LECTURE

11

Chapter 4.6 – 4.7

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Third Edition

BEAMS: COMPOSITE BEAMS; STRESS CONCENTRATIONS

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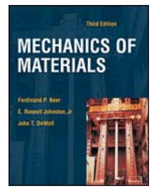

by

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SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

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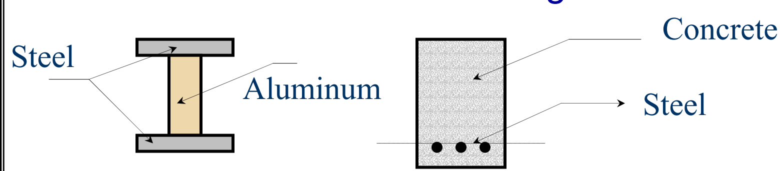
Composite Beams

- Bending of Composite Beams
  - In the previous discussion, we have considered only those beams that are fabricated from a single material such as steel.
  - However, in engineering design there is an increasing trend to use beams fabricated from two or more materials.

Composite Beams

- Bending of Composite Beams
  - These are called ***composite beams***.
  - They offer the opportunity of using each of the materials employed in their construction advantage.



Composite Beams

- Foam Core with Metal Cover Plates
  - Consider a composite beam made of metal cover plates on the top and bottom with a plastic foam core as shown by the cross sectional area of Figure 26.
  - The design concept of this composite beam is to use light-low strength foam to support the load-bearing metal plates located at the top and bottom.

Composite Beams

- Foam Core with Metal Cover Plates

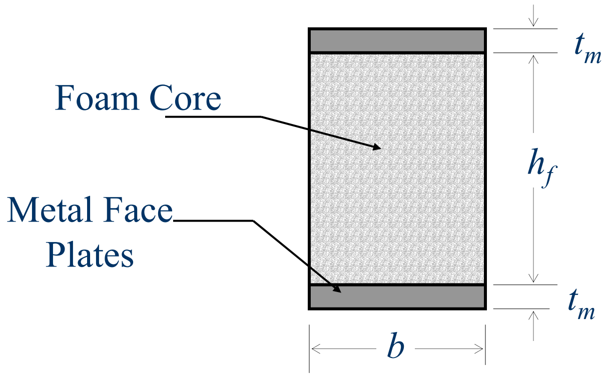


Figure 26

Composite Beams

- Foam Core with Metal Cover Plates
  - The strain is continuous across the interface between the foam and the cover plates. The stress in the foam is given by

$$\sigma_f = E_f \varepsilon \approx 0 \qquad (53)$$

  - The stress in the foam is considered zero because its modulus of elasticity E_f is small compared to the modulus of elasticity of the metal.

Composite Beams

- Foam Core with Metal Cover Plates
  - Assumptions:
    - Plane sections remain plane before and after loading.
    - The strain is linearly distributed as shown in Figure 27.

Composite Beams

- Foam Core with Metal Cover Plates

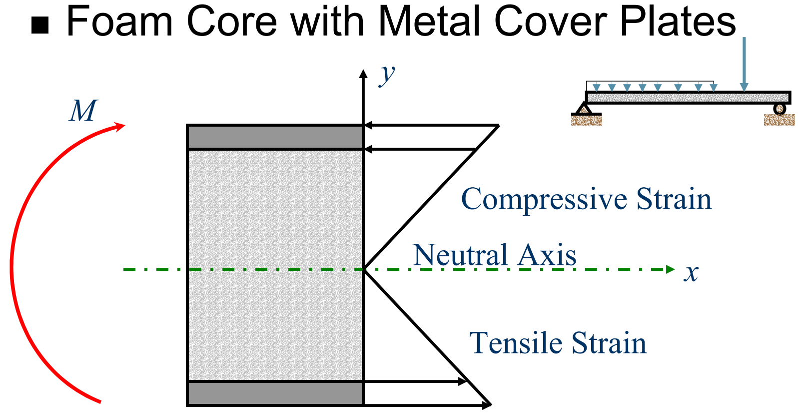


Figure 27

Composite Beams

- Foam Core with Metal Cover Plates
  - Using Hooke's law, the stress in the metal of the cover plates can be expressed as

$$\sigma_m = \varepsilon E_m = -\frac{y}{\rho} E_m \qquad (53)$$

but $E_m / \rho = M / I_x$, therefore

$$\sigma_m = -\frac{My}{I_x} \qquad (54)$$

Composite Beams

- Foam Core with Metal Cover Plates
  - The relation for the stress is the same as that established earlier; however, the foam does not contribute to the load carrying capacity of the beam because its modulus of elasticity is negligible.
  - For this reason, the foam is not considered when determining the moment of inertia I_x.

Composite Beams

- Foam Core with Metal Cover Plates
  - Under these assumptions, the moment of inertia about the neutral axis is given by

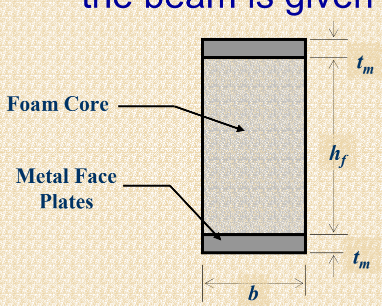
$$I_{NA} \cong 2Ad^2 = 2\left[bt_m\left(\frac{h_f t_m}{2}\right)^2\right] = \frac{bt_m}{2}\left(h_f + t_m\right)^2 \qquad (55)$$

  - Combining Eqs 54 and 55, the maximum stress in the metal is computed as

$$\sigma_{\max} = \frac{M\left(h_f + 2t_m\right)}{bt_m\left(h_f + t_m\right)^2} \qquad (56)$$

Composite Beams

- Foam Core with Metal Cover Plates
  - The maximum stress in the metal plates of the beam is given by



$$\sigma_{\max} = \frac{M\left(h_f + 2t_m\right)}{bt_m\left(h_f + t_m\right)^2} \qquad (56)$$

Composite Beams

- Example 1

  A simply-supported, foam core, metal cover plate composite beam is subjected to a uniformly distributed load of magnitude *q*. Aluminum cover plates 0.063 in. thick, 10 in. wide and 10 ft long are adhesively bonded to a polystyrene foam core. The foam is 10 in. wide, 6 in. high, and 10 ft long. If the yield strength of the aluminum cover plates is 32 ksi, determine *q*.

Composite Beams

- Example 1 (cont'd)

  The maximum moment for a simply supported beam is given by

$$M_{\max} = \frac{qL^2}{8} = \frac{q(10\times 12)^2}{8} = 1800q$$

  When the composite beam yields, the stresses in the cover plates are

$$\sigma_{\max} = F_y = 32{,}000 \text{ psi}$$

Composite Beams

- Example 1 (cont'd)

  Substituting above values for M_{max} and σ_{max} into Eq. 56, we get

$$\sigma_{max} = \frac{M(h_f + 2t_m)}{bt_m(h_f + t_m)^2}$$

$$32{,}000 = \frac{1800q(6 + 2 \times 0.063)}{10(0.063)[6 + 0.063]^2}$$

  Or

$$q = 67.2 \frac{\text{lb}}{\text{in}} = 806 \frac{\text{lb}}{\text{ft}}$$

Composite Beams

- Bending of Members Made of Several Materials
  - The derivation given for foam core with metal plating was based on the assumption that the modulus of elasticity E_f of the foam is so negligible,that is, it does not contribute to the load-carrying capacity of the composite beam.

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Composite Beams

- Bending of Members Made of Several Materials
  - When the moduli of elasticity of various materials that make up the beam structure are not negligible and they should be accounted for, then procedure for calculating the normal stresses and shearing stresses on the section will follow different approach, the *transformed section of the member.*

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Composite Beams

- Transformed Section
  - Consider a bar consisting of two portions of different materials bonded together as shown in Fig. 28. This composite bar will deform as described earlier.
  - Thus the normal strain ε_x still varies linearly with the distance *y* from the neutral axis of the section (see Fig 28b), and the following formula holds:

$$\varepsilon_x = -\frac{y}{\rho} \qquad (57)$$

Composite Beams

- Transformed Section

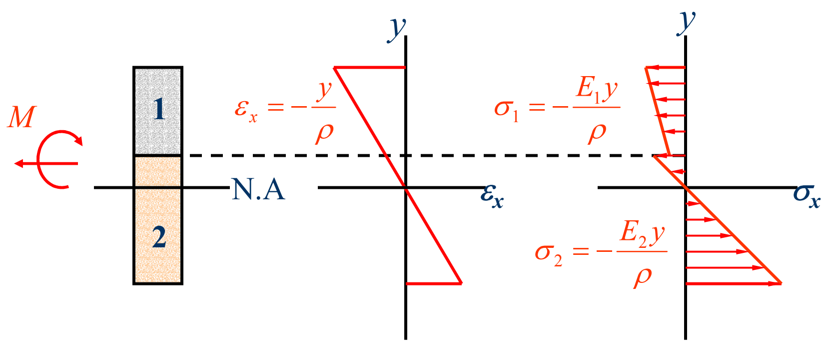


Figure 28 (a) (b) (c)

Composite Beams

- Transformed Section
  - Because we have different materials, we cannot simply assume that the neutral axis passes through the centroid of the composite section.
  - In fact one of the goal of this discussion will be to determine the location of this axis.

Composite Beams

- Transformed Section

  We can write:

$$\sigma_1 = E_1\varepsilon_x = -\frac{E_1 y}{\rho} \qquad (58a)$$

$$\sigma_2 = E_2\varepsilon_x = -\frac{E_2 y}{\rho} \qquad (58b)$$

  From Eq. 58, it follows that

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \qquad (59a)$$

$$dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA \qquad (59b)$$

Composite Beams

- Transformed Section
  - But, denoting by *n* the ratio E_2/E_1 of the two moduli of elasticity, dF_2 can expressed as

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n dA) \qquad (60)$$

  - Comparing Eqs. 59a and 60, it is noted that the same force dF_2 would be exerted on an element of area *n dA* of the first material.

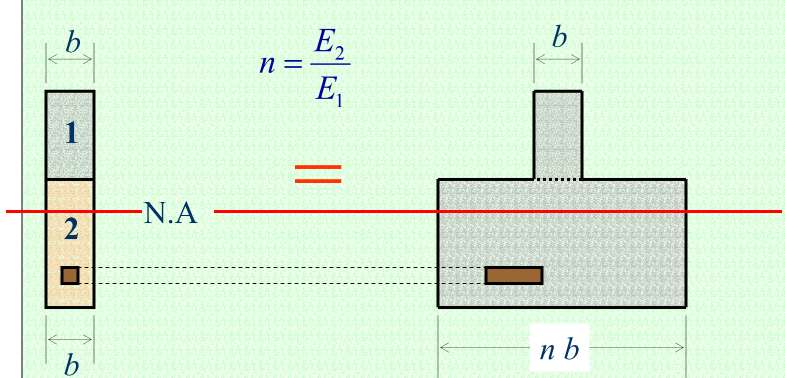
Composite Beams

- Transformed Section
  - This mean that the resistance to bending of the bar would remain the same if both portions were made of the first material, providing that the ***width*** of each element of the lower portion were multiplied by the factor ***n***.
  - The widening (if $n>1$) and narrowing ($n<1$) must be accomplished in a direction parallel to the neutral axis of the section.

Composite Beams

- Transformed Section

Figure 29



Composite Beams

- Transformed Section
  - Since the transformed section represents the cross section of a member made of a homogeneous material with a modulus of elasticity E_1,the previous method may be used to find the neutral axis of the section and the stresses at various points of the section.
  - Figure 30 shows the fictitious distribution of normal stresses on the section.

Composite Beams

- Transformed Section

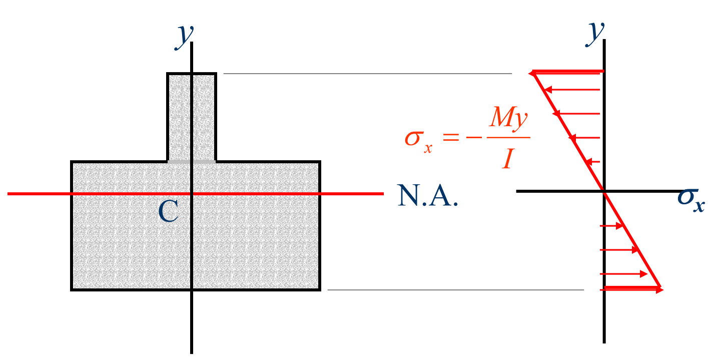


Figure 30. Distribution of Fictitious Normal Stress on Cross Section

Composite Beams

- Stresses on Transformed Section
  1. To obtain the stress σ_1 at a point located in the upper portion of the cross section of the original composite beam, the stress is simply computed from *My/I*.
  2. To obtain the stress σ_2 at a point located in the upper portion of the cross section of the original composite beam, stress σ_x computed from *My/I* is multiplied by *n*.

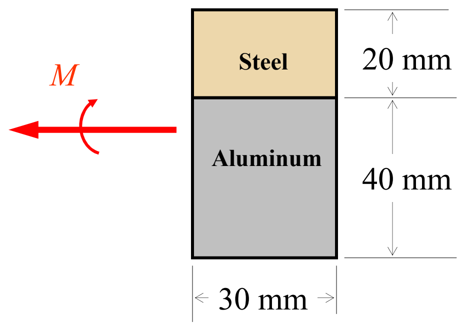
Composite Beams

- Example 2

  A steel bar and aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for streel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a moment M = 1500 N-m, determine the maximum stress in (a) the aluminum and (b) the steel.

Composite Beams

- Example 2 (cont'd)



Composite Beams

- Example 2 (cont'd)

  First, because we have different materials, we need to transform the section into a section that represents a section that is made of homogeneous material, either steel or aluminum.

  We have

$$n = \frac{E_s}{E_a} = \frac{200}{70} = 2.857$$

Composite Beams

- Example 2 (cont'd)

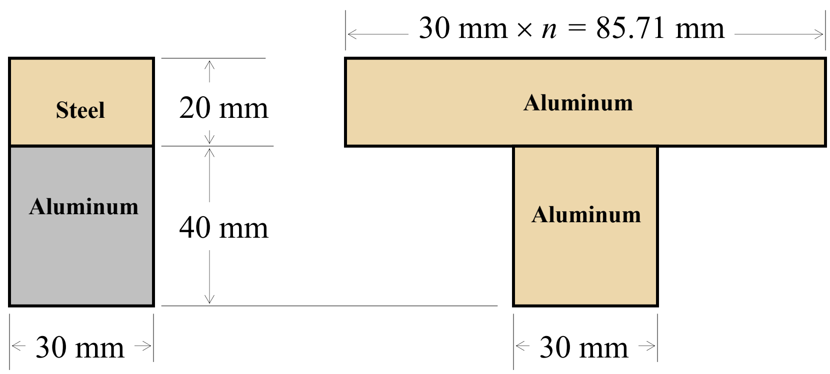


Figure 31a **Figure 31b**

Composite Beams

- Example 2 (cont'd)

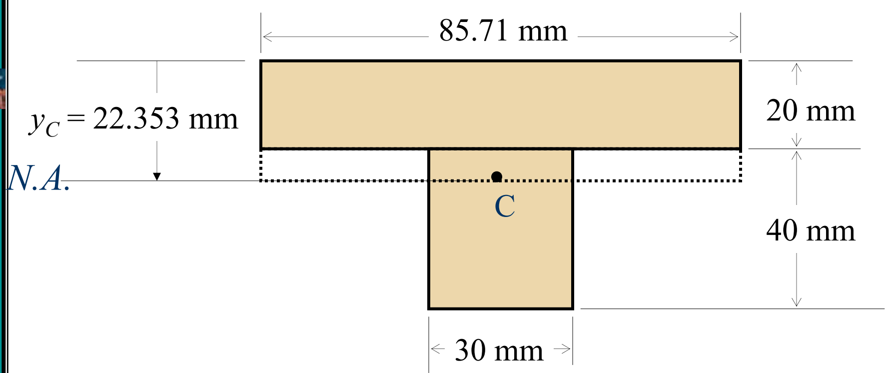
  Consider the transformed section of Fig. 31b, therefore

$$y_C = \frac{10(85.71\times 20)+40(30\times 40)}{(85.71\times 20)+(30\times 40)} = 22.353 \text{ mm from top}$$

$$I_{NA} = \frac{85.71(22.353)^3}{3} - \frac{(85.71-30)(22.353-20)^3}{3} + \frac{30(40+20-22.353)^3}{3} = 852.42\times 10^3 \text{ mm}^4 = 852.42\times 10^{-9} \text{ m}^4$$

Composite Beams

- Example 2 (cont'd)



Composite Beams

- Example 2 (cont'd)
  a) Maximum normal stress in aluminum occurs at extreme lower fiber of section, that is at y = -(20+40-22.353) = -37.65 mm.

$$\sigma_{al} = -\frac{My}{I} = -\frac{1500\left(-37.65\times10^{-3}\right)}{852.42\times10^{-9}} = 66.253\times10^{6}\,\text{Pa}$$

$$= +66.253\ \text{MPa (T)}$$

Composite Beams

- Example 2 (cont'd)
  - b) Maximum normal stress in stelel occurs at extreme upper fiber of the cross section, that is. at y =+ 22.353 mm.

$$\sigma_{St} = -n\frac{My}{I} = -(2.867)\frac{1500\left(22.353\times10^{-3}\right)}{852.42\times10^{-9}} = -112.8\times10^{6}\,\text{Pa}$$
$$= 112.8\,\text{MPa (C)}$$

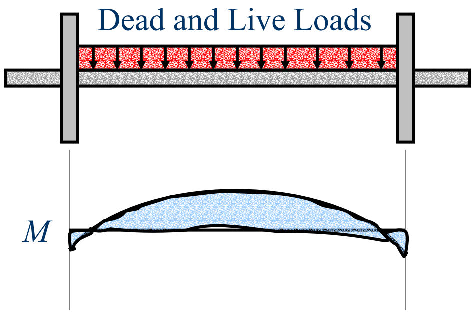
Composite Beams

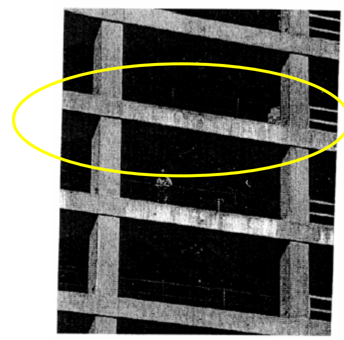
- Reinforced Concrete Beam
  - – An important example of structural members made of different materials is demonstrated by reinforced concrete beams.
  - – These beams, when subjected to positive bending moments, are reinforced by steel rods placed a short distance above their lower face as shown in Figure 33a.

Composite Beams

- Reinforced Concrete Beam

Figure 32

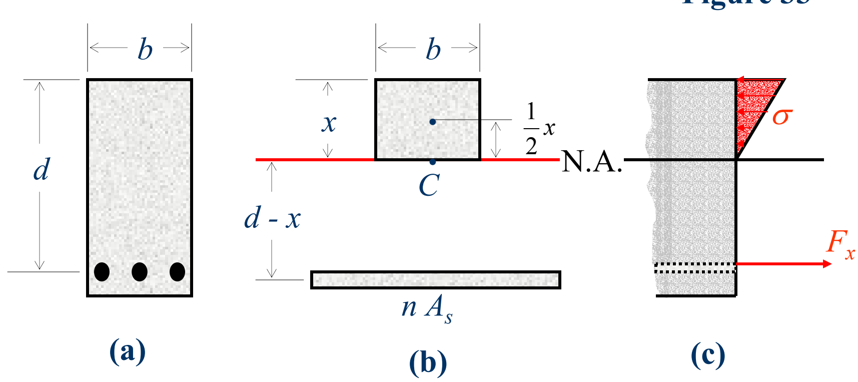




Composite Beams

- Reinforced Concrete Beam

Figure 33



Composite Beams

- Reinforced Concrete Beam
  - Concrete is very weak in tension, so it will crack below the neutral surface and the steel rods will carry the entire tensile load.
  - The upper part of the concrete beam will carry the compressive load.
  - To obtain the transformed section, the total cross-sectional area A_s of steel bar is replaced by an equivalent area nA_s.

Composite Beams

- Reinforced Concrete Beam
  - The ratio n is given by

$$n = \frac{\text{Modulus of Elasticity for Steel}}{\text{Modulus of Elasticity for Concrete}} = \frac{E_s}{E_c}$$

  - The position of the neutral axis is obtained by determining the distance x from the upper face of the beam (upper fiber) to the centroid C of the transformed section.

Composite Beams

- Reinforced Concrete Beam
  – Note that the first moment of transformed section with respect to neutral axis must be zero.
  – Since the the first moment of each of the two portions of the transformed section is obtained by multiplying its area by the distance of its own centroid from the neutral axis, we get

Composite Beams

- Reinforced Concrete Beam

$$\frac{x}{2}(bx)-(d-x)(nA_s)=0$$

or

$$\frac{1}{2}bx^2+nA_s x-nA_s d=0 \qquad (61)$$

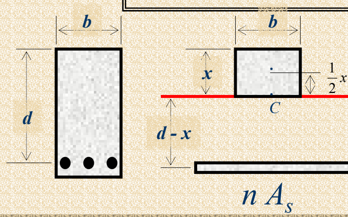
  – Solving the quadratic equation for x, both the position of the neutral axis in the beam and the portion of the cross section of the concrete beam can be obtained.

Composite Beams

- Reinforced Concrete Beam

  The neutral axis for a concrete beam is found by solving the quadratic equation:

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0 \quad (62)$$



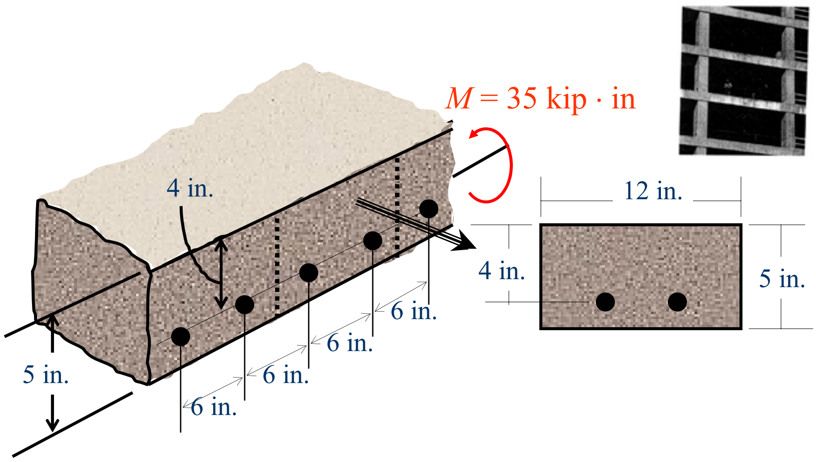
Composite Beams

- Example 3

  A concrete floor slab is reinforced by $\frac{5}{8}$-in diameter steel rods placed 1 in. above the lower face of the slab and spaced 6 in. on centers. The modulus of elasticity is 3×10^6 psi for concrete used and 30×10^6 psi for steel. Knowing that a bending moment of 35 kip·in is applied to each 1-ft width of the slab, determine (a) the maximum stress in concrete and (b) the stress in the steel.

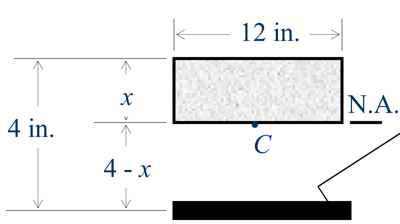
Composite Beams

- Example 3 (cont'd)



Composite Beams

- Example 3 (cont'd)
  - Transformed Section
    - Consider a portion of the slab 12 in. wide, in which there are two $\frac{5}{8}$-in diameter rods having a total cross-sectional area



$$A_s = 2\frac{\left[\pi\left(\frac{5}{8}\right)^2\right]}{4} = 0.614\ \text{in}^2$$

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$nA_s = 10(0.614) = 6.14\ \text{in}^2$$

Composite Beams

- Example 3 (cont'd)
  - Neutral Axis
    - The neutral axis of the slab passes through the centroid of the transformed section. Using Eq. 62:

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x_1 = 1.575$ take

$x_2 = -2.599$

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0$$

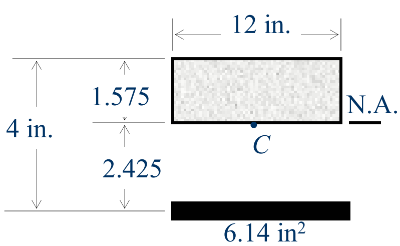
$$\frac{1}{2}(12)x^2 + 6.14x - 6.14(4) = 0$$

$$6x^2 + 6.14x - 24.56 = 0$$

$$\Longrightarrow x = 1.575 \text{ in}$$

Composite Beams

- Example 3 (cont'd)
  - Moment of Inertia
    - The centroidal moment of inertia of the transformed section is



$$I = \frac{12(1.575)^3}{3} + 6.14(2.425)^2 = 51.7 \text{ in}^4$$

Composite Beams

- Example 3 (cont'd)

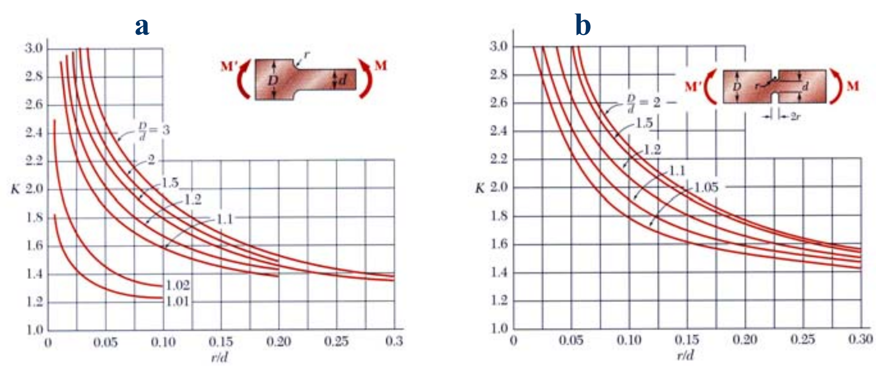
Maximum stress in concrete:

$$\sigma_c = -\frac{My}{I} = -\frac{35(1.575)}{51.7} = \boxed{-1.066 \text{ ksi (C)}}$$

Stress in steel:

$$\sigma_s = -n\frac{My}{I} = -(10)\frac{35(-2.425)}{51.7} = \boxed{+16.42 \text{ ksi (T)}}$$

Stress Concentrations



Stress concentrations may occur:

- in the vicinity of points where the loads are applied
- in the vicinity of abrupt changes in cross section

$$\sigma_m = K\frac{Mc}{I}$$

Figure 33

Stress Concentrations

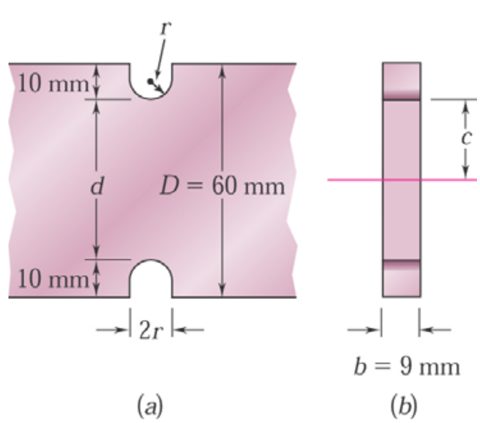
- Example 4

  Grooves 10 mm deep are to be cut in a steel bar which is 60 mm wide and 9 mm thick as shown. Determine the smallest allowable width of the grooves if the stress in the bar is not to exceed 150 MPa when the bending moment is equal to 180 N·m.

Stress Concentrations

- Example 4 (cont'd)

Figure 34



Stress Concentrations

- Example 4 (cont'd)
  - From Fig. 34a:

$$d = 60 - 2(10) = 40 \text{ mm}$$

$$c = \frac{1}{2}d = \frac{1}{2}(40) = 20 \text{ mm}$$

  - The moment of inertia of the critical cross section about its neutral axis is given by

$$I = \frac{1}{12}bd^3 = \frac{1}{12}\left(9\times10^{-3}\right)\left(40\times10^{-3}\right)^3 = 48\times10^{-9} \text{ m}^4$$

Stress Concentrations

- Example 4 (cont'd)
  - Therefore, the stress is

$$\sigma = \frac{Mc}{I} = \frac{180\left(20\times10^{-3}\right)}{48\times10^{-9}} = 75 \text{ MPa}$$

  - Using

$$\sigma_m = K\frac{Mc}{I}$$

$$150 = K(75) \Rightarrow K = 2$$

  - Also

$$\frac{D}{d} = \frac{60}{40} = 1.5$$

Stress Concentrations

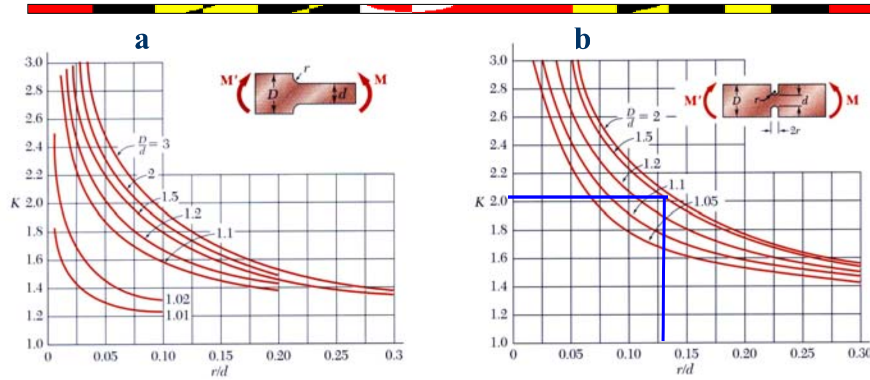
- Example 4 (cont'd)
  - From Fig. 33b, and for values of *D/d* = 1.5 and *K* = 2, therefore

$$\frac{r}{d} = 0.13$$
$$r = 0.13(d) = 0.13(40) = 5.2 \text{ mm}$$

  - Thus, the smallest allowable width of the grooves is

$$2r = 2(5.2) = \underline{10.4 \text{ mm}}$$

Stress Concentrations



Stress concentrations may occur:

- in the vicinity of points where the loads are applied
- in the vicinity of abrupt changes in cross section

$$\sigma_m = K\frac{Mc}{I}$$

Figure 33