LECTURE

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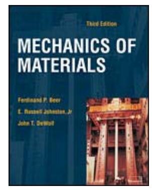
Third Edition

INTRODUCTION AND REVIEW: STATICS & STRESS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

Dr. Ibrahim A. Assakkaf

SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

1

Chapter

1.1- 1.13

Course Syllabus

- ENES 220 – Mechanics of Materials
  - Spring Semester 2003
  - Rooms
    - Lecture: EGR 0108
    - Recitation: EGR 0110
  - *Prerequisites:*
    - *ENES 102; and MATH 141; and PHYS 161*

Course Syllabus

- ENES 220 – Mechanics of Materials
  - GENERAL COURSE DESCRIPTION (UM SCHEDULE OF CLASSES)
    - Stress and deformation of solids-rods, beams, shafts, columns, tanks, and other structural, machine and vehicle members. Topics include stress transformation using Mohr's circle; shear and moment diagrams; derivation of elastic curves; and Euler's buckling formula. Design problems related to this material are given in lab.

Course Syllabus

- Instructor

Name:	Dr. Ibrahim A. Assakkaf
Office Hours:	MW 11:00 am - 12:00 pm and 1:00 pm – 2:00 pm
	F 12:00 am - 1:00 pm, and by appointment
Room:	0305, Engineering Classroom Building (EGR)
	Center for Technology and Systems Management (CTSM)
Telephone:	(W) 301-405-3279
Email:	assakkaf@eng.umd.edu
URL:	http://ctsm.umd.edu/assakkaf

Course Syllabus

- Textbook
  – Beer, Johnston, and DeWolf, Mechanics of Materials, 3rd. ed., McGraw-Hill, 2002.
- Reference
  – Riley, Sturges, and Morris, *Statics and Mechanics of Materials*, Wiley, 1995

Course Syllabus

- Instructors of Other Sections

Section	Lecture	Lec. room	Instructor	Recitation	Rec. room
0101	TuTh 8–8:50	EGR 0110	Fourney, W	F 8–9:50	CHE 2136
0102	MW 10–10:50	EGR 0108	Assakkaf, I	F 10–11:50	EGR 0110
0103	MW 10–10:50	EGL 1202	Beigel, J	F 10–11:50	CSS 2428
0104	MW 2–2:50	EGR 1104	Cárdenas, J	F 2–3:50	CHM 0122

Course Syllabus

Office Hour Schedule for Spring 2003

	Monday	Tuesday	Wednesday	Thursday	Friday
8-9			Fourney		
9-10	Cárdenas		Cárdenas	Zhao	Cárdenas
10-11				Zhao	
11-12	Assakkaf		Assakkaf	Zhao	
12-1					Assakkaf
1-2	Assakkaf		Assakkaf		
2-3				Wang	
3-4	Beigel	Park	Beigel	Wang	
4-5	Beigel	Fourney / Park	Beigel	Fourney / Wang	
5-6		Park			

(students can visit ANY of the instructors or TA’s, regardless of assigned section)

Course Syllabus

Instructors:
Ibrahim Assakkaf: EGR 0305, assakkaf@eng.umd.edu, 5-3279
Jaime Cárdenas: EGR 2139, jfcg@eng.umd.edu, 5-5322
Tom Beigel: EGR 1108, mcgyver@glue.umd.edu, 5-5314
William Fourney: EGR 3179F, four@eng.umd.edu, 5-1129

TAs:
0101 – Zhaoyang Wang: EGR 2142, zywang@glue.umd.edu, 5-5205
0102 – Kyungha Park (Kelly): EGL 1129, pdiva78@hotmail.com, 5-8718
0103 – Kyungha Park (Kelly): EGL 1129, pdiva78@hotmail.com, 5-8718
0104 – Tong Zhao: EGL 1176, zhao_tong@hotmail.com, 5-0310

Course Syllabus

Schedule for Lecture

HW #1 due

Lec.	Date	Sections	Topic	Homework				
1	W, 1/29	1.1-1.13	Introduction and review: stress	1.13	1.15	1.37	1.40	HW #1
2	M, 2/3	2.1-2.7, 2.11-2.15	Review: strain, material properties, and constitutive relations	2.1	2.63	2.65	2.68	HW #2
3	W, 2/5	2.8	Rods: axial loading and deformation	2.15	2.18	2.22	2.27	
4	M, 2/10	2.9	Rods: statically indeterminate	2.36	2.40	2.41	2.48	
5	W, 2/12	2.10, 2.18	Rods: thermal stress; stress conc.	2.51	2.53	2.59	2.100	
6	M, 2/17	3.1-3.5	Shafts: torsion loading and deformation	3.7	3.17	3.35	3.41	
7	W, 2/19	3.6	Shafts: statically indeterminate	3.53	3.54	3.56	3.58	
8	M, 2/24	3.7-3.8, 3.13	Shafts: power; stress conc.; thin-walled	3.72	3.85	3.90	3.140	

HW #2 due

Course Syllabus

Schedule for Lecture (cont'd)

Lec.	Date	Sections	Topic	Homework			
9	W, 2/26	4.1-4.5, 4.13	Beams: bending stress	4.4	4.10	4.27	4.145
10	M, 3/3		Review for Exam #1				
11	W, 3/5	4.6-4.7	Beams: composite beams; stress conc.	4.43	4.48	4.56	4.74
12	M, 3/10	5.1-5.2	Beams: V and M diagrams (formula)	5.1	5.5	5.14	5.15
13	W, 3/12	5.3	Beams: V and M diagrams (graphical)	5.41	5.48	5.49	5.64
14	M, 3/17	6.1-6.4	Beams: shearing stress	6.3	6.9	6.13	6.24
15	W, 3/19	6.6-6.7	Beams: shear flow; thin-walled	6.30	6.31	6.36	6.39
Monday, 3/24 To Sunday, 3/30			--- S P R I N G B R E A K ---				

Course Syllabus

Schedule for Lecture (cont'd)

Lec.	Date	Sections	Topic	Homework			
16	M, 3/31	9.1-9.3	Beams: deformation (integration)	9.3	9.5	9.12	9.14
17	W, 4/2	5.5, 9.6	Beams: deformation (singularity)	9.38	9.40	9.43	9.47
18	M, 4/7	9.5	Beams: statically indeterminate	9.51	9.56	9.58	9.60
19	W, 4/9	9.7-9.8	Beams: deformation (superposition)	9.68	9.74	9.84	9.92
20	M, 4/14		Review for Exam #2				
21	W, 4/16	7.1-7.3	Failure criteria: stress transformation	7.5/9	7.7/11	7.16	7.17
22	M, 4/21	7.4	Failure criteria: Mohr's circle	7.31	7.33	7.38	7.39
23	W, 4/23	7.5-7.6, 7.9	Failure criteria: multiaxial stress states	7.69	7.70	7.100	7.114

Course Syllabus

Schedule for Lecture (cont'd)

Lec.	Date	Sections	Topic	Homework			
24	M, 4/28	8.4	Components: combined loading	7.119	8.31	8.32	8.37
25	W, 4/30	8.4	Components: combined loading	8.39	8.42	8.47	8.51
26	M, 5/5	10.1-10.3	Columns: buckling (pinned ends)	10.10	10.11	10.14	10.17
27	W, 5/7	10.4	Columns: buckling (different ends)	10.19	10.21	10.25	10.27
28	M, 5/12	Varies	Advanced topics in mechanics	Announced in class			
29	W, 5/14	Varies	Advanced topics in mechanics	Announced in class			
	M, 5/19	All material	**--- FINAL EXAM - 4-6 PM - location to be announced ---**				

Course Syllabus

Schedule for Recitation

Rec.	Date	Problem Session	Project Session
1	F, 1/31	Sections 1.1-1.13	Project description and team questionnaire
2	F, 2/7	Sections 2.1-2.8, 2.11-2.15	Discussion of project guidelines
3	F, 2/14	Sections 2.9-2.10, 2.18	Group assignments / Project guidelines finalized
4	F, 2/21	Sections 3.1-3.6	Team meetings
5	F, 2/28	Section 3.7-3.8, 3.13, 4.1-4.5, 4.13	Torsion test demo / Team meetings
6	F, 3/7	--- **EXAM #1** ---	
7	F, 3/14	Sections 4.6-4.7, 5.1-5.3	Torsion test demo / Team meetings
8	F, 3/21	Sections 6.1-6.4, 6.6-6.7	Team meetings
Monday, 3/24 To Sunday, 3/30		--- **S P R I N G B R E A K** ---	

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Course Syllabus

Schedule for Recitation (cont'd)

Rec.	Date	Problem Session	Project Session
9	F, 4/4	Sections 9.1-9.3, 9.6	Team meetings
10	F, 4/11	Section 9.5, 9.7-9.8	Beam demo / Team meetings
11	F, 4/18	--- **EXAM #2** ---	
12	F, 4/25	Sections 7.1-7.6, 7.9	Beam demo / Team meetings
13	F, 5/2	Sections 8.4, 10.1-10.4	Crane demo / Team meetings
14	F, 5/9	Problems: Advanced topics	Course review

Course Syllabus

Grading Policy:

Homework / Quizzes / Class part.	20%	All exams & quizzes are closed book/notes and given during recitation
Design project	15%	
Exam #1	20%	
Exam #2	20%	
Final exam	25%	Final exam: Monday, May 10, 4- 6 PM
	100%	

Course Syllabus

Course Expectations:

Mechanics of Materials is used to answer two questions: (1) Is the material strong enough, and (2) Is the material stiff enough. That's it. Coincidentally, as an engineer, those are the two questions you want to answer whenever you design something. If the material is not strong enough, your design will break. If the material isn't stiff enough, your design probably won't function the way it's intended to. Accordingly, we will learn how to answer these questions in this course.

Course Syllabus

Homework

The lecture section meets each week on MW or TuTh. With only a short time to focus on the material, it is vital that these sessions start on time. Everyone is asked to arrive and be seated promptly, to minimize the disruption to others. The recitation section meets on Friday. This session will be conducted in two parts. The first part will consist of problem solving, discussing homework solutions, and providing occasional interactive classroom demonstrations. Periodic quizzes will also be held during this time. The second part will be devoted to the design project. During this period, students are expected to meet in groups and perform tasks necessary for completing the project. Occasionally, lectures and demonstrations will be given on material related to the project. The activities for each session are listed in the recitation schedule above. It is anticipated that the recitation time will be divided evenly between the problem session and the project session. Students are expected to regularly attend both the lecture and recitation periods. An attendance sheet will be circulated at the beginning of each class session, which will be kept as a partial record of your class participation.

Course Syllabus

Homework Policy:

Homework will be assigned as the material is covered and will be collected every Monday at the beginning of the lecture period, starting on 2/3. Assignments turned in late will be docked 10% for each day it is late past the original due date. Homework will be returned during the Friday recitation period later that week. Solutions will be available from the TAs and on the class website after the problems are returned. *No assignment will be accepted after the answers have been posted.* Students are encouraged to discuss and formulate solutions to the problems by working in teams. However, assignments *must* be completed and submitted individually. *Simply copying the answers from another student or from a solutions manual is not acceptable and will not be tolerated*

Course Syllabus

Guidelines for homework are given below:

1. Use good quality paper, such as engineering graph paper or college-ruled paper, any color, with no spiral edges
2. Write on only one side of the paper
3. Either pen or pencil is acceptable
4. Include your name, section, and page number (e.g. 1/3 means 1 of 3) on each sheet
5. Staple all pages together in the upper left corner
6. Neatly box all answers, and include appropriate units for numerical answers
7. Show all work (e.g. no work means no credit will be given)

If the above guidelines are not followed, the TA will either reject the assignment outright, for extreme cases, or deduct points for items that do not conform to the specifications

Course Syllabus

Quiz Policy:

Quizzes will be periodically given throughout the semester. Quiz problems will strictly come from class examples, book examples, or homework problems verbatim. As long as you know how to complete these problems, you are guaranteed to do well on the quizzes. Quizzes will be administered during the recitation periods.

Course Syllabus

Design Project:

The project consists of completing a design and analysis of an overhead crane structure. The details are given on a separate sheet, and will be described during the recitation periods. Students will work in teams of 4 or 5 to complete the assignment. The deliverable will be a comprehensive final report, which includes analyses of the structural members, technical drawings, and an explanation of your design decisions. Grades will be based on the completeness and professional quality of the report and drawings, as well as the accuracy of the technical analyses. Part of your grade will be based on the group design report, and the remainder will be based on your individual participation in the group project. Therefore, grades could vary among members of the same team. ***A student that does NOT contribute to the project will receive a grade of 'F' for the entire project grade.***

Introduction

- Objectives

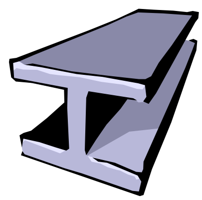
Mechanics of Materials answers two questions:

Is the material strong enough?

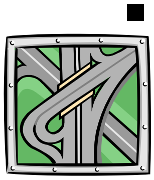
Is the material stiff enough?

Introduction

- Objectives
  - If the material is not strong enough, your design will break.
  - If the material isn't stiff enough, your design probably won't function the way it's intended to.



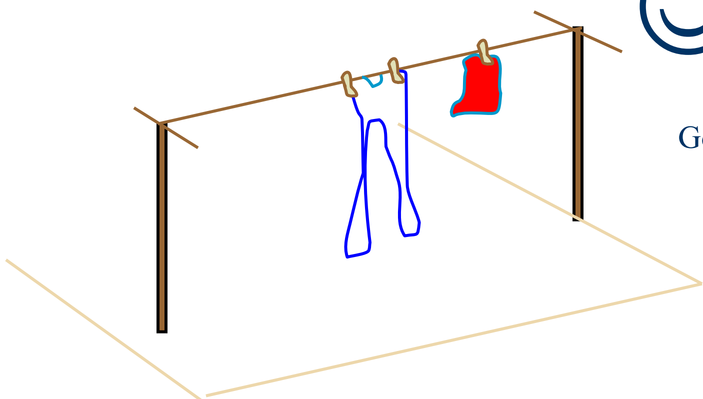
Introduction



- Objectives
  - The main objective of the study of mechanics of materials (or strength of materials) is to provide the engineer with methods of analyzing various machines and structural members.
  - Design and analysis of a given structure involve the determination of *stresses* and *deformation*

Introduction

- Objectives



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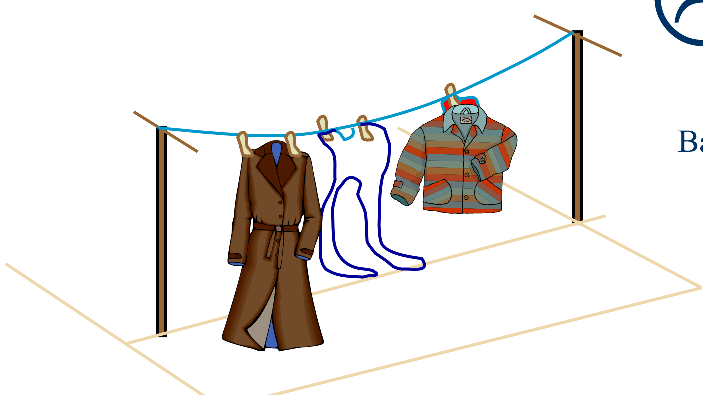
Good Design

Introduction

- Objectives



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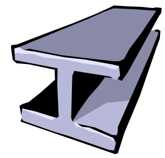
Bad Design

Introduction

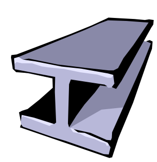
- Design Considerations
  - Safety
  - Economy



Design of engineering systems is usually a trade-off between maximizing safety and minimizing cost.

Introduction

- Typical Approach to an Engineering Solution
  - Identify the problem
  - State the objective
  - Develop alternative solutions
  - Evaluate the alternatives, and
  - Use the best alternative

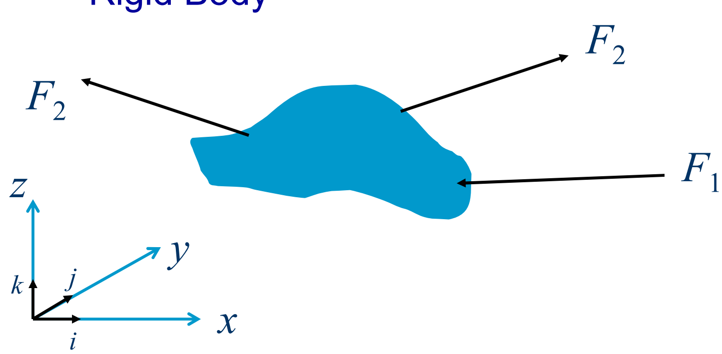


Introduction

- Methods of Analysis
  – Equations of equilibrium are used for external forces.
  – Analysis of the effect of the external forces on the structure (machine) or any component of the structure (machine).
  – Behavior of the materials under the action of forces.

Review: Statics

- Equations of Equilibrium
  – Rigid Body



Review: Statics

- Equations of Equilibrium
  - For a rigid body to be in equilibrium, both the resultant force ***R*** and a resultant moments (couples) ***C*** must vanish.
  - These two conditions can be expressed mathematically in vector form as

$$\vec{R} = \sum F_x \mathrm{i} + \sum F_y \mathrm{j} + \sum F_z \mathrm{k} = \vec{0}$$

$$\vec{C} = \sum M_x \mathrm{i} + \sum M_y \mathrm{j} + \sum M_z \mathrm{k} = \vec{0}$$

Review: Statics

- Equations of Equilibrium
  - The two conditions can also be expressed in scalar form as

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Review: Statics

- Equilibrium in Two Dimensions
  - The term "two dimensional" is used to describe problems in which the forces under consideration are contained in a plane (say the *xy*-plane)

Review: Statics

- Equilibrium in Two Dimensions
  - For two-dimensional problems, since a force in the *xy*-plane has no *z*-component and produces no moments about the *x*- or *y*-axes, hence

$$\vec{R} = \sum F_x \mathrm{i} + \sum F_y \mathrm{j} = \vec{0}$$
$$\vec{C} = \sum M_z \mathrm{k} = \vec{0}$$

Review: Statics

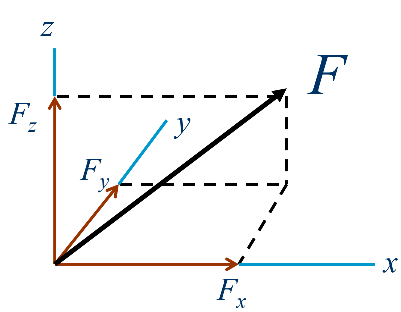
- Equilibrium in Two Dimensions
  – In scalar form, these conditions can be expressed as

$$\sum F_x = 0 \qquad \sum F_y = 0$$

$$\sum M_A = 0$$

Review: Statics

- Cartesian Vector Representation of A Force

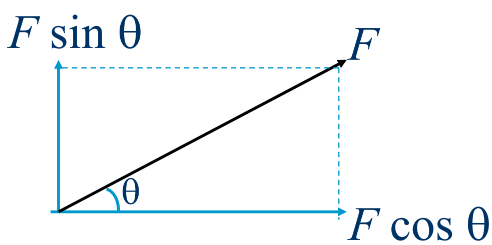


$$F = F_x \mathrm{i} + F_y \mathrm{j} + F_z \mathrm{k}$$

Review: Statics

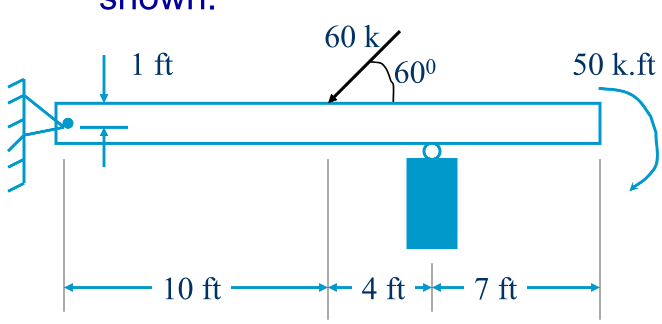
- Cartesian Vector Representation of A Force in Two Dimensions

$$\vec{F} = F_x\text{i} + F_y\text{j}$$
$$= F\cos\theta\text{i} + F\sin\theta\text{j}$$



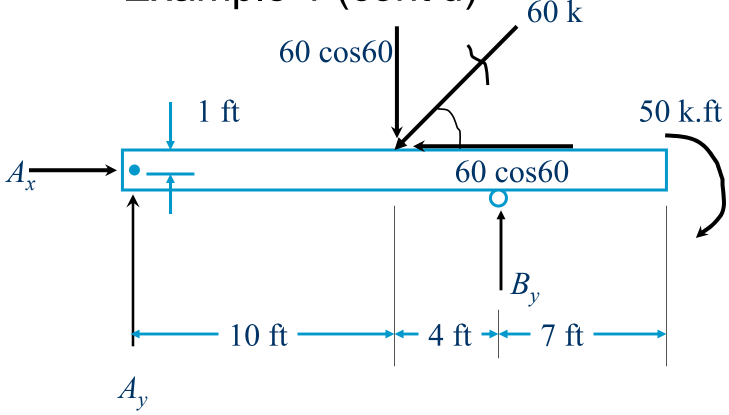
Review: Statics

- Example 1
  - Determine the reactions on the beam shown.



Review: Statics

- Example 1 (cont'd)



Review: Statics

- Example 1 (cont'd)

$$+\rightarrow \sum F_x = 0;\quad A_x - 60\cos 60 = 0 \Rightarrow A_x = \underline{30.0\text{ k}}\quad \text{ANS.}$$

$$\circlearrowleft + \sum M_A = 0;\quad 60\sin 60(10) - 60\cos 60(1) - B_y(14) + 50 = 0$$

$$\Rightarrow B_y = \underline{38.5\text{ k}}\ \text{ANS.}$$

$$+\uparrow \sum F_y = 0;\quad -60\sin 60 + 38.5 + A_y = 0 \Rightarrow A_y = \underline{13.4\text{ K}}\quad \text{ANS.}$$

Review: Vector Operations

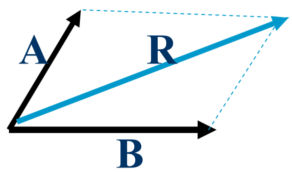
- Scalar Quantities
  - Scalar quantities can be completely described by their magnitudes.
  - Examples
    - Mass
    - Density
    - Length
    - Speed
    - Time

Review: Vector Operations

- Vector Quantities
  - A vector quantity has both magnitude and direction (line of action).
  - A vector quantity obeys the parallelogram of addition
  - Examples:
    - Force
    - Displacement
    - Velocity
    - Acceleration

Review: Vector Operations

- Addition of Vectors



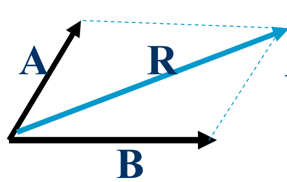
$$\vec{R} = \vec{A} + \vec{B}$$

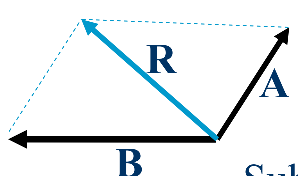
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

$$\vec{A} + \vec{B} + \vec{C} = \left(\vec{A} + \vec{B}\right) + \vec{C}$$

Review: Vector Operations

- Subtraction of Vectors





$$\vec{R} = \vec{A} - \vec{B}$$
$$= \vec{A} + (-\vec{B})$$

Review: Vector Operations

- Multiplication of Vectors by Scalars
  - The product of a vector **A** and a scalar *m* is a vector *m***A**.
  - Operations involving the products of scalars *m* and *n* and vectors **A** and **B** include the following:

$$(m+n)\vec{A} = m\vec{A} + n\vec{A}$$
$$m\left(\vec{A} + \vec{B}\right) = m\vec{A} + m\vec{B}$$
$$m\left(n\vec{A}\right) = (mn)\vec{A} = n\left(m\vec{A}\right)$$

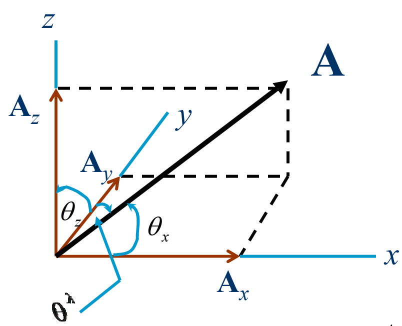
Review: Vector Operations

- Cartesian Vectors
  - Unit Vector
    - A vector of magnitude one is called a unit vector
    - Unit vectors directed along the *x*-, *y*-, and *z*-axes *of a* coordinate system are normally denoted by the symbols **i**, **j**, and **k**.
    - A unit vector is defined as

$$\vec{e}_n \text{ or } \vec{u}_n = \frac{\vec{A}}{\left|\vec{A}\right|}$$

Review: Vector Operations

- Cartesian Components of a Vector



$$\vec{A} = A_x\mathrm{i} + A_y\mathrm{j} + A_z\mathrm{k}$$

$$A_x = A\cos\theta_x$$

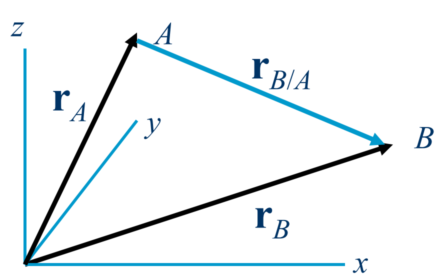
$$A_y = A\cos\theta_y$$

$$A_z = A\cos\theta_z$$

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Review: Vector Operations

- Position Vectors



$$\mathbf{r}_A = x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k}$$

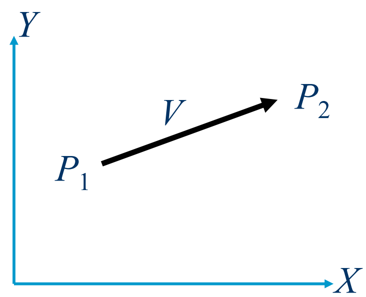
$$\mathbf{r}_B = x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \quad \Longrightarrow \quad \mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$$

Review: Vector Operations

- Vector in Two-dimensional Space
  - In general the vector in two-dimensional space is given by

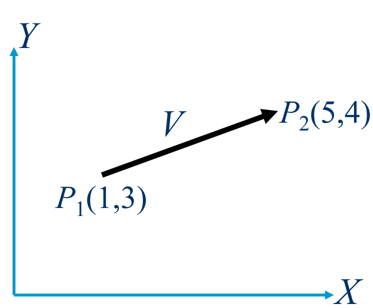
$$V_{P_2/P_1} = \left[\left(P_{2X} - P_{1X}\right) \quad \left(P_{2Y} - P_{1Y}\right) \right]$$



Review: Vector Operations

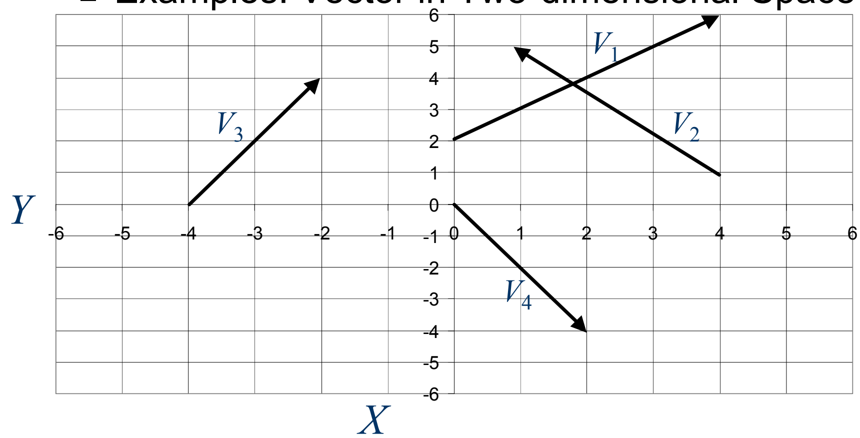
- Example: Vector in Two-dimensional Space

$$V_{P_2-P_1} = \left[\left(P_{2X_1} - P_{1X_1}\right) \quad \left(P_{2X_2} - P_{1X_2}\right) \right]$$
$$= \left[(5-1) \quad (4-3) \right]$$
$$= \left[4 \quad 1 \right]$$



Review: Vector Operations

- Examples: Vector in Two-dimensional Space



Review: Vector Operations

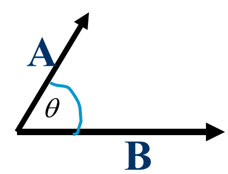
- Examples: Vector in Two-dimensional Space

$$V_1 = [(4-0) \quad (6-2)] = [4 \quad 4]$$
$$V_2 = [(1-4) \quad (5-1)] = [-3 \quad 4]$$
$$V_3 = [(-2-(-4)) \quad (4-0)] = [2 \quad 4]$$
$$V_4 = [(2-0) \quad (-4-0)] = [4 \quad -4]$$

Review: Vector Operations

- Dot or Scalar Product
  - The dot or scalar product or two intersecting vectors is defined as the product of the magnitudes of the vectors and the cosine of the angle between them.

$$\mathbf{A} \bullet \mathbf{B} = AB \cos\ \theta$$



Review: Vector Operations

- Example 2
  - Determine the angle θ between the two forces **A** and **B**:

    A = 8**i** + 9**j** + 7**k** and **B** = 6**i** - 5**j** + 3**k**

$$\left|\vec{A}\right| = A = \sqrt{8^2 + 9^2 + 7^2} = 13.93$$

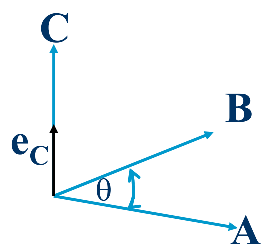
$$\left|\vec{B}\right| = B = \sqrt{6^2 + (-5)^2 + 3^2} = 8.37$$

$$\vec{A} \cdot \vec{B} = (8 \times 6) + (9 \times -5) + (7 \times 3) = 24$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{24}{13.93(8.37)} = 78.1^0$$

Review: Vector Operations

- Cross or Vector Product



$$\boxed{\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin\theta)\, \mathbf{e}_C}$$

$$\mathbf{A} \times \mathbf{B} = -\,\mathbf{B} \times \mathbf{A}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{C}$$

Review: Vector Operations

- Example 3
  - If **A** = -3.75**i** – 2.50**j** + 1.50**k** and
    B = 32**i** + 44**j** + 64 **k**
    determine the magnitude and direction of the vector **C** = **A** × **B**

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ -3.75 & -2.5 & 1.5 \\ 32 & 44 & 64 \end{vmatrix}$$

Review: Vector Operations

- Example 3 (cont'd)

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ -3.75 & -2.5 & 1.5 \\ 32 & 44 & 64 \end{vmatrix}$$

$$= [-2.5(64) - 1.5(44)]\mathrm{i} - [-3.75(64) - 1.5(32)]\mathrm{j} + [-3.75(44) - (-2.5)32]$$

$$= -226\mathrm{i} + 288\mathrm{j} - 85\mathrm{k}$$

Review: Vector Operations

- Example 3 (cont'd)

$$C = \left|\vec{C}\right| = \sqrt{(-226)^2 + (288)^2 + (-85)^2} = 376$$

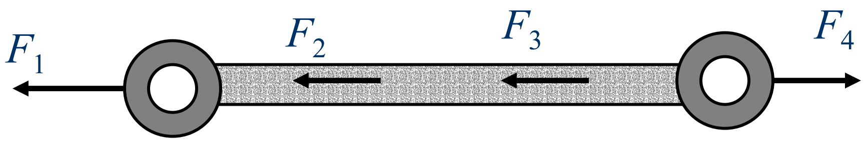
$$\theta_x = \cos^{-1}\frac{C_x}{\left|\vec{C}\right|} = \cos^{-1}\frac{-226}{376} = 126.9^0$$

$$\theta_{yx} = \cos^{-1}\frac{C_y}{\left|\vec{C}\right|} = \cos^{-1}\frac{288}{376} = 40.0^0$$

$$\theta_{zx} = \cos^{-1}\frac{C_z}{\left|\vec{C}\right|} = \cos^{-1}\frac{-85.0}{376} = 103.1^0$$

Internal Forces for Axially Loaded Members

- Analysis of Internal Forces



Assume that F_1 = 2 k, F_3 = 5 k, and F_4 = 8 k

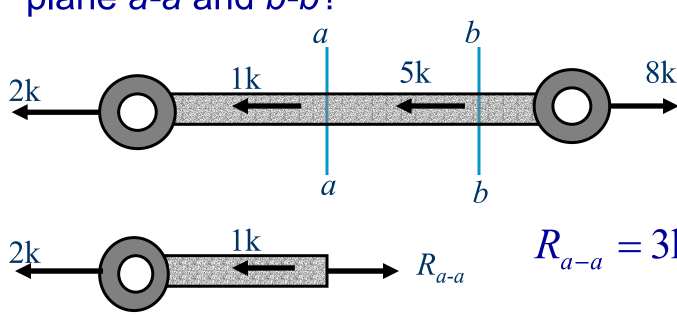
Then

$$\rightarrow +\sum -F_1 - F_2 - F_3 + F_4 = 0; \Rightarrow F_2 = F_4 - F_1 - F_3$$

or $F_2 = 8 - 2 - 5 = 1\,\text{k}$

Internal Forces for Axially Loaded Members

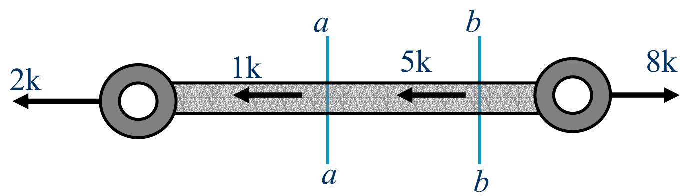
- Analysis of Internal Forces
  – What is the internal force developed on plane *a-a* and *b-b*?



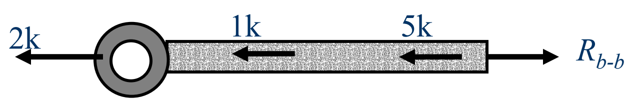
$$R_{a-a} = 3\text{k}$$

Internal Forces for Axially Loaded Members

- Analysis of Internal Forces

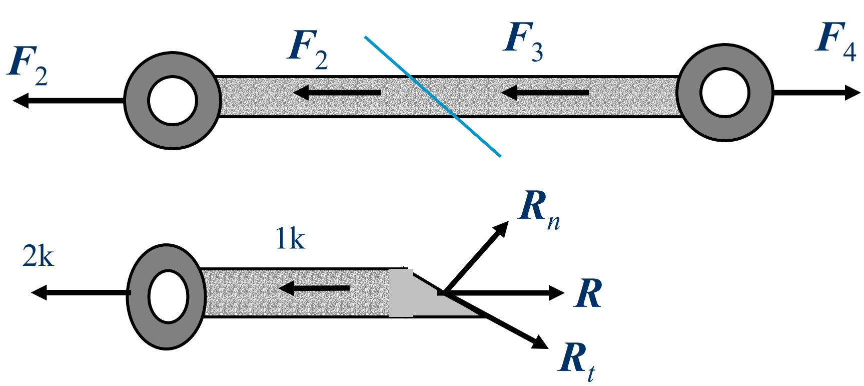


$$R_{b-b} = 8\text{ k}$$



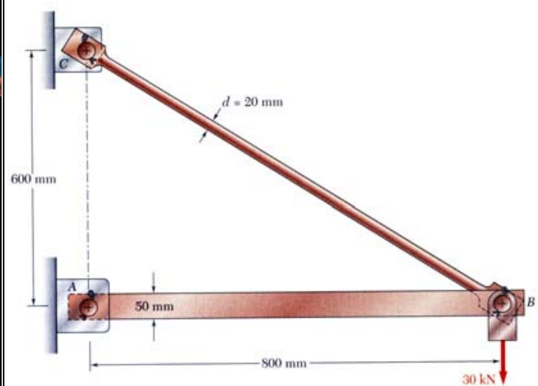
Internal Forces for Axially Loaded Members

- Analysis of Internal Forces



Internal Forces for Axially Loaded Members

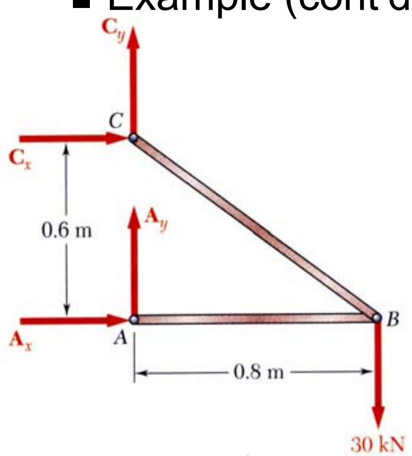
- Example



- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

Internal Forces for Axially Loaded Members

- Example (cont'd)

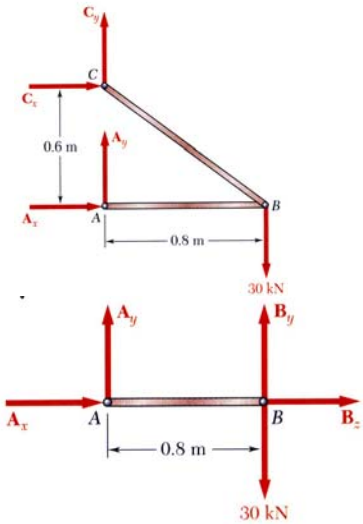


- Structure is detached from supports and the loads and reaction forces are indicated
- Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$
$$A_x = 40\,\text{kN}$$
$$\sum F_x = 0 = A_x + C_x$$
$$C_x = -A_x = -40\,\text{kN}$$
$$\sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$
$$A_y + C_y = 30\,\text{kN}$$

- A_y and C_y can not be determined from these equations

Internal Forces for Axially Loaded Members



- In addition to the complete structure, each component must satisfy the conditions for static equilibrium
- Consider a free-body diagram for the boom:

  $$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

  $$A_y = 0$$

  substitute into the structure equilibrium equation

  $$C_y = 30\,\text{kN}$$

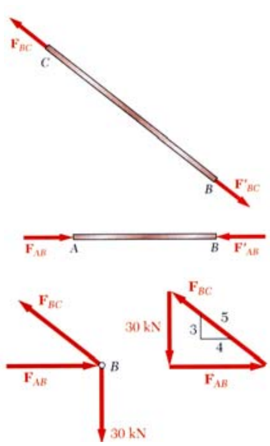
- Results:

  $$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

  Reaction forces are directed along boom and rod

Internal Forces for Axially Loaded Members

Example (cont'd)



- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

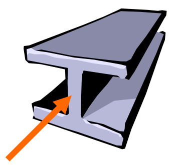
  $$\sum \vec{F}_B = 0$$

  $$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

  $$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

Axial Loading: Normal Stress

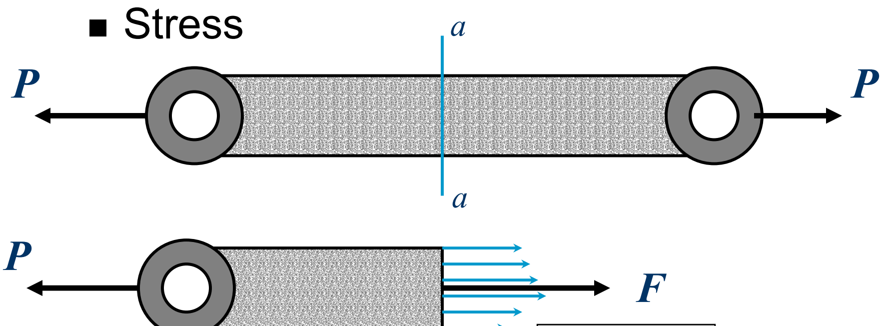
- Stress
  - Stress is the intensity of internal force.
  - It can also be defined as force per unit area, or intensity of the forces distributed over a given section.



$$\boxed{\text{Stress} = \frac{\text{Force}}{\text{Area}}} \tag{1}$$

Axial Loading: Normal Stress

- Stress



$$\boxed{\sigma_{\text{avg}} = \frac{F}{A}} \tag{2}$$

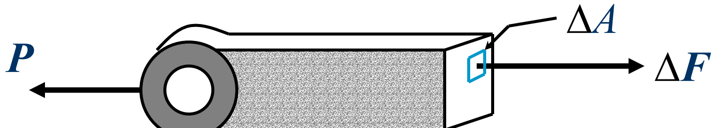
Axial Loading: Normal Stress

- Normal Stress
  - *F* = magnitude of the force ***F***
  - *A* = area of the cross sectional area of the eye bar.
- Units of Stress

SI System	U.S. Customary Units
1 kPa = 10^3 Pa = 10^3 N/m^2 1 MPa = 10^6 Pa = 10^6 N/m^2 1 GPa = 10^9 Pa = 10^9 N/m^2	lb/in^2 = psi Kip/in^2 = ksi = 1000 psi

Axial Loading: Normal Stress

- Normal Stress
  - To define the stress at a given point Q of the cross section, a small area ΔA should be considered as shown in the figure

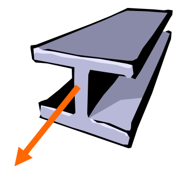


$$\boxed{\sigma = \lim_{\Delta A_s \to 0} \frac{\Delta F}{\Delta A}} \qquad (3)$$

Axial Loading: Normal Stress

- Normal Stress

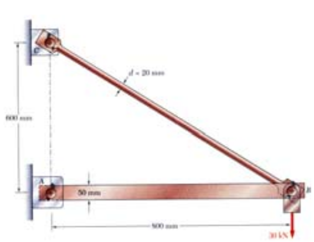
$$\boxed{\sigma = \lim_{\Delta A_s \to 0} \frac{\Delta F}{\Delta A}} \Rightarrow \sigma = \frac{dF}{dA}$$



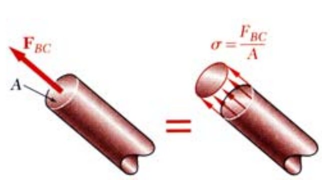
$$P = \int dF = \int_A \sigma \, dA$$

Axial Loading: Normal Stress

Example: Stress Analysis



$d_{BC} = 20$ mm



Can the structure safely support the 30 kN load?

- From a statics analysis
  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)
- At any section through member BC, the internal force is 50 kN with a force intensity or stress of

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \text{N}}{314 \times 10^{-6} \text{m}^2} = 159 \text{ MPa}$$

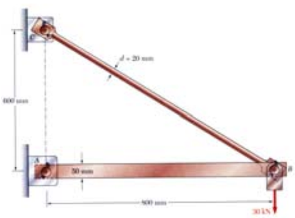
- From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

Axial Loading: Normal Stress

Example: Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements
- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^3\,\text{N}}{100\times10^6\,\text{Pa}} = 500\times10^{-6}\,\text{m}^2$$

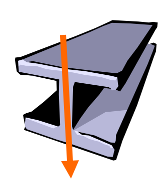
$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\text{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\text{m} = 25.2\,\text{mm}$$

- An aluminum rod 26 mm or more in diameter is adequate

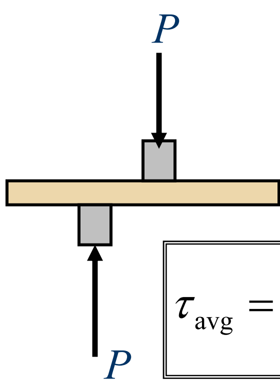
Shearing Stress

- The internal forces discussed previously and the corresponding stresses were *normal* to the section considered.
- A different type of stress can occur in a transverse cross section of a member as shown in the next slide.

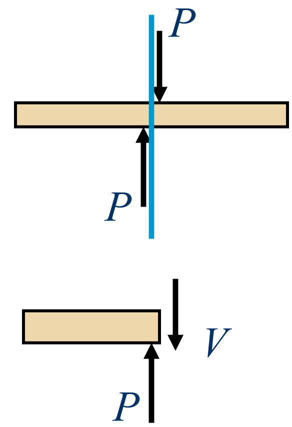


Shearing Stress

- Illustration of Shearing Stress



$$\boxed{\tau_{\text{avg}} = \frac{V}{A_s} = \frac{P}{A_s}}$$



Shearing Stress

- Shear

$$\boxed{\tau = \lim_{\Delta A_s \to 0} \frac{\Delta V}{\Delta A_s}} \qquad \tau = \frac{dV}{dA_s}$$

$$P = \int dV = \int_{A_s} \tau \, dA_s$$

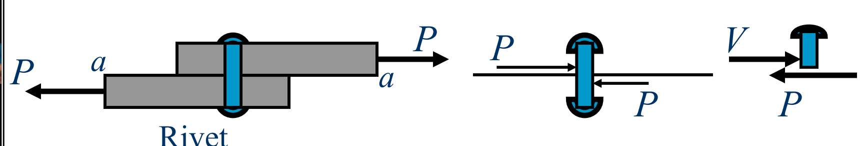
A_s = cross-sectional area of bolt or rivet

Shearing Stress

- Shearing Stress in Connection
  - Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components
  - Three Types of Shearing Stress
    1. **Single Shear**
    2. **Double Shear**
    3. **Punching Shear**

Shearing Stress

- Single Shear

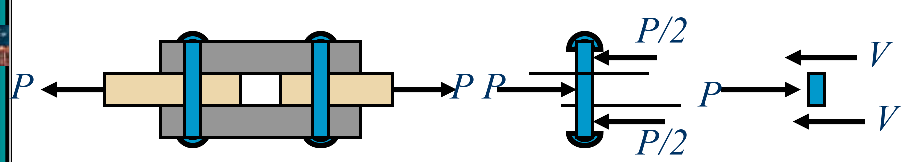


The rivet is said to in single shear V

$$\boxed{\tau_{\text{avg}} = \frac{V}{A_s} = \frac{P}{A_s}}$$

Shearing Stress

- Double Shear



The rivet is said to in double shear V

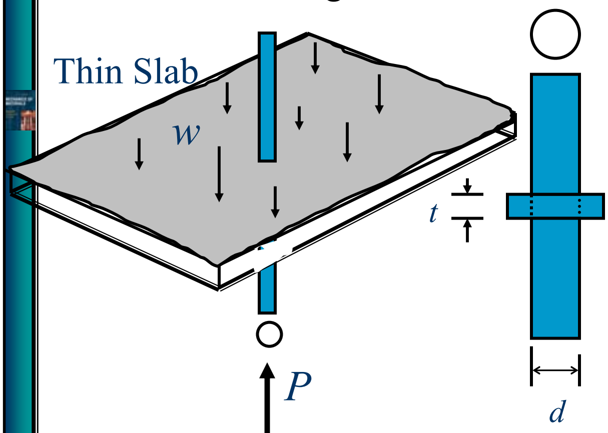
$$\boxed{\tau_{\text{avg}} = \frac{V}{A_s} = \frac{P}{2A_s}}$$

Shearing Stress

- Punching Shear
  - Examples of this type are
    - Action of punch in forming rivet hole in metal
    - Tendency of building columns to punch trough footings
    - Heavy thin-slab ceiling cause building columns to punch through slabs.

Shearing Stress

- Punching Shear



$$A_s = \pi d t$$

τ

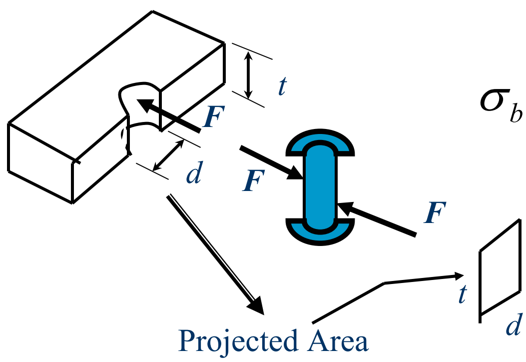
$$\tau_{\text{avg}} = \frac{P}{A_s} = \frac{P}{\pi d t}$$

Shearing Stress

- Bearing stresses (compressive normal) occur on the surface of contact between two interacting structural members.
- Bolts, pins, and rivets create stresses in the member they connect, along the bearing surface, or surface of contact.

Shearing Stress

- Illustration of Bearing Stress



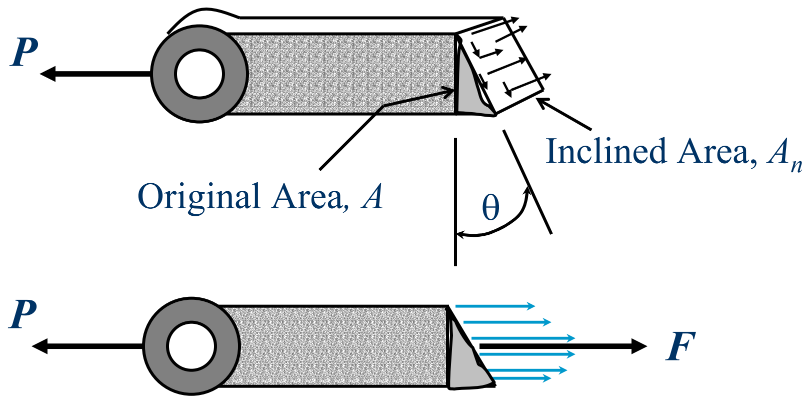
$$\sigma_b = \frac{F}{A_b} = \frac{F}{dt}$$

Stresses on an Inclined Plane in an Axially Loaded Member

- Normal, shear, and bearing stresses were introduced previously, where they usually act either normal to cross-sectional area of a member or tangential to the area.
- Stresses on planes inclined to axis of axially loaded bars will now be considered.

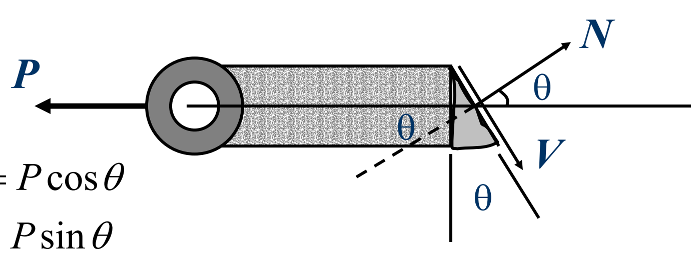
Stresses on an Inclined Plane in an Axially Loaded Member

- Illustration



Stresses on an Inclined Plane in an Axially Loaded Member

- Illustration



$$N = P\cos\theta$$

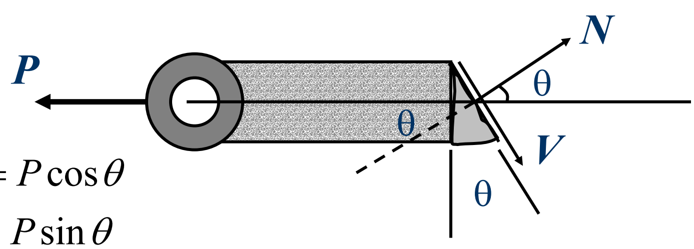
$$V = P\sin\theta$$

$$A_n = \frac{A}{\cos\theta}$$

$$\boxed{\sigma_n = \frac{N}{A_n} = \frac{P\cos\theta}{\dfrac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta = \frac{P}{2A}\left(1+\cos 2\theta\right)}$$

Stresses on an Inclined Plane in an Axially Loaded Member

- Illustration



$$N = P\cos\theta$$

$$V = P\sin\theta$$

$$A_n = \frac{A}{\cos\theta}$$

$$\boxed{\tau_n = \frac{V}{A_n} = \frac{P\sin\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta}$$

Stresses on an Inclined Plane in an Axially Loaded Member

- Maximum Normal and Shear Stresses
  - σ_n is maximum when θ = 0^0 or 180^0
  - τ_n is maximum when θ = 45^0 or 135^0
  - Also

$$\tau_{\max} = \frac{\sigma_{\max}}{2}$$

  - Therefore

$$\boxed{\sigma_{\max} = \frac{P}{A} \quad \text{and} \quad \tau_{\max} = \frac{P}{2A}}$$

Design Loads, Working Stresses, and Factor of Safety (FS)

- Failure
  - Failure is defined as the state or condition in which a member or structure no longer function as intended.
  - Types of Failure
    - Fracture
    - Excessive Deformation
    - Creep (e.g., concrete)
    - Fatigue (cyclic or repeated loading)

Design Loads, Working Stresses, and Factor of Safety (FS)

- Design Loads
  - A structural member or a machine component must be designed so that its ultimate load is considerably larger than the load the member or component will be allowed to carry under normal conditions of utilization.
  - Design loads (e.g., dead, live, snow, etc) are normally set by the classification societies (e.g., building codes)

Design Loads, Working Stresses, and Factor of Safety (FS)

- Example Live Loads

Table 5-1. Example Live Load Distribution in a Ship (Ayyub and Assakkaf 1997)

Typical Live Loads	
Type of Compartment	Live Loading (lbs/ft^2)
Living and control space, offices and passages, main deck and above	75
Living spaces below main deck	100
Offices and control spaces below main deck	150
Shop spaces	200
Storage room/Magazines	300*
Weather portions of main deck and O1 level	250**

Design Loads, Working Stresses, and Factor of Safety (FS)

- Design Loads
  - The load that a structure or machine must carry is uncertain (random), and in most cases it only estimated.
  - The actual load may vary considerably from the estimate, especially when loads at future time must be considered.

Design Loads, Working Stresses, and Factor of Safety (FS)

- Working Stress
  – The working stress (or allowable stress) is defined as the maximum stress permitted in the design computation.
- Ultimate Strength
  – The ultimate strength (stress) in tension for a material is given by

Design Loads, Working Stresses, and Factor of Safety (FS)

- Factor of Safety
  – The factor of safety (FS) can be defined as the ratio of the ultimate stress of the material to the allowable stress

$$\boxed{\mathrm{FS} = \frac{\text{ultimate stress}}{\text{allowable stress}}}$$

Design Loads, Working Stresses, and Factor of Safety (FS)

- Design Approaches
  – Deterministic, e.g., working stress or allowable stress design (ASD)
  – Probability-Based, e.g., load and resistance factor design (LRFD)

$$\frac{R_n}{\text{FS}} \geq \sum_{i=1}^{m} L_i$$

ASD

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_i$$

LRFD

Design Loads, Working Stresses, and Factor of Safety (FS)

- Probability Based-design Approach Versus Deterministic Approach

$$\frac{R_n}{\text{FS}} \geq \sum_{i=1}^{m} L_i$$

ASD

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_i$$

LRFD

  - According to ASD, one factor of safety (FS) is used that accounts for the entire uncertainty in loads and strength.
  - According to LRFD (probability-based), different partial safety factors for the different load and strength types are used.

Design Loads, Working Stresses, and Factor of Safety (FS)

- Several design codes have recently been revised to incorporate probabilistic design and analysis
  - AISC LRFD (1994)
  - AASHTO
  - API
  - ABS
  - Other structural and marine codes