ENES 220 – Mechanics of Materials
Spring 2003

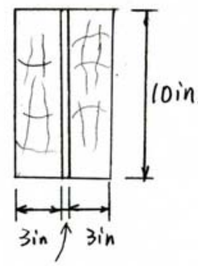
Solutions to Homework #6

Problem 4.43

Given: Wooden beams and steel plates

$E_w = 2\times10^6$ psi $\quad E_s = 30\times10^6$ psi

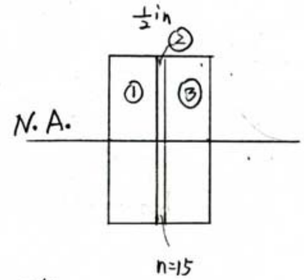
$\sigma_{w,all} = 2000$ psi $\quad \sigma_{s,all} = 22$ Ksi



Find: the largest permissible moment

FBD: not required

FPU: $M = \dfrac{\sigma_w I}{c} \quad M = \dfrac{\sigma_s I}{nc}$



Solution:

Transformed Section.

$$n = \frac{E_s}{E_w} = \frac{30\times10^6\text{ psi}}{2\times10^6\text{ psi}} = 15$$

Multiplying the horizontal dimensions of the steel portion of the section by n=15, we obtained a transformed section made entirely of wood.

Neutral axis

Due to symmetry, neutral axis is shown in the figure, or $\bar{Y}=0$.

Centroidal Moment of Inertia

$$I_1 = \frac{1}{12}b_1h_1^3 = \frac{1}{12}(3in)(10in)^3 = 250\ in^4$$

$$I_2 = \frac{n}{12}b_2h_2^3 = \frac{(15)}{12}(\tfrac{1}{2}in)(10in)^3 = 625\ in^4$$

$$I_3 = I_1 = 250\ in^4$$

$$I = I_1 + I_2 + I_3 = (250in^4) + (625in^4) + (250in^4) = 1125\ in^4$$

Largest permissible moment

$$\text{Wood:}\quad M = \frac{\sigma_w I}{c} = \frac{(2000psi)(1125in^4)}{5in} = 450\times10^3\ lb\cdot in$$

$$\text{Steel:}\quad M = \frac{\sigma_s I}{nc} = \frac{(22\times10^3psi)(1125in^4)}{(15)(5in)} = 330\times10^3\ lb\cdot in$$

Choose the smaller value $\quad M = 330\times10^3\ lb\cdot in = \underline{\underline{300\ kip\cdot in}}$

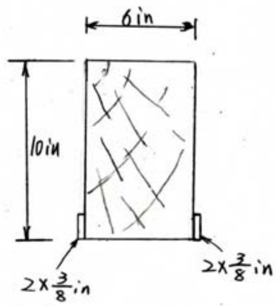
Problem 4.48

Given: A 6x10-in timber beam bolted by steel straps.

$E_w = 1.5\times10^6$ psi $\quad E_s = 30\times10^6$ psi

$M = 200$ Kip·in

Find: the maximum stress in (a) the wood (b) the steel



FBD: not required

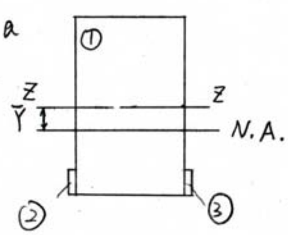
FPU: $\sigma_s = \dfrac{n\,My}{I}$, $\sigma_w = \dfrac{My}{I}$

Solution:

Transformed section

$$n = \frac{E_s}{E_w} = \frac{30\times10^6\,\text{psi}}{1.5\times10^6\,\text{psi}} = 20$$

Multiplying the horizontal dimensions of the steel portion of the section by n=20, we obtained a transformed section made entirely of wood



Neutral axis

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2\cdot(20)\cdot(4\,\text{in})(2\,\text{in}\times\frac{3}{8}\,\text{in}) + 0}{2\cdot(20)(2\,\text{in}\times\frac{3}{8}\,\text{in}) + (10\,\text{in}\times6\,\text{in})}$$

$$= 1.333 \text{ in}$$

Centroidal moment of inertia

Using the parallel-axis theorem:

$$I = \frac{1}{12}(6\,\text{in})(10\,\text{in})^3 + (6\,\text{in})(10\,\text{in})(1.333\,\text{in})^2$$
$$+ 2\left[\frac{1}{12}(\tfrac{3}{8}\,\text{in}\times20)(2\,\text{in})^3 + (\tfrac{3}{8}\,\text{in}\times20)(2\,\text{in})(2.667\,\text{in})^2\right]$$
$$= 830 \text{ in}^4$$

Maximum stress

(a) Wood: $y = 1.333\,\text{in} + 5\,\text{in} = 6.333\,\text{in}$

$$\sigma = -\frac{My}{I} = -\frac{(200\,\text{Kip}\cdot\text{in})(6.333\,\text{in})}{830\,\text{in}^4} = -1.526\ \text{Ksi}$$

(b) Steel $n=20$, $y = 1.333\,\text{in} - 5\,\text{in} = -3.667\,\text{in}$

$$\sigma = -\frac{n\,My}{I} = -\frac{(20)(200\,\text{Kip}\cdot\text{in})(-3.667\,\text{in})}{830\,\text{in}^4} = 17.67\ \text{Ksi}$$

Problem 4.56

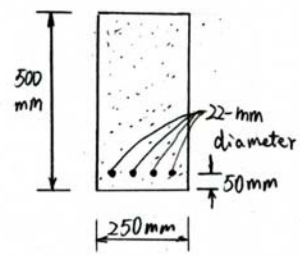
Given: Reinforced concrete beam, $M = 175\ kN\cdot m$

$E_c = 25 GPa \quad E_s = 200 GPa$

Find: (a) the stress in the steel (b) the maximum stress in the concrete

FBD: not required

FPU: $\sigma_s = -\dfrac{nMy}{I} \qquad \sigma_c = -\dfrac{My}{I}$



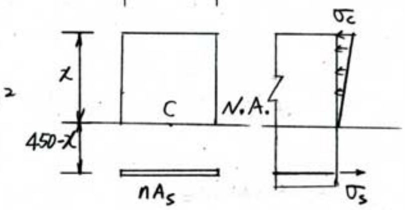
Solution:

Transformed Section.

$A_s = 4 \cdot \frac{\pi}{4} d^2 = 4 \cdot \frac{\pi}{4} \cdot (22mm)^2 = 1520.5\ mm^2$

$n = \dfrac{E_s}{E_c} = \dfrac{200 GPa}{25 GPa} = 8.0$

$nA_s = 8.0\,(1520.5\ mm^2) = 12.164 \times 10^3\ mm^2$



Neutral axis

$250 \cdot x \cdot \frac{x}{2} - (12.164 \times 10^3)(450 - x) = 0$

$x = 166.19\ mm$

Moment of Inertia

$I = \frac{1}{3}(250mm)(166.19mm)^3 + (12.164 \times 10^3\ mm^2)(450\ mm - 166.19mm)^2$

$= 1.3623 \times 10^{-3}\ m^4$

(a) Stress in the steel

$y = 166.19mm - 450mm = -283.81\ mm = -0.28381\ m$

$\sigma_s = -\dfrac{nMy}{I} = -\dfrac{(8.0)(175 \times 10^3\ N\cdot m)(-0.28381 m)}{1.3623 \times 10^{-3}\ m^4} = 292 \times 10^6 Pa$

$= \underline{\underline{292 MPa}}$

(b) Maximum stress in the concrete

$y = 166.19mm = 0.16619m$

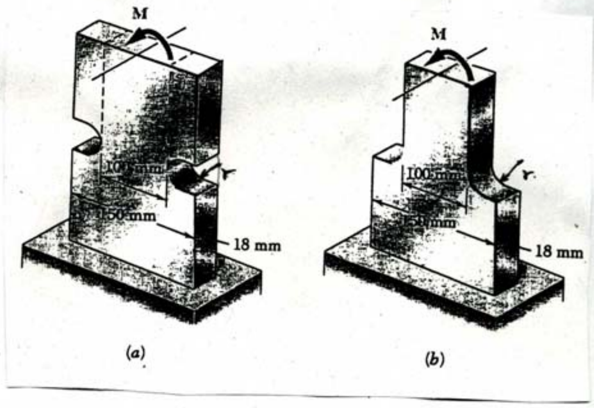
$\sigma_c = -\dfrac{My}{I} = -\dfrac{(175 \times 10^3\ N\cdot m)(0.16619m)}{1.3623 \times 10^{-3}\ m^4} = -21.3 \times 10^6 Pa$

$= \underline{\underline{-21.3 MPa}}$

Problem 4.74

Given: $M = 2\ KN \cdot m$, $r = 10mm$

Find: the maximum stress in the bar show in figure (a) and (b)



FBD: not required

FPU: stress concentration

$$\sigma = \frac{KMc}{I}$$

Solution

For both configurations

$D = 150mm$, $d = 100mm$. $r = 10mm$

$$\frac{D}{d} = \frac{150\,mm}{100\,mm} = 1.50$$

$$\frac{r}{d} = \frac{10\,mm}{100\,mm} = 0.10$$

Stress – concentration factors

For configuration (a), from Figure 4.32, we have $K_a = 2.21$

For configuration (b), from Figure 4.31, we have $K_b = 1.79$

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(18mm)(100mm)^3 = 1.5 \times 10^6 mm^4 = 1.5 \times 10^{-6} m^4$$

$$c = \frac{1}{2} d = 50mm = 0.05m$$

Maximum stress

(a) $$\sigma = \frac{K_a M c}{I} = \frac{(2.21)(2 \times 10^3 N \cdot m)(0.05m)}{1.5 \times 10^{-6} m^4} = 147 \times 10^6 Pa = \underline{\underline{147 MPa}}$$

(b) $$\sigma = \frac{K_b M c}{I} = \frac{(1.79)(2 \times 10^3 N \cdot m)(0.05m)}{1.5 \times 10^{-6} m^4} = 119 \times 10^6 Pa = \underline{\underline{119 MPa}}$$