ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #3

Problem 2.36

Given $P = 450\,kN$ $h = 10mm$ $E_b = 105\,GPa$ $E_a = 70\,GPa$ $L = 300mm$

To Find: (a) the normal stress in the brass core σ_b

(b) the normal stress in the aluminum plates σ_a

FBD: figure (a)

FPU: $\delta = \dfrac{PL}{EA}$ $\varepsilon = \dfrac{\delta}{L}$ $\sigma = E\varepsilon$

Solution: Let P_b = portion of axial force carried by brass core

P_a = portion of axial force carried by two aluminum plates

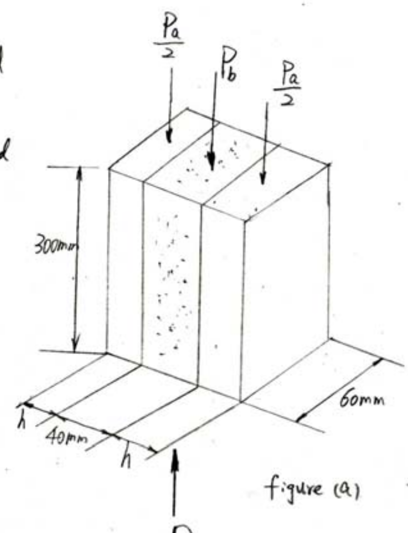


figure (a)

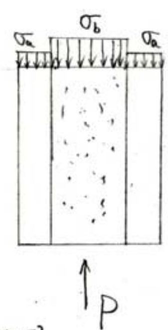
Equilibrium: $P = P_a + P_b$

Compatibility $\delta = \dfrac{P_b L}{E_b A_b} = \dfrac{P_a L}{E_a A_a}$

$$P_b = \frac{E_b A_b \delta}{L} \quad P_a = \frac{E_a A_a \delta}{L}$$

$$P = (E_b A_b + E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$



$A_b = (60mm)(40mm) = 2400\,mm^2 = 2400 \times 10^{-6} m^2$

$A_a = (2)(60mm)(10mm) = 120\,mm^2 = 1200 \times 10^{-6} m^2$

$$\varepsilon = \frac{450 \times 10^3 N}{(105\times10^9 Pa)(2400\times10^{-6} m^2) + (70\times10^9 Pa)(1200\times10^{-6} m^2)} = 1.3393\times10^{-3}$$

(a) $\sigma_b = E_b \varepsilon = (105\times10^9 Pa)(1.3393\times10^{-3}) = 140.6\times10^6 Pa = \underline{\underline{140.6\,MPa}}$

(b) $\sigma_a = E_a \varepsilon = (70\times10^9 Pa)(1.3393\times10^{-3}) = 93.75\times10^6 Pa = \underline{\underline{93.75\,MPa}}$

Problem 2.40

Given a brass bolt $d = \frac{3}{8}$ in. $E_b = 15 \times 10^6$ Psi

a steel tube $d_o = \frac{7}{8}$ in. $t = \frac{1}{8}$ in $E_s = 29 \times 10^6$ Psi $d_i = \frac{5}{8}$ in

$\delta = 0.025$ in.

To Find: (a) the normal stress in the bolt σ_b

(b) the normal stress in the tube σ_s

FBD : figure (a)

FPU : $\delta = \frac{PL}{EA}$, $\sigma = \frac{P}{A}$, compatibility

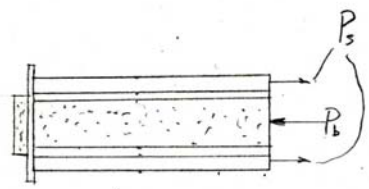


figure (a)

Solution:

Equilibrium $P_s = P_b = P$ (1)

Compatibility $\delta = \delta_b + \delta_t$ (2)

$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8} in\right)^2 = 0.11045 \text{ in}^2$

$A_t = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left(\left(\frac{7}{8}in\right)^2 - \left(\frac{5}{8}in\right)^2\right) = 0.29452 \text{ in}^2$

$$\delta_b = \frac{P_b L}{E_b A_b} = \frac{P\,(12\text{ in})}{(15\times10^6\text{ Psi})(0.11045\text{ in}^2)} = 7.2431 \times 10^{-6} P \quad \text{in.}$$

$$\delta_t = \frac{P_t L}{E_t A_t} = \frac{P\,(12\text{ in})}{(29\times10^6\text{ psi})(0.29452\text{ in}^2)} = 1.4050 \times 10^{-6} P \quad \text{in}$$

From (2), we have $0.025\text{in} = (7.2431\times10^{-6} P + 1.4050\times10^{-6} P)$ in.

$$P = 2.8908 \times 10^3 \text{ lb}$$

(a) $\sigma_b = \frac{P}{A_b} = \frac{2.8908\times10^3 \text{ lb}}{0.11045 \text{ in}^2} = 26.2\times10^3$ psi $= \underline{\underline{26.2 \text{ ksi (tension)}}}$

(b) $\sigma_t = -\frac{P}{A_t} = -\frac{2.8908\times10^3 \text{ lb}}{0.29452 \text{ in}^2} = -9.82\times10^3$ psi $= \underline{\underline{-9.82 \text{ ksi (compression)}}}$

Problem 2.41

Given cylindrical steel rod CD $E_s = 29\times10^6$ psi
cylindrical aluminum rod $E_a = 10.4\times10^6$ psi

To Find (a) the Reactions at A and D R_A, R_D.
(b) the deflection of point C δ_C.

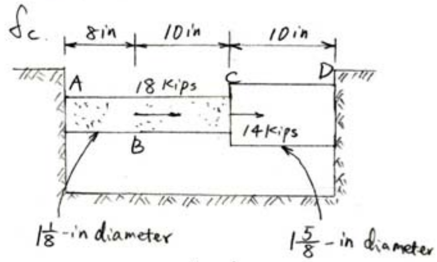


figure (a)

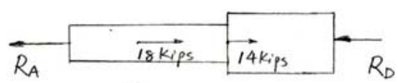


figure (b)

FBD: figure (b)

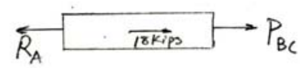
FPU: $\delta = \dfrac{PL}{EA}$, compatibility

Solution:

<u>AB:</u> $P = R_A$ $L = 8$ in

$$A = A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(1.125\,\text{in})^2 = 0.99402\ \text{in}^2$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A\,(8\,\text{in})}{(10.4\times10^6\,\text{psi})(0.99402\,\text{in}^2)} = 0.77386\times10^{-6}R_A$$



<u>BC:</u> $P = P_{BC} = R_A - 18\times10^3$ ksi, $L = 10$ in

$A = 0.99402$ in^2

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18\times10^3\,\text{lb})\,(10\,\text{in})}{(10.4\times10^6\,\text{psi})(0.99402\,\text{in}^2)} = (0.96732\times10^{-6}\,\text{in/lb})R_A - (17.412\times10^{-3}\ \text{in})$$

<u>CD:</u> $P = -R_D$ $L = 10$ in $A = \frac{\pi}{4}d_{CD}^2 = \frac{\pi}{4}(1.625\,\text{in})^2 = 2.0739\ \text{in}^2$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(-R_D)\,(10\,\text{in})}{(29\times10^6\,\text{psi})(2.0739\,\text{in}^2)} = -(0.16627\times10^{-6}\,\text{in/lb})\,R_D$$

<u>Compatibility</u> $\delta_{AD} = 0$

$$\therefore\ \delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = (1.74118\times10^{-6}\,\text{in/lb})R_A - (0.16627\times10^{-6}\,\text{in/lb})R_D - (17.412\ \text{in}) = 0 \quad (1)$$

<u>Equilibrium</u> $R_A - 18\text{kip} - 14\text{kip} + R_D = 0$ (2)

(a) From (1) (2), We have $R_A = \underline{\underline{11.92\times10^3\,\text{lb}}}$ $R_D = \underline{\underline{20.08\times10^3\,\text{lb}}}$

(b) $\delta_C = \delta_{AB} + \delta_{CD} = (1.7412\times10^{-6}\,\text{in/lb})R_A - (17.412\times10^{-3}\,\text{in})$

$$= (1.7412\times10^{-6}\,\text{in/lb})(11.92\times10^{-3}\,\text{lb}) - (17.412\times10^{-3}\,\text{in}) = \underline{\underline{3.34\times10^{-3}\ \text{in}}}$$

Problem 2.48

Given the rigid bar ABCD shown in figure (a)

To Find: the tension in each wire

FBD: figure (b)

FBU: $\delta = \frac{PL}{EA}$

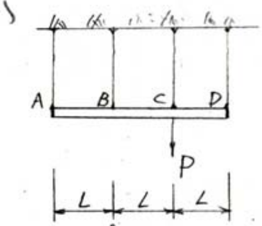


figure (a)

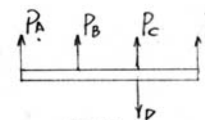


figure (b)

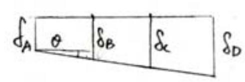


figure (c)

Solution:

Let θ be the slope of bar ABCD after deformation

$$\delta_B = \delta_A + L\theta$$
$$\delta_C = \delta_A + 2L\theta$$
$$\delta_D = \delta_A + 3L\theta$$
$$P_A = \frac{EA}{\ell}\delta_A$$
$$P_B = \frac{EA}{\ell}\delta_B = \frac{EA}{\ell}\delta_A + \frac{EAL}{\ell}\theta$$
$$P_C = \frac{EA}{\ell}\delta_C = \frac{EA}{\ell}\delta_A + \frac{2EAL}{\ell}\theta$$
$$P_D = \frac{EA}{\ell}\delta_D = \frac{EA}{\ell}\delta_A + \frac{3EAL}{\ell}\theta$$

Equilibrium

$+\uparrow \Sigma F_y = 0 \quad P_A + P_B + P_C + P_D - P = 0$

$$\frac{4EA}{\ell}\delta_A + \frac{6EA}{\ell}L\theta = P \qquad (1)$$

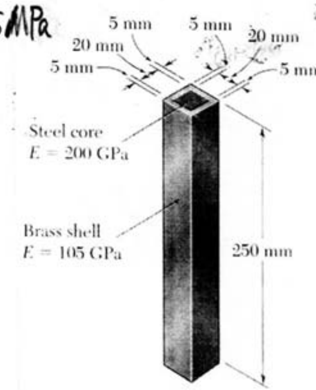
$+\circlearrowleft \Sigma M_A = 0 \quad LP_B + 2LP_C + 3LP_D - 2LP = 0$

$$\frac{6EA}{\ell}\delta_A + \frac{14EA}{\ell}L\theta = 2LP \qquad (2)$$

Solving (1) and (2) simultaneously $\delta_A = \frac{1}{10}\frac{P\ell}{EA}$ $\quad L\theta = \frac{1}{10}\frac{P\ell}{EA}$

$$P_A = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \underline{\underline{\frac{1}{10}P}}$$
$$P_B = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \underline{\underline{\frac{1}{5}P}}$$
$$P_C = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 2\frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \underline{\underline{\frac{3}{10}P}}$$
$$P_D = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 3\frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \underline{\underline{\frac{2}{5}P}}$$

Problem 2.51



Given: $\alpha_b = 20.9\times10^{-6}/°C$, $\alpha_s = 11.7\times10^{-6}/°C$, $\sigma_s = 55\,MPa$
$E_b = 105\ GPa$, $E_s = 200\,GPa$.

To Find: $\Delta T = ?$

FBD: Not required

FPU: $\varepsilon = \frac{\sigma}{E} + \alpha\cdot\Delta T$, $P = \sigma A$

Solution:

Let P_s = tensile force developed in the steel. For equilibrium with zero total force, the compressive force in the brass $P_b = -P_s$

Area: $A_s = (20mm)(20mm) = 400\,mm^2$
$A_b = (30mm)(30mm) - (20mm)(20mm) = 500\,m^2$

Force: $\left.\begin{array}{l} P_s = \sigma_s A_s \\ P_b = \sigma_b A_b \end{array}\right\} \Rightarrow \sigma_b = -\frac{A_s}{A_b}\sigma_s = -\frac{400\,mm^2}{500\,mm^2}\cdot 55\,MPa = -44\,MPa$

Strain: $\varepsilon_s = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T)$
$\varepsilon_b = \frac{\sigma_b}{E_b} + \alpha_b(\Delta T)$

Matching: $\varepsilon_s = \varepsilon_b$

$$\frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_b}{E_b} + \alpha_b(\Delta T)$$

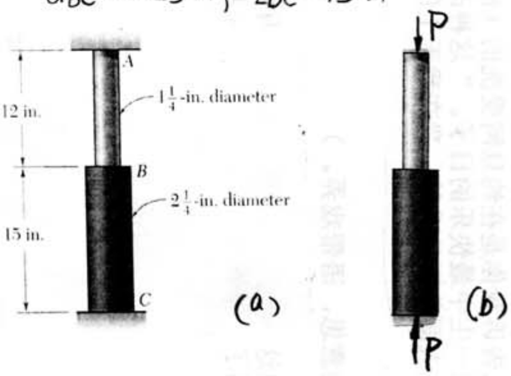
$$\therefore \Delta T = \frac{\frac{\sigma_s}{E_s} - \frac{\sigma_b}{E_b}}{\alpha_b - \alpha_s} = \frac{\frac{55\times10^6\,Pa}{200\times10^9\,Pa} - \frac{-44\times10^6\,Pa}{105\times10^9\,Pa}}{20.9\times10^{-6}/°C - 11.7\times10^{-6}/°C} = \boxed{75.44\,°C}$$

Problem 2.53

Given: $E_s = 29 \times 10^6\ psi$, $\alpha_s = 6.5 \times 10^{-6}/°F$, $d_{AB} = 1.25\ in$, $L_{AB} = 12\ in$

$E_b = 15 \times 10^6\ psi$, $\alpha_b = 10.4 \times 10^{-6}/°F$, $d_{BC} = 2.25\ in$, $L_{BC} = 15\ in$

$\Delta T = 65°F$



To Find: (a) σ_{AB} and σ_{BC} (b) δ_B

FBD: Figure (b)

FPU: $\delta_T = \alpha L \Delta T$, $\delta_P = \frac{PL}{EA}$

Solution:

Area: $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25\ in)^2 = 1.227185\ in^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25\ in)^2 = 3.976078\ in^2$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (12 in)(6.5 \times 10^{-6}/°F)(65°F) + (15\ in)(10.4 \times 10^{-6}/°F)(65°F)$$
$$= 1.521 \times 10^{-2}\ in$$

shortening due to induced compressive force

$$\delta_P = \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}} = \frac{(P)(12 in)}{(29 \times 10^6 psi)(1.227185\ in^2)} + \frac{(P)(15\ in)}{(15 \times 10^6 psi)(3.976078\ in^2)}$$
$$= 5.88693 \times 10^{-7} P \quad (in)$$

For zero net deflection $\delta_T = \delta_P$

$$\therefore 5.88693 \times 10^{-7} P = 1.521 \times 10^{-2}$$
$$\Rightarrow P = 2.58369 \times 10^4\ lb$$

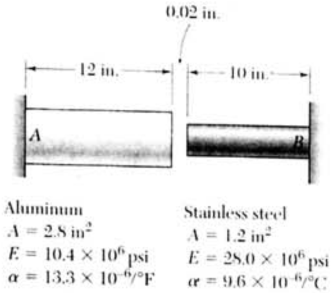
(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{2.58369 \times 10^4\ lb}{1.227185\ in^2} = -2.105 \times 10^4\ psi = \boxed{-21.05\ ksi}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{2.58369 \times 10^4 lb}{3.976078\ in^2} = -6.50 \times 10^3\ psi = \boxed{-6.50\ ksi}$$

(b) $$\delta_B = \frac{PL_{AB}}{E_s A_{AB}} - L_{AB}\alpha_s(\Delta T) = \frac{(2.58369 \times 10^4\ lb)(12\ in)}{(29 \times 10^6\ psi)(1.227185\ in^2)} - (12 in)(6.5 \times 10^{-6}/°F)(65°F)$$
$$= \boxed{3.642 \times 10^{-3}\ in\ (\uparrow)}$$

Problem 2.59

Given:



$T_r = 70°F$, $T = 320°F$

To Find: (a) σ_a (b) δ_a

FBD:

FPU: $\delta = L\alpha(\Delta T)$, $\delta = \dfrac{PL}{EA}$, $\sigma = \dfrac{P}{A}$

Solution: $\Delta T = T - T_r = 320°F - 70°F = 250°F$

Free thermal expansion:

$$\delta_T = L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T)$$
$$= (12\text{in})(13.3\times10^{-6}/°F)(250°F) + (10\text{in})(9.6\times10^{-6}/°F)(250°F)$$
$$= 63.9\times10^{-3}\text{ in} = 0.0639\text{ in}$$

shortening due to P to meet constraint

$$\delta_P = 0.0639\text{ in} - 0.02\text{ in} = 0.0439\text{ in}$$

$$\delta_P = \frac{PL_a}{E_a A_a} + \frac{PL_s}{E_s A_s} = \frac{(12\text{ in})(P)}{(10.4\times10^6\text{psi})(2.8\text{in}^2)} + \frac{(10\text{in})(P)}{(28.0\times10^6\text{psi})(1.2\text{in}^2)}$$
$$= 709.71\times10^{-9} P$$

$$\therefore\ 709.71\times10^{-9} P = 0.0439 \Rightarrow P = 61.857\times10^3\text{ lb}$$

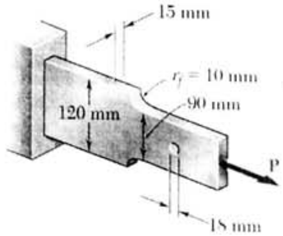
(a) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{61.857\times10^3\text{ lb}}{2.8\text{ in}^2} = -22.09\times10^3\text{psi} = \boxed{-22.09\text{ ksi}}$

(b) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (12\text{in})(13.3\times10^{-6}/°F)(250°F) - \frac{(61.857\times10^3\text{lb})(12\text{ in})}{(10.4\times10^6\text{ psi})(2.8\text{in}^2)} = \boxed{0.0144\text{ in}}$$

Problem 2.100

Given:



$\sigma_{all} = 135$ MPa

To Find: The maximum allowable load P

FBD: Not required

FPU: $P = \frac{\sigma_{max} A}{K}$

Solution:

(1) At the hole: $r = 9$ mm, $d = 90\text{mm} - 18\text{mm} = 72$ mm

$$\frac{r}{d} = \frac{9\text{mm}}{72\text{mm}} = 0.125$$

From Fig 2.64a, $K = 2.65$

$$A_{net} = td = (15\text{ mm})(72\text{ mm}) = 1.08 \times 10^3 \text{ mm}^2 = 1.08 \times 10^{-3} \text{ m}^2$$

$$\therefore P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.08 \times 10^{-3}\text{ m}^2)(135 \times 10^6 \text{ Pa})}{2.65} = 55 \times 10^3 N = \underline{55 \text{ kN}}$$

(2) At the fillet: $D = 120$ mm, $d = 90$ mm

$$\frac{D}{d} = \frac{120\text{mm}}{90\text{mm}} = 1.333$$

$r = 10$ mm, $\frac{r}{d} = \frac{10\text{mm}}{90\text{mm}} = 0.1111.$

From Fig 2.64b. $K = 2.02$

$$A_{min} = td = (15\text{mm})(90\text{mm}) = 1.35 \times 10^3 \text{ mm}^2 = 1.35 \times 10^{-3} \text{ m}^2$$

$$\therefore P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.35 \times 10^{-3}\text{m}^2)(135 \times 10^6 \text{ Pa})}{2.02} = 90 \times 10^3 N = \underline{90 \text{ kN}}$$

From (1) and (2), the maximum allowable load $\boxed{P = 55 \text{ kN}}$