LECTURE

2

Chapter
2.1 - 2.7
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Third Edition

REVIEW: STRAIN, MATERIAL PROPERTIES, & CONST. RELATIONS

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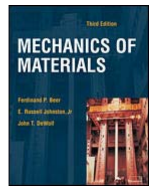

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SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering
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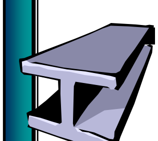
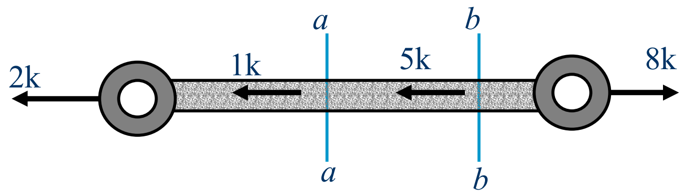
Displacement, Deformation, and Strain

- In the previous sections, focus was concentrated on the stresses created in various members and connections by the loads applied to a structure or machine.
- Also, methods for design of simple members using the ASD and LRFD were briefly discussed.

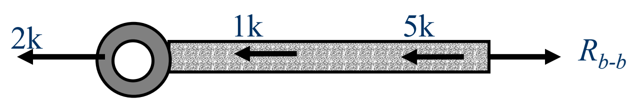


Displacement, Deformation, and Strain

- Analysis of Internal Forces

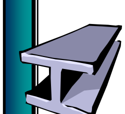


$$R_{b-b} = 8\text{ k}$$



Displacement, Deformation, and Strain

- Another important aspect of the analysis and design of a structures relates to the deformation caused by the loads applied to a structure.



- Excessive deformation that prevents a structure or machine from fulfilling the purpose for which it was intended should be avoided in the design.

Displacement, Deformation, and Strain

- Sometimes, it is not possible to determine the forces on a structure by applying only the equations of equilibrium.
- The analysis of deformation can help us in the determination of stresses, and consequently these forces.

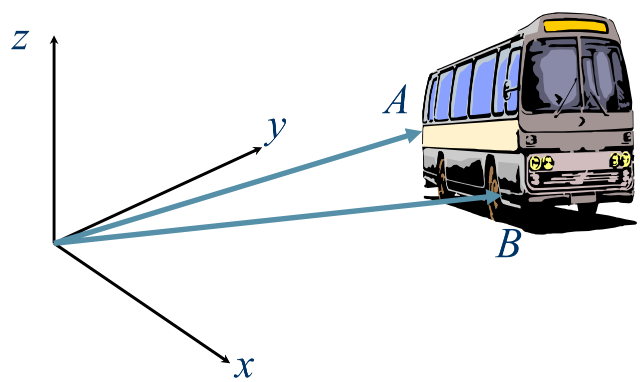
Displacement, Deformation, and Strain

- Displacement
  – Definition

Displacement can be defined as the movement of individual points on a structural system due to various external loads.

Displacement, Deformation, and Strain

- Displacement of Points on a Body



Displacement, Deformation, and Strain

- Displacement
  - This movement of a point with respect to some reference system of axes is a vector quantity known as *displacement.*
  - Displacement can be classified into:
    - Translation of points
    - Rotation of lines
    - Change of length;i,e., elongation and contraction.
    - Distortion; i,e., angle change between lines.

Displacement, Deformation, and Strain

- Displacement
  - The first two types of displacement occur because of movement of the entire body.
  - While, the latter two are associated with the *local deformation* of the body.
  - Our concern is primarily with displacement associated with local deformation.

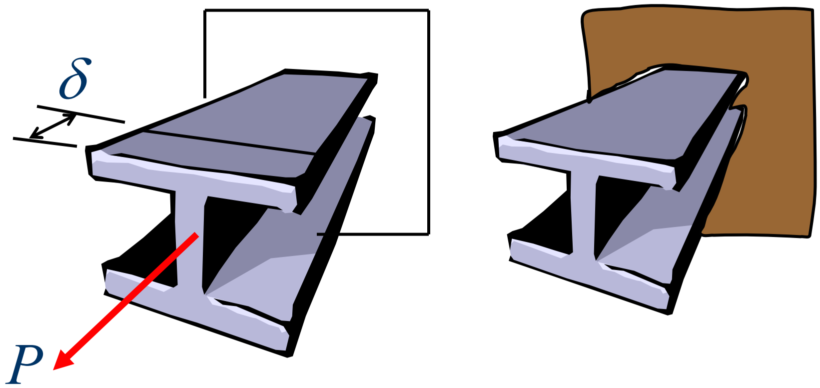
Displacement, Deformation, and Strain

- Deformation
  - Definition

When displacements induced by applied loads cause the size and/or shape of a body to be altered, individual points of the body move relative to one another. The change in any dimension associated with these relative displacements is defined as deformation.

Displacement, Deformation, and Strain

- Deformation : Body size change

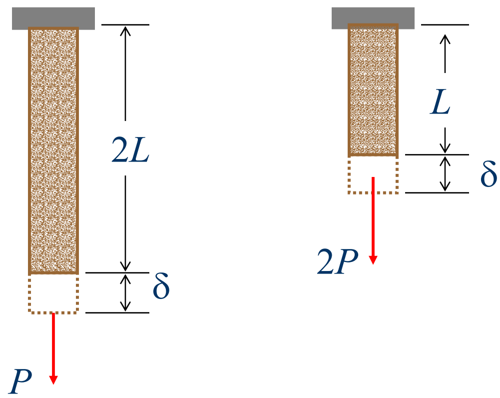


Displacement, Deformation, and Strain

- Deformation
  - Deformation is not uniquely related to forces or stress.
  - Two rods of identical material and identical cross-sectional area subjected to different loads, can have the same deformation δ, if the second is half in length.

Displacement, Deformation, and Strain

- Deformation of Two Rods

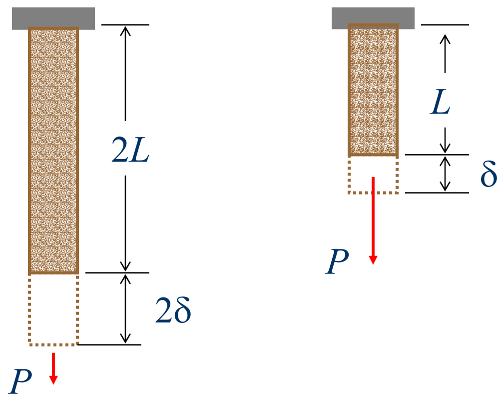


Displacement, Deformation, and Strain

- Deformation of Two Rods
  – If two rods of identical material and identical cross-sectional area subjected to identical loads, then deformation in the 2*L*-long rod, will be twice as large as the deformation in the *L*-long rod.

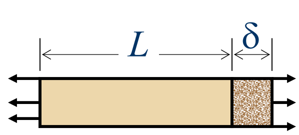
Displacement, Deformation, and Strain

- Deformation of Two Rods

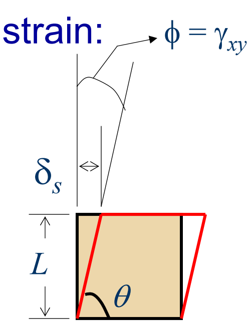


Displacement, Deformation, and Strain

- Strain
  - Two general types of strain:
    - Axial (normal) Strain
    - Shearing Strain



Normal Strain



Shearing Strain

Displacement, Deformation, and Strain

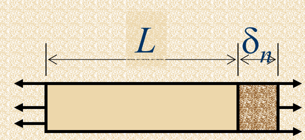
- Normal Strain
  - Definition

Normal strain can be defined as the deformation per unit length of a body when the body is subjected to normal axial loading

Displacement, Deformation, and Strain

- Average Axial Strain

$$\varepsilon_{\text{avg}} = \frac{\delta_n}{L}$$



- True Axial Strain

$$\varepsilon(p) = \lim_{\Delta L \to 0} \frac{\Delta \delta_n}{\Delta L} = \frac{d\delta_n}{dL}$$

Displacement, Deformation, and Strain

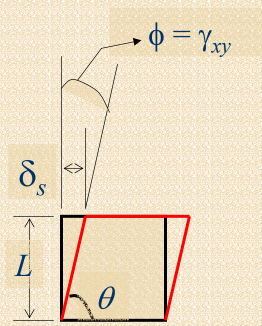
- Average Shearing Strain

$$\gamma_{\text{avg}} = \tan\phi$$

Since δ_s is vary small,

sin ϕ = tan ϕ = ϕ, therefore,

$$\gamma_{\text{avg}} = \frac{\delta_s}{L}$$



Displacement, Deformation, and Strain

- Units of Strain
  - Normal Strain
    - Normal strains are usually expressed in units of in/in or micro in/in (μin/in)
  - Shearing Strains
    - Shearing strains are frequently expressed in radians or micro radians (μradians)

Displacement, Deformation, and Strain

- Example 1

A 50-ft length of steel wire is subjected to a tensile load that produces a change in length of 1.25 in. Determine the axial strain in the wire

$$\varepsilon = \frac{\delta_n}{L} = \frac{\Delta L}{L} = \frac{1.25}{50(12)} = 0.002083 \text{ in/in} = 2083\mu$$

Displacement, Deformation, and Strain

- Example 2

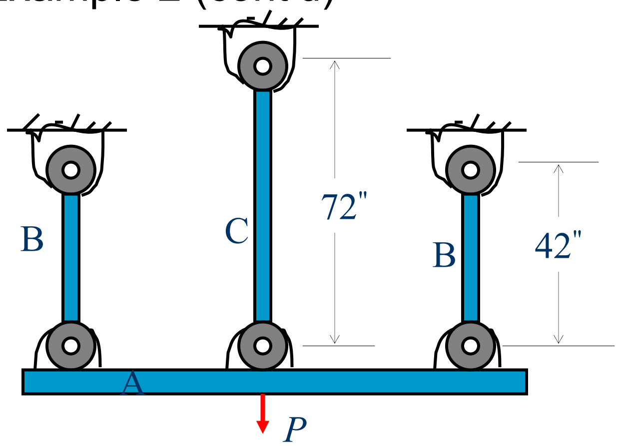
A rigid steel plate A is supported by three rods as shown. There is no strain in the rods before the load *P* is applied. After *P* is applied, the axial strain in rod *C* is 900μ in/in. Determine

(a) The axial strain in rods B.

(b) The axial strain in rods B if there is a 0.006-in clearance in the connections between A and B before the load is applied.

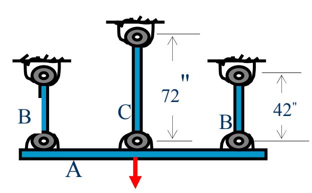
Displacement, Deformation, and Strain

- Example 2 (cont'd)



Displacement, Deformation, and Strain

- Example 2 (cont'd)



$$\varepsilon_C = \frac{\delta_C}{L_C} \Rightarrow \delta_C = \varepsilon_C L_C = 900\times10^{-6}(72) = 0.0648\text{ in}$$

a) $$\varepsilon_B = \frac{\delta_B}{L_B} = \frac{0.0648}{42} = 0.001543 = 1543\mu$$

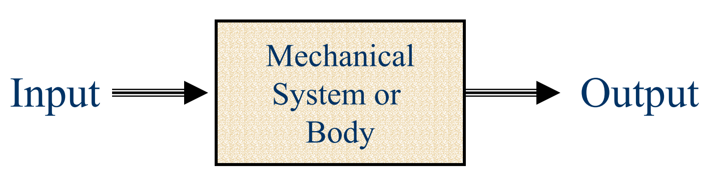
a) $$\varepsilon_B = \frac{\delta_B}{L_B} = \frac{0.0648-0.006}{42} = 0.001400 = 1400\mu$$

Stress-Strain-Temperature Relationships

- Introduction
  - The thing that distinguishes one system from another is the relationship of the **input** to the **output.**
  - A fundamental problem in engineering is to determine the input-output relation for a given system.

Stress-Strain-Temperature Relationships

- Introduction
  - One one way to do this would be to take the system and measure experimentally the inputs and outputs.



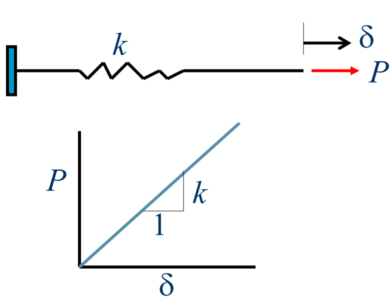
Stress-Strain-Temperature Relationships

- Mechanical Models
  - If the material is a solid the *output* is measured in terms of the *deformation* produced by the force.
  - If the material id fluid, the corresponding *output* is measured in terms of some flow rate.

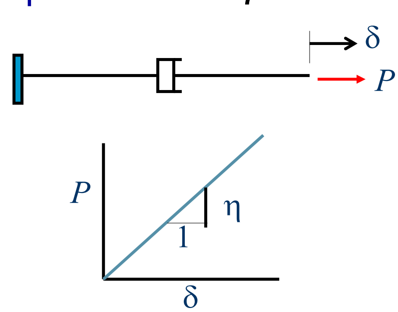
Input (force) ⇒ output (deformation, flow rate)

Stress-Strain-Temperature Relationships

- Linear Models
  - Linear elastic Response $P = k\delta$
  - Linear Viscous response $P = \eta\delta$



Elastic element (spring)



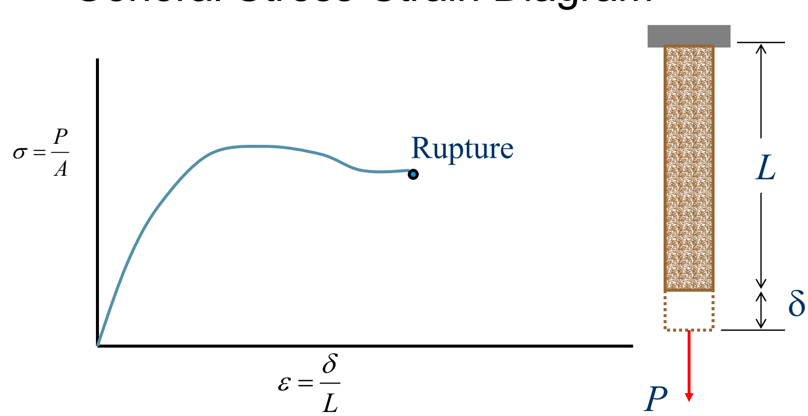
Viscous element (dashpot)

Stress-Strain-Temperature Relationships

- Stress-Strain Diagrams (curves)
  - The simplest and most common experiment for measuring the mechanical response of engineering structural materials is the uniaxial tensile or compression test.
  - In this test, a lowly increasing (quasi-static) load is applied to a specimen of material and the resulting deformation data are recorded.

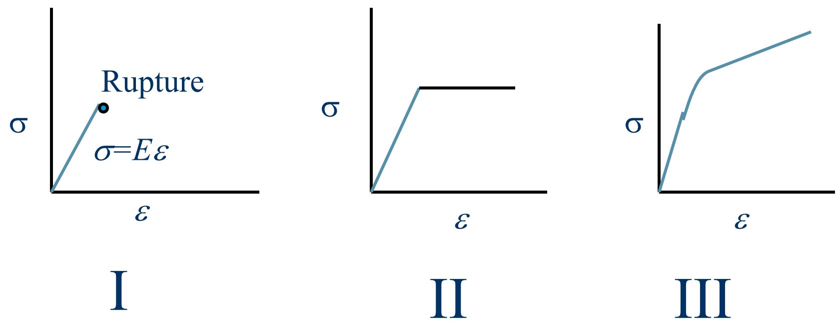
Stress-Strain-Temperature Relationships

- General Stress-Strain Diagram



Stress-Strain-Temperature Relationships

- Idealized Stress-Strain Curves

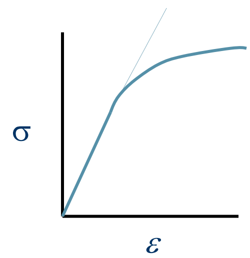


Stress-Strain-Temperature Relationships

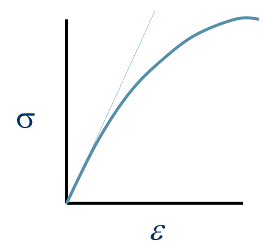
- Idealized Stress-Strain Curves
  - Glass and some other brittle material have curves that are practically linear as illustrated by type I.
  - The first part of the curve for structural or mild steel (common material) is like that of type II.
  - A commonly used aluminum alloy, 2024-T4, has a curve similar to that of type III.

Stress-Strain-Temperature Relationships

- Idealized Stress-Strain Curves



IV

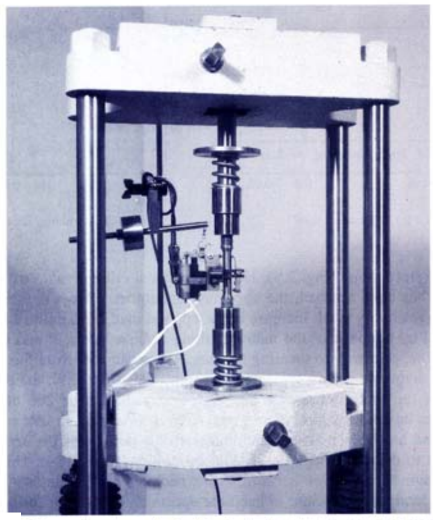


V

Stress-Strain-Temperature Relationships

- Idealized Stress-Strain Curves
  - Stress-strain curves for cast iron and concrete sometimes do not have any initial linear range, as illustrated by type V.
  - The curves for many other engineering materials approximate that of type IV.
  - Each specific material has a different value of the modulus of elasticity *E*.

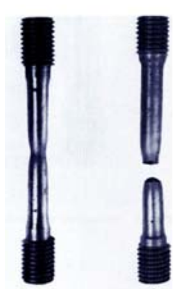
Stress-Strain Test

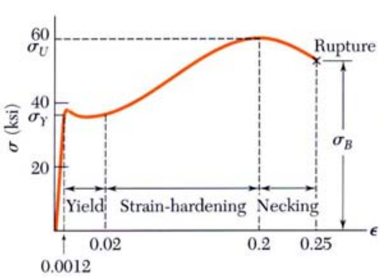


This machine is used to test tensile test specimens, such as those shown in this chapter.

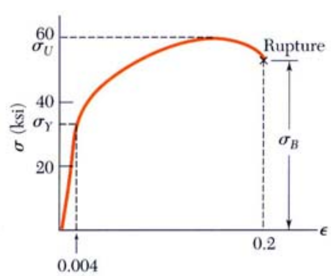
Test specimen with tensile load.

Stress-Strain Diagram: Ductile Materials



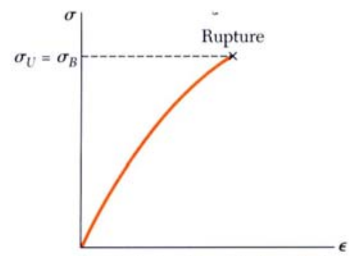


(a) Low-carbon steel



(b) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



Stress-strain diagram for a typical brittle material.

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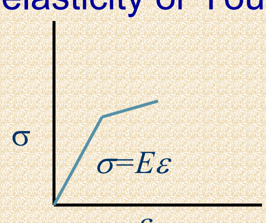
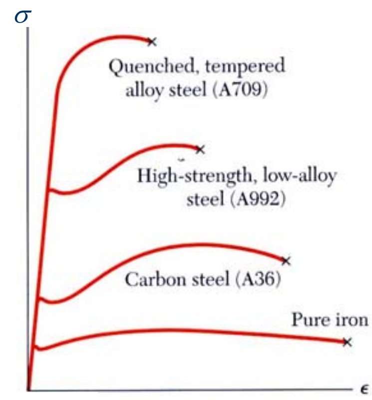
Stress-Strain-Temperature Relationships

- Modulus of Elasticity, *E*
  - The initial portion of the stress-strain curve (diagram) is a straight line. The equation for this straight line is called the modulus of elasticity or Young's Modulus *E*



$$\sigma = E\varepsilon$$

Hooke's Law: Modulus of Elasticity



Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

  $\sigma = E\varepsilon$

  E = Youngs Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

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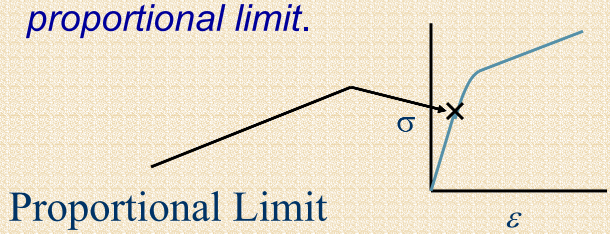
Stress-Strain-Temperature Relationships

- Shear Modulus of Elasticity, *G*
  - The shear modulus is similar to the modulus of elasticity. However it is applied to shear stress-strain.

$$\tau = G\gamma$$

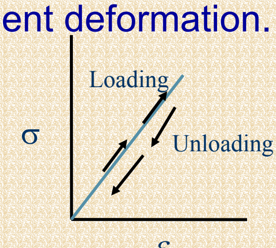
Stress-Strain-Temperature Relationships

- Proportional Limit
  – The maximum stress for which stress and strain are proportional is called the *proportional limit*.

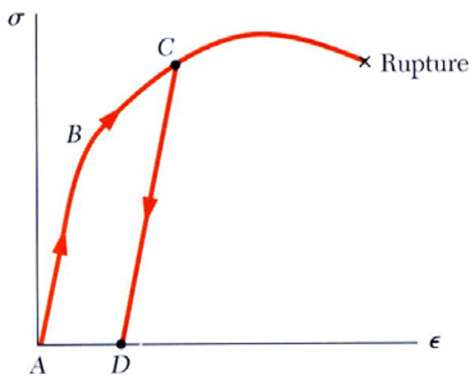


Stress-Strain-Temperature Relationships

- Elastic Limit
  – The elastic limit is the maximum stress for which the material acts elastically, and in which the maximum load does not cause permanent deformation.



Elastic vs. Plastic Behavior



- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

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Stress-Strain-Temperature Relationships

- Yield Strength
  - The yield strength is defined as the stress that will induce permanent set, usually 0.05 to 0.3 percent (which is equivalent to a strain of 0.0005 to 0.003).
- Ultimate Strength
  - The maximum stress developed in a material before rupture is called ultimate strength.

Stress-Strain-Temperature Relationships

- Poisson's Ratio
  - A material loaded in one direction will undergo strains perpendicular to the direction of the load in addition to those parallel to the load. The ratio of the lateral or perpendicular strain to the longitudinal or axial strain is called Poisson's ratio.

$$\nu = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = \frac{\varepsilon_l}{\varepsilon_a} \qquad E = 2(1+\nu)G$$

Stress-Strain-Temperature Relationships

- Temperature Strain
  - Most materials when unstrained expand when heated and contract when cooled.
  - The thermal strain due to one degree (1^0) change in temperature is given by α and is known is the coefficient of thermal expansion

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Stress-Strain-Temperature Relationships

- Thermal Strain
  - The thermal strain due a temperature change of ΔT degrees is given by

$$\varepsilon_T = \alpha \Delta T$$

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Stress-Strain-Temperature Relationships

- Total Strain
  - The sum of the normal strain caused by the loads and the thermal strain is called the total strain, and it is given by

$$\varepsilon_{\text{total}} = \varepsilon_\sigma + \varepsilon_T = \frac{\sigma}{E} + \alpha \, \Delta T$$

Stress-Strain-Temperature Relationships

- Example 3

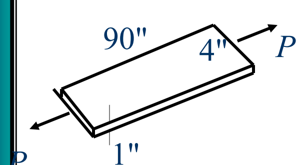
A 100-kip axial load is applied to a 1×4 ×90 -in. rectangular bar. When loaded , the 4-in. side measures 3.9986 in., and the length has increased 0.09 in. Determine Poisson's ratio, Young's modulus, and the modulus of rigidity of the material.

Stress-Strain-Temperature Relationships

- Example 3 (cont'd)

The lateral and longitudinal strains and the axial stress for the bar can be determined as follows:

$$\delta_{\text{lat}} = 3.9986 - 4 = -0.0014 \text{ in}$$



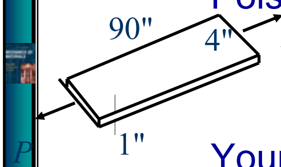
$$\varepsilon_{\text{lat}} = \frac{\delta_{\text{lat}}}{L} = \frac{-0.0014}{4} = -0.00035$$

$$\varepsilon_{\text{long}} = \frac{\delta_{\text{long}}}{L} = \frac{0.09}{90} = 0.00100$$

$$\sigma = \frac{P}{A} = \frac{100}{4(1)} = 25 \text{ ksi}$$

Stress-Strain-Temperature Relationships

- Example 3 (cont'd)



Poisson's ratio:

$$\nu = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = \frac{0.00035}{0.00100} = 0.35$$

Young's Modulus and Modulus of Rigidity:

$$E = \frac{\sigma}{\varepsilon} = \frac{25}{0.00100} = 25{,}000 \text{ ksi}$$

$$G = \frac{E}{2(1+\nu)} = \frac{25{,}000}{2(1+0.35)} = 9260 \text{ ksi}$$