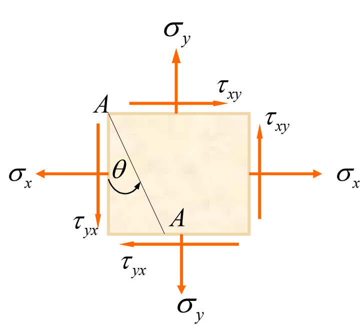
- Plane Stresses using Mohr's Circle

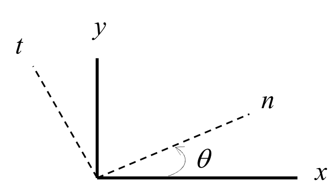
  plotted above the σ-axis, and those tending to produce counterclockwise rotation of the stress element plotted below the σ-axis.

  – Sign conventions for interpreting the normal and shearing stresses will be provided, and illustrated through examples.

Mohr's Circle for Plane Stress

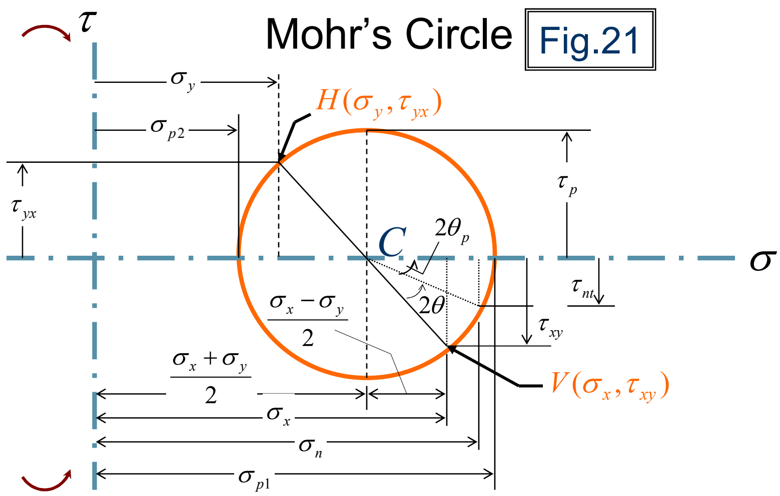
- Plane Stresses using Mohr's Circle
  - Mohr's circle for any point subjected to plane stress can be drawn when stresses on two mutually perpendicular planes through the point are known.





Mohr's Circle for Plane Stress

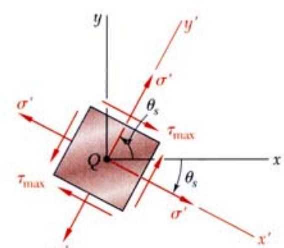
Mohr's Circle Fig.21

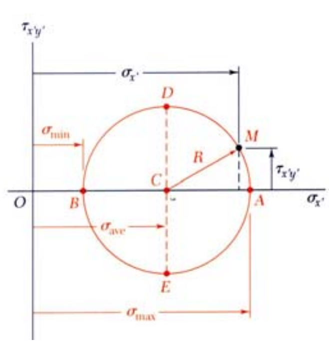


Mohr's Circle for Plane Stress

- Mohr's Circle

Maximum shearing stress occurs for $\sigma_{x'} = \sigma_{ave}$





$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90° and offset from θ_p by 45°

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Mohr's Circle for Plane Stress

- Plane Stresses using Mohr's Circle

Drawing Procedure for Mohr's Circle

1. Choose a set of *x*-*y* coordinate axes.
2. Identify the stresses σ_x, σ_y and $\tau_{xy} = \tau_{yx}$ and list them with proper sign.
3. Draw a set of $\sigma\tau$-coordinate axes with σ and τ positive to the right and upward, respectively.
4. Plot the point (σ_x, -τ_{xy}) and label it point *V* (vertical plane).

Mohr's Circle for Plane Stress

- Plane Stresses using Mohr's Circle

Drawing Procedure for Mohr's Circle (cont'd)

5. Plot the point (σ_y, τ_{yx}) and label it point *H* (horizontal plane).
6. Draw a line between *V* and *H*. This establishes the center and the radius *R* of Mohr's circle.
7. Draw the circle.
8. An extension of the radius between *C* and *V* can be identified as the *x*-axis or reference line for the angle measurements (I.e., θ =0).

Mohr's Circle for Plane Stress

- Sign Conventions
  - In a given face of the stressed element, the shearing stresses that tends to rotate the element *clockwise* will be plotted above the σ-axis in the circle.
  - In a given face of the stressed element, the shearing stresses that tends to rotate the element counter*clockwise* will be plotted below the σ-axis in the circle.

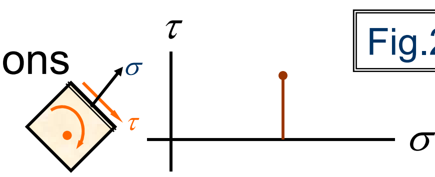
Mohr's Circle for Plane Stress

- Sign Conventions

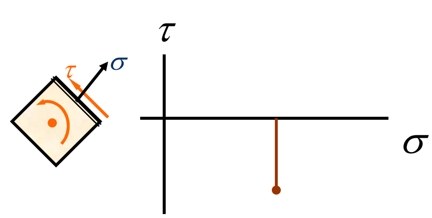
Fig.22

The following jingle may be helpful in remembering this conventions:
"In the kitchen, the clock is above, and the counter is below."

Beer and Johnston (1992)



(a) Clockwise → **Above**

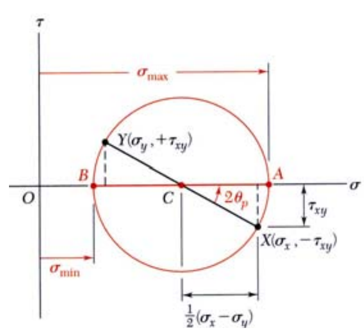


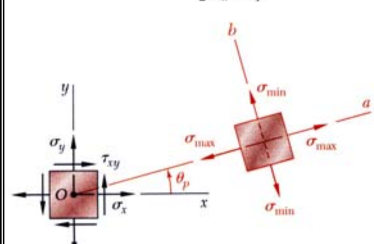
(b) Counterclockwise → **Below**

Mohr's Circle for Plane Stress

- Points of Interests on Mohr's Circle
  1. Point *D* that provides the principal stress σ_{p1}.
  2. Point *E* that gives the principal stress σ_{p2}.
  3. Point *A* that provides the maximum in-plane shearing stress $-\tau_p$ and the accompanied normal stress σ_{avg} that acts on the plane.

Mohr's Circle for Plane Stress





- With the physical significance of Mohr's circle for plane stress established, it may be applied with simple geometric considerations. Critical values are estimated graphically or calculated.
- For a known state of plane stress $\sigma_x, \sigma_y, \tau_{xy}$ plot the points *X* and *Y* and construct the circle centered at *C*.

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

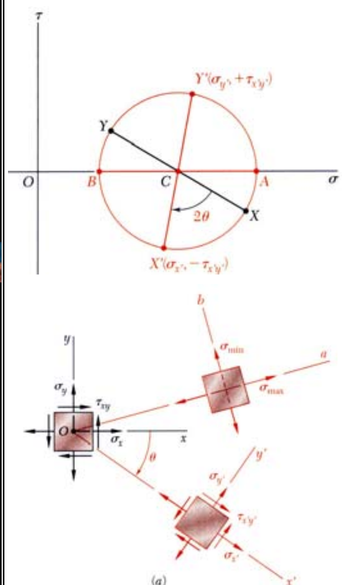
- The principal stresses are obtained at *A* and *B*.

$$\sigma_{\max,\min} = \sigma_{ave} \pm R$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The direction of rotation of *Ox* to *Oa* is the same as *CX* to *CA*.

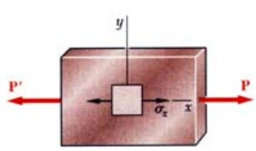
Mohr's Circle for Plane Stress



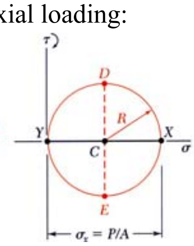
- With Mohr's circle uniquely defined, the state of stress at other axes orientations may be depicted.
- For the state of stress at an angle θ with respect to the *xy* axes, construct a new diameter *X'Y'* at an angle 2θ with respect to *XY*.
- Normal and shear stresses are obtained from the coordinates *X'Y'*.

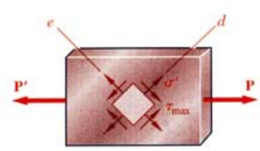
Mohr's Circle for Plane Stress

- Mohr's circle for centric axial loading:



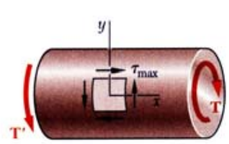
$$\sigma_x = \frac{P}{A}, \quad \sigma_y = \tau_{xy} = 0$$



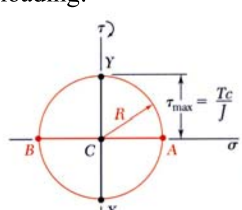


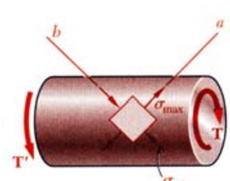
$$\sigma_x = \sigma_y = \tau_{xy} = \frac{P}{2A}$$

- Mohr's circle for torsional loading:



$$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = \frac{Tc}{J}$$





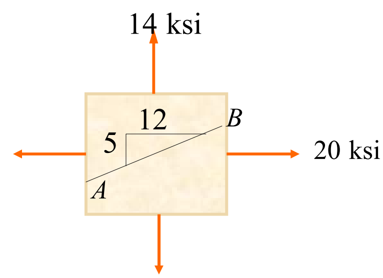
$$\sigma_x = \sigma_y = \frac{Tc}{J} \quad \tau_{xy} = 0$$

Mohr's Circle for Plane Stress

- Example 5

The stresses shown in Figure 23 act at a point on the free surface of a stressed body. Use Mohr's circle to determine the normal and shearing stresses at this point on the inclined plane *AB* shown in the figure.

Fig.23



Mohr's Circle for Plane Stress

- Example 5 (cont'd)

The given values for use in drawing Mohr's circle are:

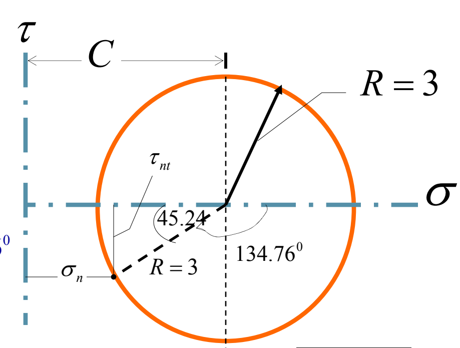
$$\sigma_x = \sigma_{p1} = 20\text{ ksi}$$

$$\sigma_y = \sigma_{p2} = 14\text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \qquad 2\theta = 2\tan^{-1}\left(\frac{-12}{5}\right) = -134.76^0$$

$$C = \frac{20+14}{2} = 17\text{ ksi}$$

$$R = \text{radius} = \frac{20-14}{2} = 3$$



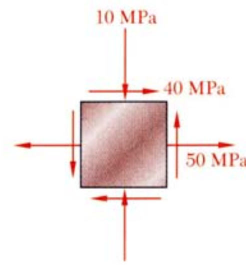
$$\tau_{nt} = R\sin(45.24) = 3\sin(45.24) = \boxed{2.13\text{ ksi}}$$

$$\sigma_n = C - R\cos(45.24) = 17 - 3(0.7) = \boxed{14.89\text{ ksi}}$$

Mohr's Circle for Plane Stress

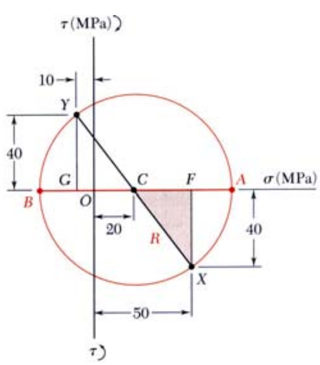
- Example 6

For the state of plane stress shown (a) construct Mohr's circle, determine (b) the principal planes (c) the principal stresses, (d) the maximum shearing stress and the corresponding normal stress.



Mohr's Circle for Plane Stress

- Example 6 (cont'd)



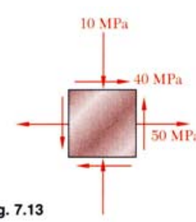


Fig. 7.13

SOLUTION:

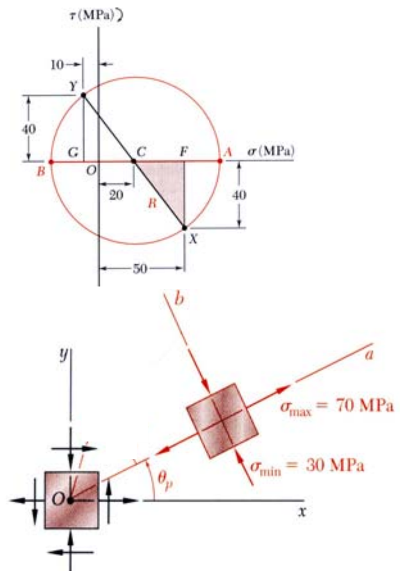
- Construction of Mohr's circle

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(50) + (-10)}{2} = 20\,\text{MPa}$$

$$CF = 50 - 20 = 30\,\text{MPa} \qquad FX = 40\,\text{MPa}$$

$$R = CX = \sqrt{(30)^2 + (40)^2} = 50\,\text{MPa}$$

Example 6 (cont'd)



- Principal planes and stresses

$\sigma_{max} = OA = OC + CA = 20 + 50$

$\boxed{\sigma_{max} = 70\,\text{MPa}}$

$\sigma_{max} = OB = OC - BC = 20 - 50$

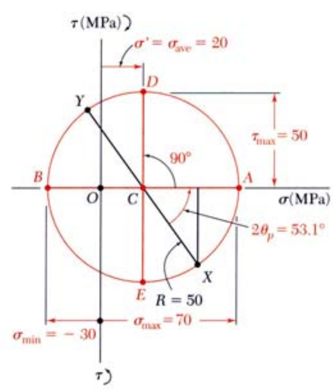
$\boxed{\sigma_{max} = -30\,\text{MPa}}$

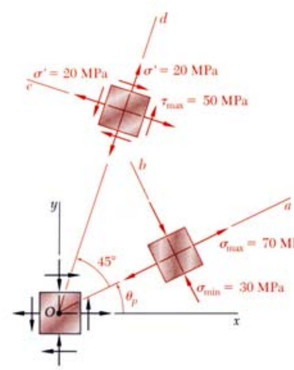
$$\tan 2\theta_p = \frac{FX}{CP} = \frac{40}{30}$$

$2\theta_p = 53.1°$

$\boxed{\theta_p = 26.6°}$

Example 6 (cont'd)





- Maximum shear stress

$\theta_s = \theta_p + 45°$ | $\tau_{max} = R$ | $\sigma' = \sigma_{ave}$

$\boxed{\theta_s = 71.6°}$ | $\boxed{\tau_{max} = 50\,\text{MPa}}$ | $\boxed{\sigma' = 20\,\text{MPa}}$