ENES 220 – Mechanics of Materials
Spring 2003

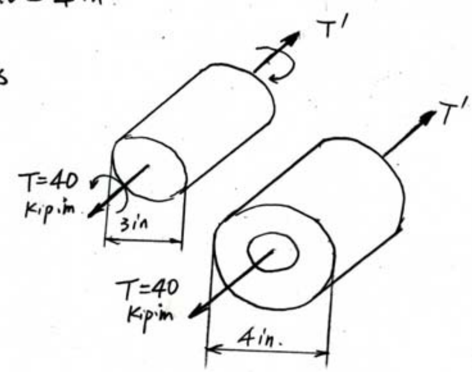
Solutions to Homework #4

Problem 3.7

Given: $T = 40\ \text{kip}\cdot\text{in}$ $\quad d_s = 3\text{in}$ $\quad d_o = 4\text{in}$

Find: (a) the maximum shearing stress

(b) the inner diameter of the hollow cylinder



FBD: not required

FPU: $\tau_{max} = \frac{Tc}{J}$

Solution:

(a) Solid shaft

$$c = \frac{1}{2}d = \frac{1}{2}(3.0\text{in}) = 1.5\text{ in}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{Tc}{\frac{1}{2}\pi c^4} = \frac{2T}{\pi c^3}$$

$$= \frac{(2)\ (40\text{Kip}\cdot\text{in})}{\pi\ (1.5\text{ in})^3} = \underline{\underline{7.545\ \text{Ksi}}}$$

(b) Hollow shaft

$$C_o = \frac{1}{2}d_o = \frac{1}{2}(4\text{ in}) = 2.0\text{in}$$

$$J = \frac{\pi}{2}(C_o^4 - C_i^4)$$

$$\tau_{max} = \frac{T\,C_o}{J}$$

$$C_i^4 = C_o^4 - \frac{2TC_o}{\pi\,\tau_{max}} = (2.0\text{in})^4 - \frac{2\cdot(40\cdot\text{Kip.in})(2\text{in})}{\pi\ (7.545\text{Ksi})}$$

$$= 9.25\text{ in}^4$$

$$C_i = 1.74\text{ in}$$

$$d_i = 2C_i = 2\cdot(1.74\text{in}) = \underline{\underline{3.48\text{ in}}}$$

Problem 3.17

Given $d_{AB} = 60mm$ $d_{CD.o} = 90mm$. $t = 6mm$

$\tau_{all} = 75MPa$

To Find: the largest torque

FBD : not required

FPU : $T = \frac{J\tau_{all}}{c}$

Solution :

Rod AB: $c = \frac{1}{2}d = 0.030m$

$J = \frac{\pi}{2}c^4$

$T_{all} = \frac{J\tau_{all}}{c} = \frac{\frac{1}{2}\pi c^4 \cdot \tau_{all}}{c}$

$= \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.030m)^3(75 \times 10^6 Pa) = 3.181 \times 10^3 N \cdot m$

Pipe CD: $c_o = \frac{1}{2}d_{CD,o} = 0.045m$

$c_i = c_o - t = (0.045m) - (0.006m) = 0.039m$

$J = \frac{\pi}{2}(c_i^4 - c_o^4) = \frac{\pi}{2}((0.045m)^4 - (0.039m)^4) = 2.8073 \times 10^{-6} m^4$

$T_{all} = \frac{J\tau_{all}}{c_o} = \frac{(2.8073 \times 10^{-6} m^4) \cdot (75 \times 10^6 Pa)}{0.045m} = 4.679 \times 10^3 N \cdot m$

Allowable torque is the smaller value $3.181 \times 10^3 N \cdot m$

Problem 3.35

Given $T_A = 300\ N\cdot m$ $T_B = 400\ N\cdot m$ $d_{AB} = 30mm$ $d_{CD} = 46mm$

To Find : (a) the angle of twist between A and B

(b) the angle of twist between A and C

FBD : Figure (a) (b)

FPU: $\varphi = \dfrac{TL}{GJ}$

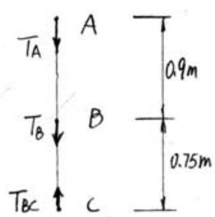


Figure (a)

Figure (b)

Solution:

(a) Twist angle between A and B

$T_{AB} = T_A = 300\ N\cdot m$ $L_{AB} = 0.9m$

$C_{AB} = \frac{1}{2} d_{AB} = 0.015m$

$J_{AB} = \frac{\pi}{2} C_{AB}^4 = \frac{\pi}{2}(0.015m)^4 = 79.522 \times 10^{-9}\ m^4$

$$\varphi_{AB} = \frac{T_{AB} L_{AB}}{G J} = \frac{(300 N\cdot m)(0.9m)}{(77\times10^9 Pa)(79.522\times10^{-9} m^4)}$$

$= 44.095 \times 10^{-3}$ rad

$\varphi_{AB} = \underline{\underline{2.53^\circ}}$

(b) Twist angle between A and C

$T_{BC} = T_A + T_B = (300 N\cdot m) + (400 N\cdot m) = 700\ N\cdot m$ $L_{BC} = 0.75m$

$C_{BC} = \frac{1}{2} d_{BC} = 0.023\ m$

$J_{BC} = \frac{\pi}{2} C_{BC}^4 = \frac{\pi}{2}(0.023m)^4 = 439.573 \times 10^{-9}\ m^4$

$$\varphi_{BC} = \frac{T_{BC} L_{BC}}{G J_{BC}} = \frac{(700 N\cdot m)(0.75m)}{(77\times10^9 Pa)(439.573\times10^{-9} m^4)} = 15.511\times10^{-3}\ rad$$

$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 59.606 \times 10^{-3}$ rad

$\varphi_{AC} = \underline{\underline{3.42^\circ}}$

Problem 3.41

Given two shafts $d = \frac{3}{4}$ in , two gears $r_B = 3in$, $r_E = 4in$

$L_{AB} = 11 in$ $L_{EF} = 8 in$, $T = 750$ lb·in

To Find: φ_A

FBD : Figure (a) (b)

FPU : $\varphi = \frac{TL}{GJ}$

Solution: $F = \frac{T}{r_B} = \frac{T_{EF}}{r_E}$

$T_{EF} = \frac{r_E}{r_B} T = \frac{4in}{3in}(750 \text{ lb·in}) = 1000 \text{ lb·in}$

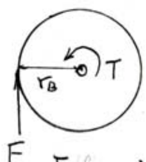


Figure (a)

Twist in shaft FE

$J_{FE} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.375 \text{ in})^4$
$= 31.063 \times 10^{-3} \text{ in}^4$

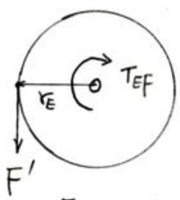


Figure (b)

$\varphi_{FE} = \frac{T_{FE} L_{FE}}{G J_{FE}} = \frac{(1000 \text{ lb·in})(8 \text{ in})}{(11.2 \times 10^6 \text{ Pa})(31.063 \times 10^{-3} \text{ in}^4)}$
$= 22.995 \times 10^{-3}$ rad

Rotation at E $\varphi_E = 22.995 \times 10^{-3}$ rad

$\delta = r_E \varphi_E = r_B \varphi_B$

$\varphi_B = \frac{r_E}{r_B} \varphi_E = \frac{4in}{3in}(22.935 \times 10^{-3} \text{ rad}) = 30.660 \times 10^{-3}$ rad

Twist in shaft BA

$J_{BA} = 31.063 \times 10^{-3} \text{ in}^4$

$\varphi_{AB} = \frac{T_{AB} L_{BA}}{G J_{BA}} = \frac{(750 \text{ lb·in})(11 \text{ in})}{(11.2 \times 10^6 \text{ Pa})(31.063 \times 10^{-3} \text{ in}^4)} = 23.713 \times 10^{-3}$ rad

Rotation at A

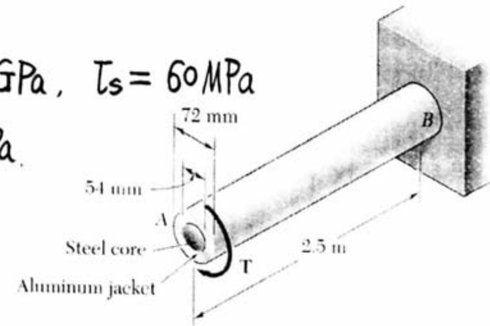
$\varphi_A = \varphi_B + \varphi_{AB} = (30.660 \times 10^{-3} \text{ rad}) + (23.713 \times 10^{-3} \text{ rad}) = 54.373 \times 10^{-3}$ rad
$= \underline{\underline{3.12^0}}$

Problem 3.53

Given: Steel core: $d_s = 54$ mm, $G_s = 77$ GPa, $\tau_s = 60$ MPa

Aluminum: $d_a = 72$ mm, $G_a = 27$ GPa

$\tau_a = 45$ MPa.



To Find: ϕ_A

FBD: Not required

FPU: $\tau_{max} = Gc\dfrac{\phi}{L}$, $\phi = \dfrac{\tau_{max} L}{Gc}$

Solution:

(1) Steel core: $c_s = \dfrac{d_s}{2} = 0.027$ m

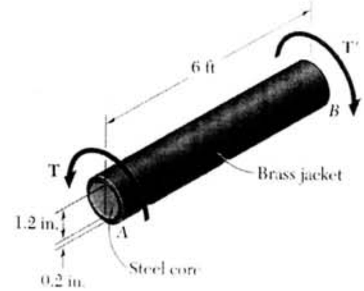
$$\phi_s = \frac{\tau_s L}{G_s c_s} = \frac{(60\times10^6)(2.5)}{(77\times10^9)(0.027)} = 0.07215 \text{ rad} = \boxed{4.134^\circ}$$

(2) Aluminum jacket: $c_a = \dfrac{d_a}{2} = 0.036$ m

$$\phi_a = \frac{\tau_a L}{G_a c_a} = \frac{(45\times10^6)(2.5)}{(27\times10^9)(0.036)} = 0.11574 = 6.631^\circ$$

$\therefore$ The largest angle may be rotated: 4.134°

Problem 3.54



Given: $G_b = 5.6 \times 10^6$ psi, $d_b = 1.6$ in
$G_s = 11.2 \times 10^6$ psi, $d_s = 1.2$ in
$T = 5$ kip·in

To Find: (1) The maximum τ_b
(2) The maximum τ_s
(3) ϕ_{BA}.

FBD: Not required

FPU: $T = GJ\frac{\phi}{L}$, $\tau = Gc\frac{\phi}{L}$

Solution:

① Steel core: $c_s = \frac{1}{2}d_s = 0.6$ in

$J_s = \frac{\pi}{2}c_s^4 = 0.203575 \text{ in}^4$

Torque carried by steel core:

$$T_s = G_s J_s \frac{\phi}{L}$$

② Brass jacket: $c_b = \frac{1}{2}d_b = 0.8$ in

$J_b = \frac{\pi}{2}(c_b^4 - c_s^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.439823 \text{ in}^4$

Torque carried by brass jacket:

$$T_b = G_b J_b \frac{\phi}{L}$$

③ Total torque: $T = T_s + T_b = (G_s J_s + G_b J_b)\frac{\phi}{L}$

$$\therefore \frac{\phi}{L} = \frac{T}{G_s J_s + G_b J_b} = \frac{5 \times 10^3 \text{ lb·in}}{(11.2 \times 10^6 \text{psi})(0.203575 \text{ in}^4) + (5.6 \times 10^6 \text{psi})(0.439823 \text{in}^4)}$$
$$= 1.054174 \times 10^{-3} \text{ rad/in}$$

(a) The maximum shearing stress in steel core:

$$\tau_s = G_s c_s \frac{\phi}{L} = (11.2 \times 10^6 \text{psi})(0.6 \text{in})(1.054174 \times 10^{-3}/\text{in}) = 7.084 \times 10^3 \text{ lb} = \boxed{7.084 \text{ ksi}}$$

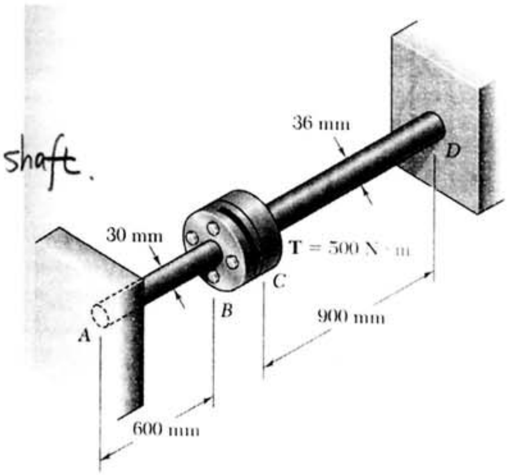
(b) The maximum shearing stress in brass jacket:

$$\tau_b = G_b c_b \frac{\phi}{L} = (5.6 \times 10^6 \text{psi})(0.8 \text{in})(1.054174 \times 10^{-3}/\text{in}) = 4.723 \times 10^3 \text{lb} = \boxed{4.723 \text{ ksi}}$$

(c)

$$\phi_{BA} = \left(\frac{\phi}{L}\right)L = (1.054174 \times 10^{-3}/\text{in})(6 \times 12 \text{ in}) = 0.0759 \text{ rad} = \boxed{4.349°}$$

Problem 3.56



Given: $G = 77\,GPa$, $T = 500\,N\cdot m$

To Find: The maximum shearing stress in each shaft.

FBD: Not required

FPU: $T = \frac{GJ\phi}{L}$, $\tau = \frac{Tc}{J}$

Solution:

① Shaft AB

$L_{AB} = 0.6\,m$, $c_{AB} = \frac{1}{2} d_{AB} = 0.015\,m$

$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.015m)^4 = 79.52 \times 10^{-9}\,m^4$

$$T_{AB} = \frac{G_{AB} J_{AB}}{L_{AB}} \phi_B = \frac{(77 \times 10^9 Pa)(79.52 \times 10^{-9} m^4)}{(0.6m)} \phi_B = 10.205 \times 10^3 \phi_B$$

② Shaft CD

$L_{CD} = 0.9\,m$, $c_{CD} = \frac{1}{2} d_{CD} = 0.018\,m$

$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.018m)^4 = 164.896 \times 10^{-9}\,m^4$

$$T_{CD} = \frac{G_{CD} J_{CD}}{L_{CD}} \phi_C = \frac{(77 \times 10^9 Pa)(164.896 \times 10^{-9} m^4)}{(0.9m)} \phi_C = 14.108 \times 10^3 \phi_C$$

③ Matching rotation at the flanges $\phi_B = \phi_C = \phi$.

Total torque: $T = T_{AB} + T_{CD} = 500\,N\cdot m$

$(10.205 \times 10^3 + 14.108 \times 10^3)\phi = 500 \quad \Rightarrow \phi = 20.565 \times 10^{-3}\,rad$

$\therefore T_{AB} = (10.205 \times 10^3)(20.565 \times 10^{-3}) = 209.87\,N\cdot m$

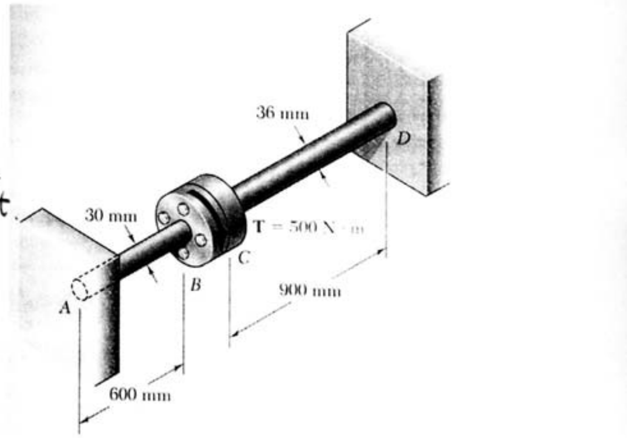
$T_{CD} = (14.108 \times 10^3)(20.565 \times 10^{-3}) = 290.13\,N\cdot m$

(a) Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB} c_{AB}}{J_{AB}} = \frac{(209.87 N\cdot m)(0.015 m)}{79.52 \times 10^{-9} m^4} = 39.59 \times 10^6\,Pa = \boxed{39.6\,MPa}$$

(b) Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD} c_{CD}}{J_{CD}} = \frac{(290.13 N\cdot m)(0.018 m)}{164.896 \times 10^{-9} m^4} = 31.67 \times 10^6\,Pa = \boxed{31.7\,MPa}$$



Problem 3.58

Given: $G = 77 GPa$, $\phi_0 = 1.5°$

$T = 500 N\cdot m$, applied to flange C.

To Find: The maximum shearing stress in each shaft.

FBD: Not required

FPU: $T = \frac{GJ\phi}{L}$, $\tau = \frac{Tc}{J}$

Solution:

① Shaft AB

$L_{AB} = 0.6m$, $C_{AB} = \frac{1}{2}d_{AB} = 0.015m$, $J_{AB} = \frac{\pi}{2}C_{AB}^4 = \frac{\pi}{2}(0.015m)^4 = 79.52\times10^{-9}m^4$

$T_{AB} = \frac{G_{AB}J_{AB}}{L_{AB}}\phi_B = \frac{(77\times10^9 Pa)(79.52\times10^{-9}m^4)}{(0.6m)}\phi_B = 10.205\times10^3\phi_B$

② Shaft CD

$L_{CD} = 0.9m$, $C_{CD} = \frac{1}{2}d_{CD} = 0.018m$, $J_{CD} = \frac{\pi}{2}C_{CD}^4 = \frac{\pi}{2}(0.018m)^4 = 164.896\times10^{-9}m^4$

$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\phi_C = \frac{(77\times10^9 Pa)(164.896\times10^{-9}m^4)}{(0.9m)}\phi_C = 14.108\times10^3\phi_C$

③ Clear rotation for flange C: $\phi_0 = 1.5° = 26.18\times10^{-3}$ rad.

Torque to remove clearance: $T_{CD}'' = (14.108\times10^3)(26.18\times10^{-3}) = 369.347 N\cdot m$

Torque to cause additional rotation ϕ': $T' = T_{total} - T_{CD}'' = 500 - 369.347 = 130.653 N\cdot m$

$T' = T_{AB}' + T_{CD}' \Rightarrow 130.653 N\cdot m = (10.205\times10^3 + 14.108\times10^3)\phi'$

$\therefore \phi' = \frac{130.653}{10.205\times10^3 + 14.108\times10^3} = 5.374\times10^{-3}$ rad

$T_{AB}' = (10.205\times10^3)(5.374\times10^{-3}) = 54.84 N\cdot m$

$T_{CD}' = (14.108\times10^3)(5.374\times10^{-3}) = 75.81 N\cdot m$

(a) Maximum shearing stress in AB

$\tau_{AB} = \frac{T_{AB}C_{AB}}{J_{AB}} = \frac{(54.84 N\cdot m)(0.015m)}{(79.52\times10^{-9}m^4)} = 10.34\times10^6 Pa = \boxed{10.34 MPa}$

(b) Maximum shearing stress in CD

$\tau_{CD} = \frac{T_{CD}C_{CD}}{J_{CD}} = \frac{(75.81 N\cdot m + 369.35 N\cdot m)(0.018m)}{(164.896\times10^{-9}m^4)} = 48.59\times10^6 Pa = \boxed{48.59 MPa}$