

ENES 220 – Mechanics of Materials  
Spring 2003

Solutions to Homework #5

Problem 3.72

Given :  $L = 125 \text{ ft}$   $d_1 = 16 \text{ in}$   $d_2 = 8 \text{ in}$   $\tau_{\text{all}} = 8500 \text{ psi}$   
 $G = 11.2 \times 10^6 \text{ psi}$   $f = 165 \text{ rpm}$

To Find: (a) the maximum power that can be transmitted  $P$   
          (b) the angle of twist of the shaft  $\phi$

FBD : not required

FPU :  $T = \frac{J\tau}{C}$   $P = 2\pi f T$ ,  $\phi = \frac{TL}{GJ}$

Solution :

Polar moment of inertia  $J$

$$C_2 = \frac{1}{2}d_2 = 8 \text{ in}, C_1 = \frac{1}{2}d_1 = 4 \text{ in}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}((8 \text{ in})^4 - (4 \text{ in})^4) = 6031.86 \text{ in}^4$$

Allowable Torque  $T$

$$T = \frac{J \tau_{\text{all}}}{C_2} = \frac{(6031.86 \text{ in}^4) \cdot (8500 \text{ psi})}{8 \text{ in}} = 6.4088 \times 10^6 \text{ lb}\cdot\text{in}$$

(a) Maximum power that can be transmitted  $P$

$$f = \frac{165 \text{ rpm}}{60 \text{ sec.}} = 2.75 \text{ Hz}$$

$$P = 2\pi f T = 2\pi (2.75 \text{ Hz})(6.4088 \times 10^6 \text{ lb}\cdot\text{in}) \\ = 110.74 \times 10^6 \text{ lb}\cdot\text{in/s} = \underline{\underline{16.78 \times 10^3 \text{ hp}}}$$

(b) Twist angle of the shaft  $\phi$

$$L = 125 \text{ ft} = 1500 \text{ in}$$

$$\phi = \frac{TL}{GJ} = \frac{(6.4088 \times 10^6 \text{ lb}\cdot\text{in}) \cdot (1500 \text{ in})}{(11.2 \times 10^6 \text{ psi})(6031.86 \text{ in}^4)}$$

$$= 0.1423 \text{ rad} = \underline{\underline{8.15^\circ}}$$

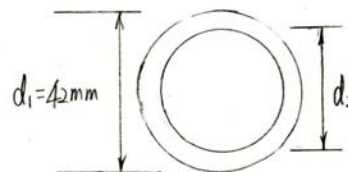
Problem 3.85

Given :  $d_1 = 42 \text{ mm}$   $\tau_{\text{all}} = 75 \text{ MPa}$   $G = 77 \text{ GPa}$   $T = 900 \text{ N}\cdot\text{m}$   
 $\phi_{\text{all}} = 4^\circ$   $L = 1.6 \text{ m}$

To Find:  $d_2$

FBD : not required

FPU :  $\tau = \frac{Tc}{J}$   $\phi = \frac{TL}{GJ}$



Solution: Polar moment of Inertia J

Based on stress limit  $\tau_{\text{all}} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$c_1 = \frac{1}{2} d_1 = 0.021 \text{ m}$$

$$\tau = \frac{Tc}{J}$$

$$\therefore J = \frac{Tc_1}{\tau_{\text{all}}} = \frac{(900 \text{ N}\cdot\text{m})(0.021 \text{ m})}{75 \times 10^6 \text{ Pa}} = 252 \times 10^{-9} \text{ m}^4$$

Based on angle of twist limit  $\phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ}, \quad G = 77 \text{ GPa} = 77 \times 10^9 \text{ Pa}$$

$$\therefore J = \frac{TL}{G\phi} = \frac{(900 \text{ N}\cdot\text{m})(1.6 \text{ m})}{(77 \times 10^9 \text{ Pa})(69.813 \times 10^{-3} \text{ rad})} = 267.88 \times 10^{-4} \text{ m}^4$$

Larger value for J govern,  $\therefore J = 267.88 \times 10^{-4} \text{ m}^4$

Inner diameter  $d_2$

$$J = \frac{\pi}{2} (c_1^4 - c_2^4)$$

$$\therefore c_2 = \left( c_1^4 - \frac{2J}{\pi} \right)^{\frac{1}{4}} = \left( (0.021 \text{ m})^4 - \frac{2 \cdot (267.88 \times 10^{-4} \text{ m}^4)}{\pi} \right)^{\frac{1}{4}}$$

$$= 12.44 \times 10^{-3} \text{ m} = 12.44 \text{ mm}$$

$$\therefore d_2 = 2c_2 = \underline{\underline{24.9 \text{ mm}}}$$



Problem 3.90

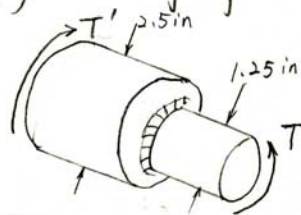
Given:  $P = 60 \text{ hp}$   $\tau_{\text{all}} = 6000 \text{ psi}$   $r = 0.25 \text{ in}$

$D = 2.5 \text{ in}$   $d = 1.25 \text{ in}$

To Find: the smallest permissible speed of the shaft  $f$

FBD: not required

FPU:  $\tau = \frac{K T c}{J}$   $P = 2\pi f T$



Solution:

Stress concentration factor  $K$

$$\frac{r}{d} = \frac{0.25 \text{ in}}{1.25 \text{ in}} = 0.200$$

$$\frac{D}{d} = \frac{2.5 \text{ in}}{1.25 \text{ in}} = 2.00$$

From Figure 3.32  $K = 1.26$

Allowable Torque  $T$

For smaller shaft  $c = \frac{1}{2}d = \frac{1}{2}(1.25 \text{ in}) = 0.625 \text{ in}$

$$\tau = \frac{K T c}{J} = \frac{2 K T}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau_{\text{all}}}{2 K} = \frac{\pi (0.625 \text{ in})^3 (6000 \text{ psi})}{2 \cdot (1.26)} = 1.826 \times 10^3 \text{ lb}\cdot\text{in}$$

The smallest permissible speed of the shaft  $f$

$$P = 60 \text{ hp} = (60 \text{ hp}) (6600 \text{ lb}\cdot\text{in/s}\cdot\text{hp}) = 396 \times 10^3 \text{ lb}\cdot\text{in/s}$$

$$P = 2\pi f T$$

$$\therefore f = \frac{P}{2\pi T} = \frac{396 \times 10^3 \text{ lb}\cdot\text{in/s}}{2\pi (1.826 \times 10^3 \text{ lb}\cdot\text{in})} = 34.5 \text{ Hz}$$

$$= \underline{\underline{2071 \text{ rpm}}}$$

Problem 3.140

Given:  $T = 50 \text{ kip}\cdot\text{in}$      $t_a = 0.25 \text{ in}$      $t_b = 0.375 \text{ in}$

$h = 5 \text{ in}$      $w = 3 \text{ in}$

To Find: shearing stress at point a and b

FBD: not required

FPU:  $\tau = \frac{T}{2tQ}$

Solution:

Area bounded by centerline

$$Q = (4.625 \text{ in})(2.75 \text{ in}) = 12.719 \text{ in}^2$$

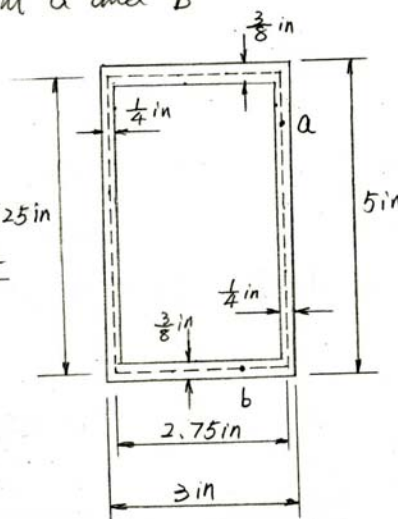
Shearing Stresses

At point a     $t_a = 0.25 \text{ in}$

$$\tau = \frac{T}{2t_a Q} = \frac{50 \text{ kip}\cdot\text{in}}{2 \cdot (0.25 \text{ in})(12.719 \text{ in}^2)} = \underline{\underline{7.86 \text{ ksi}}}$$

At point b     $t_b = 0.375 \text{ in}$

$$\tau = \frac{T}{2t_b Q} = \frac{50 \text{ kip}\cdot\text{in}}{2 \cdot (0.375 \text{ in})(12.719 \text{ in}^2)} = \underline{\underline{5.24 \text{ ksi}}}$$



**Problem 4.4**

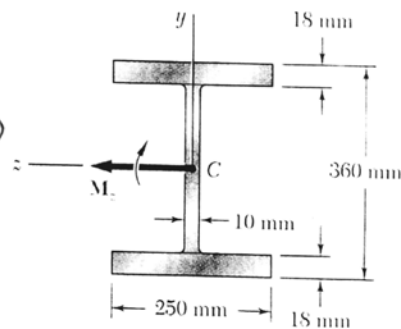
[Given]  $\sigma_Y = 345 \text{ MPa}$ ,  $\sigma_U = 450 \text{ MPa}$ ,  $F.S. = 3.0$ .

[To Find] largest couple can be applied. (bent about  $y$  axis)

[FBD] Not required

[FPU]  $I = \frac{1}{12}bh^3$ ,  $\sigma_{all} = \frac{\sigma_U}{F.S.}$   $M = \frac{\sigma I}{C}$

[Solution]



$$I_1 = \frac{1}{12}bh_1^3 = \frac{1}{12}(18\text{mm})(250\text{mm})^3 = 23437500\text{mm}^4$$

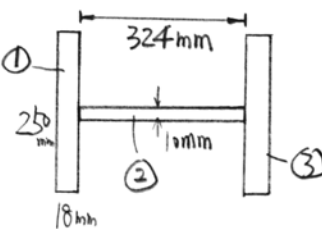
$$= 2.34375 \times 10^{-5} \text{ m}^4$$

$$I_3 = I_1 = 2.34375 \times 10^{-5} \text{ m}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(324\text{mm})(10\text{mm})^3 = 27000\text{mm}^4 = 2.7 \times 10^{-8} \text{ m}^4$$

$$I = I_1 + I_2 + I_3 = 2.34375 \times 10^{-5} \text{ m}^4 + 2.7 \times 10^{-8} \text{ m}^4 + 2.34375 \times 10^{-5} \text{ m}^4$$

$$= 4.6902 \times 10^{-5} \text{ m}^4$$



$$\sigma = \frac{Mc}{I}, \text{ where } c = \frac{250\text{mm}}{2} = 125\text{mm} = 0.125 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \text{ MPa}}{3.0} = 150 \times 10^6 \text{ Pa}$$


$$\therefore M_{all} = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6 \text{ Pa})(4.6902 \times 10^{-5} \text{ m}^4)}{0.125 \text{ m}} = 56282.4 \text{ N}\cdot\text{m}$$

$$= \boxed{56.3 \text{ kN}\cdot\text{m}}$$

**Problem 4.10**

[Given]  $\longrightarrow$

[To Find] Maximum stresses in BC portion.

[FBD]   
 (Fig. 1)

[FPU]  $I = \frac{1}{12}bh^3 + Ad^2$ ,  $\sigma = -\frac{My}{I}$  (Fig. 2)

[Solution]

(1)

	$A$ (in <sup>2</sup> )	$\bar{y}_o$ (in)	$A\bar{y}_o$ (in <sup>3</sup> )
①	$3 \cdot 6 = 18$	$3 + 2 = 5$	$18 \cdot 5 = 90$
②	$9 \cdot 2 = 18$	1	$18 \cdot 1 = 18$
$\Sigma$	36		108

$$\bar{y}_o = \frac{108 \text{ in}^3}{36 \text{ in}^2} = 3 \text{ in}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3 \text{ in}) (6 \text{ in})^3 + (18 \text{ in}^2) (2 \text{ in})^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (9 \text{ in}) (2 \text{ in})^3 + (18 \text{ in}^2) (2 \text{ in})^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 \text{ in}^4 + 78 \text{ in}^4 = 204 \text{ in}^4$$

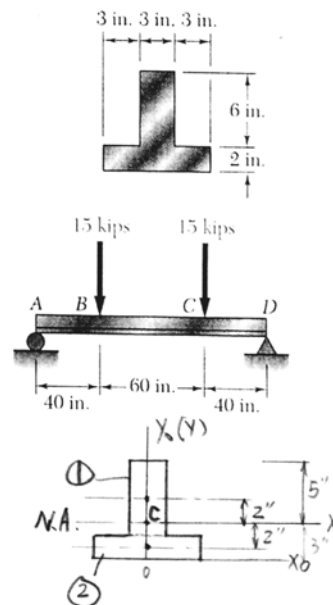
$$y_{\text{top}} = 3 \text{ in} + 2 \text{ in} = 5 \text{ in}, \quad y_{\text{bot}} = -(1 \text{ in} + 2 \text{ in}) = -3 \text{ in}$$

(2) From statics, using the symmetry feature.  $R_A = R_D = 15 \text{ kips}$

From Fig. 2,  $\sum M_A = 0 \quad M - P(40 \text{ in}) = 0 \Rightarrow M = (15 \text{ kips})(40 \text{ in}) = 600 \text{ kip}\cdot\text{in}$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(600 \text{ kip}\cdot\text{in})(5 \text{ in})}{204 \text{ in}^4} = \boxed{-14.71 \text{ ksi}}$$

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(600 \text{ kip}\cdot\text{in})(-3 \text{ in})}{204 \text{ in}^4} = \boxed{8.82 \text{ ksi}}$$



**Problem 4.27**

[Given]  $E = 29 \times 10^6 \text{ psi}$ ,  $\rho = \frac{3}{8} \text{ in}$

[To Find]  $\sigma_{\max}$ ,  $M$

[FBD] Not required

[FPU]  $\frac{1}{\rho} = \frac{\sigma_m}{Ec}$ ,  $\frac{1}{\rho} = \frac{M}{EI}$ ,  $I = \frac{1}{12}bh^3$

[Solution]

(1)  $c = \frac{0.005 \text{ in}}{2} = 0.0025 \text{ in}$

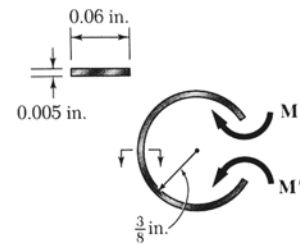
$$\frac{1}{\rho} = \frac{\sigma_m}{Ec}$$

$$\therefore \sigma_m = \frac{Ec}{\rho} = \frac{(29 \times 10^6 \text{ psi})(0.0025 \text{ in})}{(0.375 \text{ in})} = 193333.3 \text{ psi} = \boxed{193.33 \text{ ksi}}$$

(2)  $I = \frac{1}{12}bh^3 = \frac{1}{12}(0.06 \text{ in})(0.005 \text{ in})^3 = 6.25 \times 10^{-10} \text{ in}^4$

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$\therefore M = \frac{EI}{\rho} = \frac{(29 \times 10^6 \text{ psi})(6.25 \times 10^{-10} \text{ in}^4)}{0.375 \text{ in}} = \boxed{4.833 \times 10^{-2} \text{ lb}\cdot\text{in}}$$





**Problem 4.145**

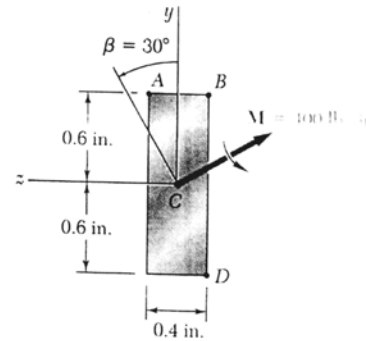
[Given]  $\longrightarrow$

[To Find]  $\sigma_A, \sigma_B, \sigma_D$

[FBD] Not required

[FPU] 
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

[Solution]



$$I_z = \frac{1}{12} (0.4 \text{ in})(1.2 \text{ in})^3 = 0.0576 \text{ in}^4$$

$$I_y = \frac{1}{12} (1.2 \text{ in})(0.4 \text{ in})^3 = 0.0064 \text{ in}^4$$

$$M_z = -400 \cos 30^\circ = -346.41 \text{ lb}\cdot\text{in}$$

$$M_y = 400 \sin 30^\circ = 200 \text{ lb}\cdot\text{in}$$

(a)  $y_A = 0.6 \text{ in}, z_A = 0.2 \text{ in}$

$$\begin{aligned} \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41 \text{ lb}\cdot\text{in})(0.6 \text{ in})}{(0.0576 \text{ in}^4)} + \frac{(200 \text{ lb}\cdot\text{in})(0.2 \text{ in})}{(0.0064 \text{ in}^4)} \\ &= 9858.4 \text{ psi} = \boxed{9.86 \text{ ksi}} \end{aligned}$$

(b)  $y_B = 0.6 \text{ in}, z_B = -0.2 \text{ in}$

$$\begin{aligned} \sigma_B &= -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41 \text{ lb}\cdot\text{in})(0.6 \text{ in})}{(0.0576 \text{ in}^4)} + \frac{(200 \text{ lb}\cdot\text{in})(-0.2 \text{ in})}{(0.0064 \text{ in}^4)} \\ &= -2641.6 \text{ psi} = \boxed{-2.64 \text{ ksi}} \end{aligned}$$

(c)  $y_D = -0.6 \text{ in}, z_D = -0.2 \text{ in}$

$$\begin{aligned} \sigma_D &= -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41 \text{ lb}\cdot\text{in})(-0.6 \text{ in})}{(0.0576 \text{ in}^4)} + \frac{(200 \text{ lb}\cdot\text{in})(-0.2 \text{ in})}{(0.0064 \text{ in}^4)} \\ &= -9858.4 \text{ psi} = \boxed{-9.86 \text{ ksi}} \end{aligned}$$