

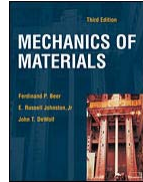


15

Chapter 6.6 – 6.7

BEAMS: SHEAR FLOW, THIN WALLED MEMBERS

A. J. Clark School of Engineering • Department of Civil and Environmental Engineering



by

Dr. Ibrahim A. Assakkaf

SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

LECTURE 15. BEAMS: SHEAR FLOW, THIN-WALLED MEMBERS (6.6 – 6.7)
Slide No. 1

Shear on the Horizontal Face of a Beam Element

The diagrams illustrate the derivation of shear flow. The top diagram shows a beam of length \$x\$ between points A and B, with point C at distance \$x\$. It is subjected to point loads \$P_1\$ and \$P_2\$, and a distributed load \$w\$. The middle diagram shows a beam element of length \$\Delta x\$ between points C and D, with a neutral axis (N.A.) at a distance \$y_1\$ from the centerline. Shear stresses \$\sigma_c\$ and \$\sigma_d\$ are shown on the vertical faces of the element. The bottom diagram shows the same beam element with shear forces \$V_C\$ and \$V_D\$ acting on its ends, and a shear flow \$q\$ acting on its horizontal face.

- Consider prismatic beam
- For equilibrium of beam element

$$\sum F_x = 0 = \Delta H + \int_A (\sigma_D - \sigma_C) dA$$

$$\Delta H = \frac{M_D - M_C}{I} \int_A y dA$$
- Note,

$$Q = \int_A y dA$$

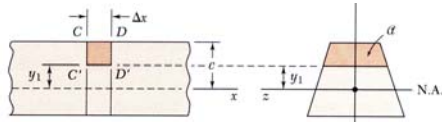
$$M_D - M_C = \frac{dM}{dx} \Delta x = V \Delta x$$
- Substituting,

$$\Delta H = \frac{VQ}{I} \Delta x$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = \underline{\underline{\textit{shear flow}}}$$



Shear on the Horizontal Face of a Beam Element



- Shear flow,

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = \text{shear flow}$$

- where

$$Q = \int_A y dA$$

= first moment of area above y_1

$$I = \int_{A+A'} y^2 dA$$

= second moment of full cross section

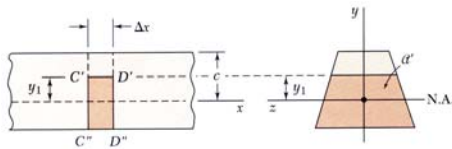
- Same result found for lower area

$$q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I} = -q'$$

$$Q + Q' = 0$$

= first moment with respect to neutral axis

$$\Delta H' = -\Delta H$$



Shearing Stress in Beams

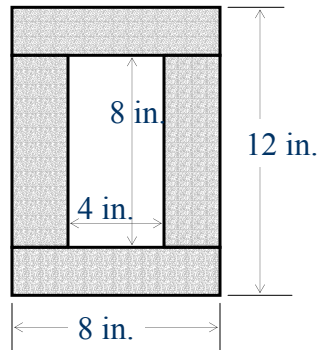
■ Example 16

The transverse shear V at a certain section of a timber beam is 600 lb. If the beam has the cross section shown in the figure, determine (a) the vertical shearing stress 3 in. below the top of the beam, and (b) the maximum vertical stress on the cross section.



Shearing Stress in Beams

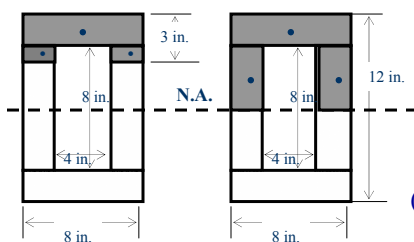
■ Example 16 (cont'd)



Shearing Stress in Beams

■ Example 16 (cont'd)

From symmetry, the neutral axis is located 6 in. from either the top or bottom edge.



$$I = \frac{8(12)^3}{12} - \frac{4(8)^3}{12} = 981.3 \text{ in}^4$$

$$Q_{3'} = 8(2)(5) + 2[1(2)(3.5)] = 94.0 \text{ in}^3$$

$$Q_{NA} = 8(2)(5) + 2[2(2)(4)] = 112.0 \text{ in}^3$$

$$(a) \tau_{Q_{3'}} = \frac{VQ_{3'}}{It} = \frac{6000(94)}{981.3(4)} = 143.7 \text{ psi}$$

$$(b) \tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6000(112)}{981.3(4)} = 171.2 \text{ psi}$$



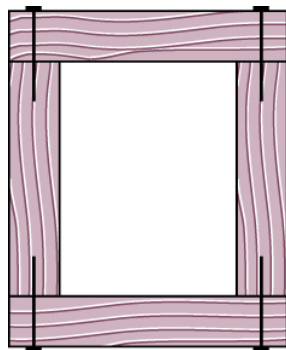
Longitudinal Shear on a Beam Element of Arbitrary Shape

- Consider a box beam obtained by nailing together four planks as shown in Fig. 1.
- The shear per unit length (Shear flow) q on a horizontal surfaces along which the planks are joined is given by

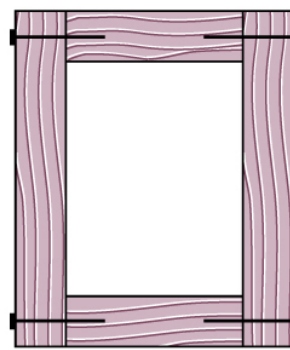
$$q = \frac{VQ}{I} = \text{shear flow} \quad (1)$$



Longitudinal Shear on a Beam Element of Arbitrary Shape



(a)



(b)

Figure 1

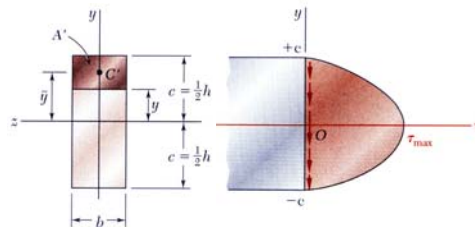


Longitudinal Shear on a Beam Element of Arbitrary Shape

- But could q be determined if the planks had been joined along *vertical surfaces*, as shown in Fig. 1b?
- Previously, we had examined the distribution of the vertical components τ_{xy} of the stresses on a transverse section of a W-beam or an S-beam as shown in the following viewgraph.



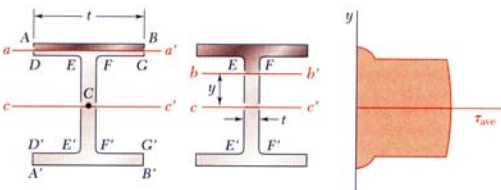
Shearing Stresses τ_{xy} in Common Types of Beams



- For a narrow rectangular beam,

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{3V}{2A} \left(1 - \frac{y^2}{c^2} \right)$$

$$\tau_{\max} = \frac{3V}{2A}$$



- For American Standard (S-beam) and wide-flange (W-beam) beams

$$\tau_{\text{ave}} = \frac{VQ}{It}$$

$$\tau_{\max} = \frac{V}{A_{\text{web}}}$$

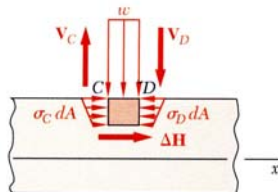
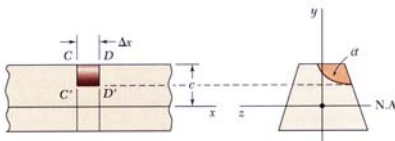
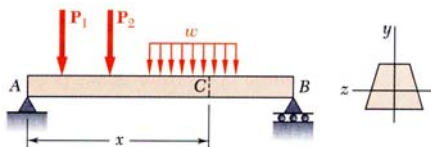


Longitudinal Shear on A Beam Element of Arbitrary Shape

- But what about the *horizontal* component τ_{xz} of the stresses in the flanges?
- To answer these questions, the procedure developed earlier must be extended for the determination of the shear per unit length q so that it will apply to the cases just described.



Longitudinal Shear on a Beam Element of Arbitrary Shape



- We have examined the distribution of the vertical components τ_{xy} on a transverse section of a beam. We now wish to consider the horizontal components τ_{xz} of the stresses.
- Consider prismatic beam with an element defined by the curved surface CDD'C'.

$$\sum F_x = 0 = \Delta H + \int_a (\sigma_D - \sigma_C) dA$$

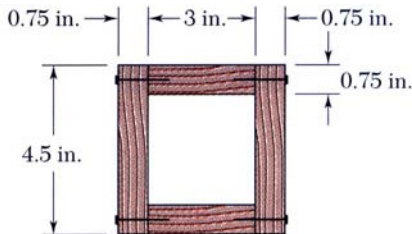
- Except for the differences in integration areas, this is the same result obtained before which led to

$$\Delta H = \frac{VQ}{I} \Delta x \quad q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$



Shearing Stress in Beams

■ Example 17



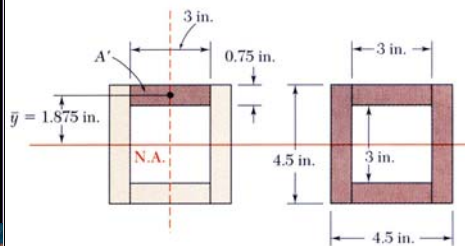
A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.75 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.
- Based on the spacing between nails, determine the shear force in each nail.



Example 17 (cont'd)



For the upper plank,

$$Q = A'y = (0.75 \text{ in.})(3 \text{ in.})(1.875 \text{ in.}) = 4.22 \text{ in}^3$$

For the overall beam cross-section,

$$I = \frac{1}{12}(4.5 \text{ in.})^3 - \frac{1}{12}(3 \text{ in.})^3 = 27.42 \text{ in}^4$$

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.

$$q = \frac{VQ}{I} = \frac{(600 \text{ lb})(4.22 \text{ in}^3)}{27.42 \text{ in}^4} = 92.3 \frac{\text{lb}}{\text{in}}$$

$$f = \frac{q}{2} = 46.15 \frac{\text{lb}}{\text{in}} = \text{edge force per unit length}$$

- Based on the spacing between nails, determine the shear force in each nail.

$$F = f \ell = \left(46.15 \frac{\text{lb}}{\text{in}}\right)(1.75 \text{ in.})$$

$$F = 80.8 \text{ lb}$$



Shearing Stress in Thin-Walled Members

- It was noted earlier that Eq. 1 can be used to determine the shear flow in an arbitrary shape of a beam cross section.
- This equation will be used in this section to calculate both the shear flow and the average shearing stress in thin-walled members such as flanges of wide-flange beams (Fig. 2) and box beams or the walls of structural tubes.



Shearing Stress in Thin-Walled Members

Wide-Flange Beams



Figure 2



Shearing Stress in Thin-Walled Members

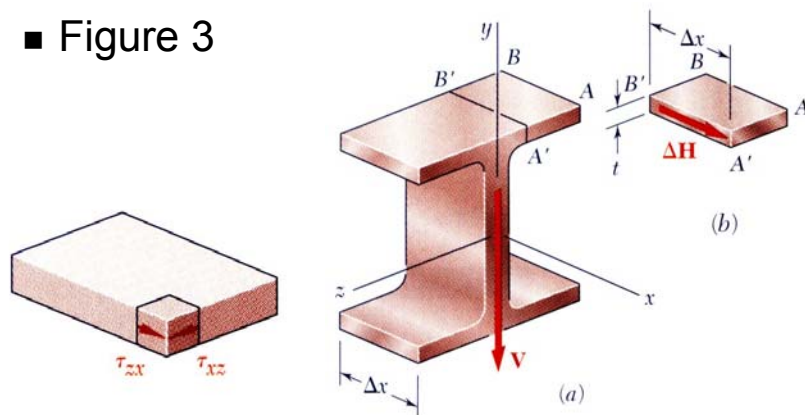
- Consider a segment of a wide-flange beam subjected to the vertical shear V .
- The longitudinal shear force on the element is

$$\Delta H = \frac{VQ}{I} \Delta x \quad (2)$$



Shearing Stress in Thin-Walled Members

- Figure 3





Shearing Stress in Thin-Walled Members

- The corresponding shear stress is

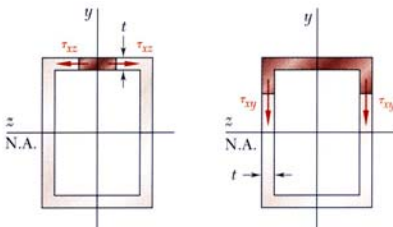
$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t \Delta x} = \frac{VQ}{It} \quad (3)$$

- Previously found a similar expression for the shearing stress in the web

$$\tau_{xy} = \frac{VQ}{It} \quad (4)$$



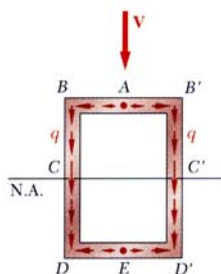
Shearing Stress in Thin-Walled Members



- The variation of shear flow across the section depends only on the variation of the first moment.

$$q = \tau t = \frac{VQ}{I}$$

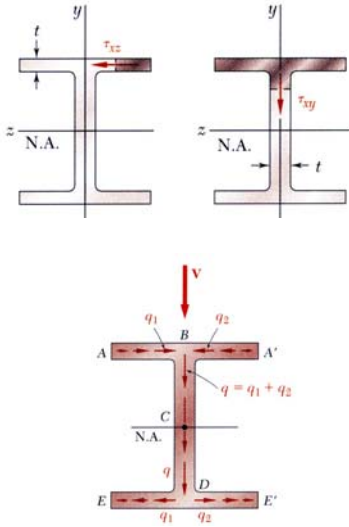
- For a box beam, q grows smoothly from zero at A to a maximum at C and C' and then decreases back to zero at E .



- The sense of q in the horizontal portions of the section may be deduced from the sense in the vertical portions or the sense of the shear V .



Shearing Stress in Thin-Walled Members



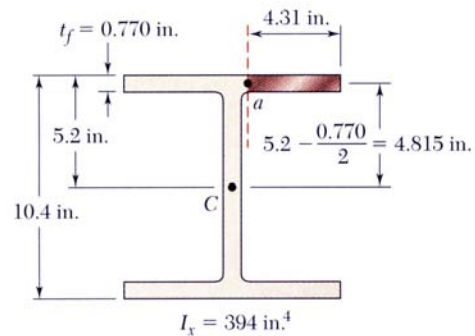
- For a wide-flange beam, the shear flow increases symmetrically from zero at A and A' , reaches a maximum at C and the decreases to zero at E and E' .
- The continuity of the variation in q and the merging of q from section branches suggests an analogy to fluid flow.



Shearing Stress in Thin-Walled Members

■ Example 18

Knowing that the vertical shear is 50 kips in a W10 × 68 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point a .





Shearing Stress in Thin-Walled Members

■ Example 18 (cont'd)

SOLUTION:

- For the shaded area,

$$Q = (4.31 \text{ in})(0.770 \text{ in})(4.815 \text{ in}) \\ = 15.98 \text{ in}^3$$

- The shear stress at a ,

$$\tau = \frac{VQ}{It} = \frac{(50 \text{ kips})(15.98 \text{ in}^3)}{(394 \text{ in}^4)(0.770 \text{ in})}$$

$$\tau = 2.63 \text{ ksi}$$

