ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #2

Problem 2.1

Given: a steel rod $L = 2.2\,m$; $E = 200\,GPa$

$\delta \le 1.2\,mm$ when $P = 8.5\,KN$

To Find: (a) the smallest diameter d

(b) the corresponding σ

FBD : not required

FPU : $\delta = \frac{PL}{AE}$, $\sigma = \frac{P}{A}$

Solution :

(a) $$\delta = \frac{PL}{AE} \quad \therefore A = \frac{PL}{E\delta} = \frac{(8.5\times10^3)(2.2)}{(200\times10^9)(1.2\times10^{-3})} = 77.92\times10^{-6}\,m^2$$

$$A = \frac{\pi}{4}d^2 \quad \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\,m = \underline{\underline{9.96\,mm}}$$

(b) $$\sigma = \frac{P}{A} = \frac{8.5\times10^3}{77.92\times10^{-6}} = 109.1\times10^6\,Pa = \underline{\underline{109.1\,MPa}}$$

Problem 2.63

Given: a rod with $d = \frac{7}{8}$ in. is subject to a tension force $P = 17$ kips

$\nu = 0.3$, $E = 29 \times 10^6$ psi, $L = 8$ in.

To Find: (a) the elongation of the rod δ_x

(b) the change in diameter δ_y

FBD: $P \longleftarrow\!\!=\!=\!=\!\!\longrightarrow 17$ kips

FPU: $\sigma = \dfrac{P}{A}$, $\varepsilon_x = \dfrac{\sigma}{E}$, $\varepsilon_y = -\nu\varepsilon_x$

Solution:

$$P = 17 \text{ Kips} = 17 \times 10^3 \text{ lb}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{0.60132} = 28.27 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{28.27 \times 10^3}{29 \times 10^6} = 974.9 \times 10^{-6}$$

(a) $\delta_x = L\varepsilon_x = (8.0)(974.9 \times 10^{-6}) = 7.80 \times 10^{-3} \text{ in} = \underline{\underline{0.00780 \text{ in.}}}$

(b) $\varepsilon_y = -\nu\varepsilon_x = -(0.3)(974.9 \times 10^{-6}) = -292.5 \times 10^{-6}$

$\delta_y = d\,\varepsilon_y = \left(\frac{7}{8}\right)(-292.5 \times 10^{-6}) = -256 \times 10^{-6} \text{ in} = \underline{\underline{-0.000256 \text{ in.}}}$

Problem 2.65

Given: an aluminum pipe, length $L = 2m$, outer diameter $d_o = 240mm$

wall thickness $t = 10mm$ $P = -640kN$

$E = 73\,GPa$, $\nu = 0.33$

To Find: (a) the change in length δ.

(b) the change in its outer diameter Δd_o

(c) the change in its wall thickness

FBD: not required

FPU: $\delta = \frac{PL}{AE}$, $\varepsilon_x = \frac{\delta}{L}$, $\varepsilon_y = -\nu\varepsilon_x$

Solution: $d_o = 240mm$ $t = 10mm$ $d_i = d_o - 2t = 220mm$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3\,mm^2 = 7.2257 \times 10^{-3}\,m^2$$

$$P = -640 \times 10^3 N$$

(a) $$\delta = \frac{PL}{AE} = \frac{(-640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3}\,m = \underline{\underline{-2.43\,mm}}$$

$$\varepsilon_x = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

(b) $$\Delta d_o = d_o\varepsilon_y = (240)(400.4 \times 10^{-6}) = \underline{\underline{0.0961\,mm}}$$

(c) $$\Delta t = t\varepsilon_y = (10)(400.4 \times 10^{-6}) = \underline{\underline{0.0400\,mm}}$$

Problem 2.68

Given: $P = 600\,lb$ $t_0 = \frac{1}{16}$ in. $w_0 = \frac{1}{2}$ in.

$E = 29 \times 10^6$ psi, $\nu = 0.30$ $L_0 = 2.00$

To Find: (a) change in length δ_x

(b) change in with δ_y

(c) change in thickness δ_z

(d) change in cross-sectional area of portion AB

FBD: not required

FPU: $\sigma = \frac{P}{A}$, $\varepsilon_x = \frac{\sigma}{E}$, $\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$

Solution: $A_0 = (\frac{1}{2})(\frac{1}{16}) = 0.03125$ in.^{2}

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

(a) $\delta_x = L_0 \varepsilon_x = (2.0)(662.07 \times 10^{-6}) = \underline{\underline{1.324 \times 10^{-8} \text{ in.}}}$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

(b) $\delta_y = w_0 \varepsilon_y = (\frac{1}{2})(-198.62 \times 10^{-6}) = \underline{\underline{-99.3 \times 10^{-6} \text{ in.}}}$

(c) $\delta_z = t_0 \varepsilon_z = (\frac{1}{16})(-198.62 \times 10^{-6}) = -12.41 \times 10^{-6}$ in.

(d) $A = wt = w_0(1+\varepsilon_y)\, t_0(1+\varepsilon_z)$

$$= w_0 t_0 (1 + \varepsilon_y + \varepsilon_z + \varepsilon_y \varepsilon_z)$$

$$\Delta A = A - A_0 = w_0 t_0 (\varepsilon_y + \varepsilon_z + \varepsilon_y \varepsilon_z)$$

$$= (\tfrac{1}{2})(\tfrac{1}{16})(-198.62 \times 10^{-6} - 198.62 \times 10^{-6} + (-198.62 \times 10^{-6})^2)$$

$$= -12.41 \times 10^{-6} \text{ in}^2$$

Problem 2.15

Given: $l_{AB} = 2\text{ in}$, $d_{AB} = 1.5\text{ in}$, $l_{BC} = 3\text{ in}$, $d_{BC} = 1\text{ in}$, $l_{CD} = 2\text{ in}$, $d_{CD} = 1.5\text{ in}$

$E = 29 \times 10^6\text{ psi}$

To Find: (a) $\delta_{AD} = 0.002\text{ in}$, $P = ?$

(b) $\delta_{BC} = ?$

FBD: Not required

FPU: $\delta = \frac{PL}{AE}$

Solution:

(a) $$\delta_{AB} = \frac{P_{AB} l_{AB}}{A_{AB} E} = \frac{(P)(2\text{ in})}{\frac{\pi}{4}(1.5\text{ in})^2 (29 \times 10^6\text{ psi})} = 3.90265 \times 10^{-8} P \quad (\text{in})$$

$$\delta_{BC} = \frac{P_{BC} l_{BC}}{A_{BC} E} = \frac{(P)(3\text{ in})}{\frac{\pi}{4}(1\text{ in})^2 (29 \times 10^6\text{ psi})} = 1.317144 \times 10^{-7} P \quad (\text{in})$$

$$\delta_{CD} = \frac{P_{CD} l_{CD}}{A_{CD} E} = \frac{(P)(2\text{ in})}{\frac{\pi}{4}(1.5\text{ in})^2 (29 \times 10^6\text{ psi})} = 3.90265 \times 10^{-8} P \quad (\text{in})$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$0.002(\text{in}) = 3.90265 \times 10^{-8} P + 1.317144 \times 10^{-7} P + 3.90265 \times 10^{-8} P \quad (\text{in})$$

$$\therefore \; P = 9.534 \times 10^3\text{ lb} = \boxed{9.534\text{ kips}}$$

(b) $$\delta_{BC} = 1.317144 \times 10^{-7} \times 9.534 \times 10^3\text{ in} = \boxed{1.256 \times 10^{-3}\text{ in}}$$

Problem 2.18

Given: steel rod ABC : $E = 200\,GPa$, $d = 36\,mm$, $L_{AB} = 2m$, $L_{BC} = 3m$, $F_B = 50\,kN$

brass rod CD : $E = 105\,GPa$, $d = 36\,mm$, $L_{CD} = 2.5m$, $F_D = 100\,kN$

To Find: (a) δ_C ; (b) δ_D

FBD: N/A

FPU: $\delta = \frac{PL}{AE}$

Solution:

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.036m)^2 = 1.01787 \times 10^{-3} m^2$$

Portion	P_i	L_i	E_i	$\delta_i = P_i L_i / AE_i$
AB	150 KN	2m	200GPa	$1.474 \times 10^{-3} m$
BC	100KN	3m	200GPa	$1.474 \times 10^{-3} m$
CD	100KN	2.5m	105 GPa	$2.339 \times 10^{-3} m$

(a) $\delta_C = \delta_{AB} + \delta_{BC} = 1.474 \times 10^{-3} m + 1.474 \times 10^{-3} m = 2.948 \times 10^{-3} m$

$= \boxed{2.95\ mm}$

(b) $\delta_D = \delta_C + \delta_{CD} = 2.948 \times 10^{-3} m + 2.339 \times 10^{-3} m = 5.287 \times 10^{-3} m$

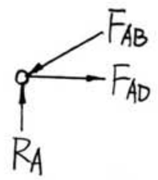
$= \boxed{5.29\ mm}$

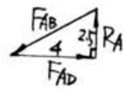
Problem 2.22

Given: steel truss, $E = 200\text{ GPa}$, $A_{AB} = 2400\text{ mm}^2$, $A_{AD} = 1800\text{ mm}^2$, $F_B = 228\text{ kN}$

To Find: δ_{AB} and δ_{AD}

FBD:





(FBD of joint A)

FPU: $\delta = \dfrac{PL}{AE}$

Solution:

Statics: Reactions are 114 kN upward at A and C.

Member BD is a zero force member.

Use joint A as a free body, draw the force triangle and solve for F_{AB} and F_{AD} by proportions:

$$l_{AB} = \sqrt{(4.0\text{m})^2 + (2.5\text{m})^2} = 4.717\text{ m}$$

$$F_{AB} = \frac{4.717}{2.5} R_A = \frac{4.717}{2.5} \times 114\text{ kN} = 215.10\text{ kN}$$

$$F_{AD} = \frac{4}{2.5} \times R_A = \frac{4}{2.5} \times 114\text{ kN} = 182.4\text{ kN}$$

(a) $$\delta_{AB} = \frac{F_{AB}\, l_{AB}}{A_{AB} E} = \frac{(215.10\text{kN})(4.717\text{m})}{(2400\text{ mm}^2)(200\text{GPa})} = \frac{(215.10 \times 10^3\text{N})(4.717\text{m})}{(2400 \times 10^{-6}\text{m}^2)(200 \times 10^9\text{Pa})}$$

$$= 2.11 \times 10^{-3}\text{ m} = \boxed{2.11\text{ mm}}$$

(b) $$\delta_{AD} = \frac{F_{AD}\, l_{AD}}{A_{AD} E} = \frac{(182.4\text{kN})(4.0\text{m})}{(1800\text{ mm}^2)(200\text{GPa})} = \frac{(182.4 \times 10^3\text{N})(4.0\text{ m})}{(1800 \times 10^{-6}\text{m}^2)(200 \times 10^9\text{Pa})}$$

$$= 2.03 \times 10^{-3}\text{m} = \boxed{2.03\text{ mm}}$$

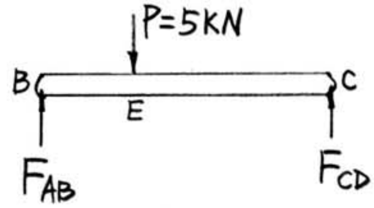
Problem 2.27

Given: $l_{AB} = 0.36\,m$, $l_{CD} = 0.36\,m$, $l_{BE} = 0.20\,m$, $l_{EC} = 0.44\,m$

$P = 5\,KN$, $E = 75\,GPa$, $A_{AB} = A_{CD} = 125\,mm^2$

To Find: δ_E

FBD:



(free body: BEC)

FPU: $\delta = \frac{PL}{AE}$

Solution:

Use member BEC as a free body

$+\circlearrowleft \Sigma M_C = 0 \qquad (0.44\,m)(P) - (0.64\,m)(F_{AB}) = 0$

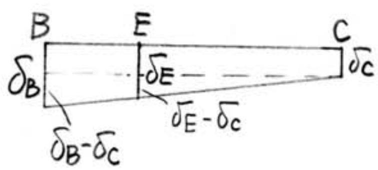
$$\therefore F_{AB} = \frac{0.44}{0.64} P = \frac{0.44}{0.64} \times 5\,KN = 3.4375\,KN$$

$+\circlearrowleft \Sigma M_B = 0 \qquad (0.64\,m)(F_{CD}) - (0.20\,m)(P) = 0$

$$\therefore F_{CD} = \frac{0.20}{0.64} P = \frac{0.20}{0.64} \times 5\,KN = 1.5625\,KN$$

$$\delta_B = \delta_{AB} = \frac{F_{AB} l_{AB}}{A_{AB} E} = \frac{(3.4375 \times 10^3 N)(0.36\,m)}{(125 \times 10^{-6} m^2)(75 \times 10^9 Pa)} = 1.32 \times 10^{-4}\,m$$

$$\delta_C = \delta_{CD} = \frac{F_{CD} l_{CD}}{A_{CD} E} = \frac{(1.5625 \times 10^3 N)(0.36\,m)}{(125 \times 10^{-6} m^2)(75 \times 10^9 Pa)} = 6 \times 10^{-5}\,m$$



Deformation Diagram

$$\frac{\delta_E - \delta_C}{\delta_B - \delta_C} = \frac{l_{EC}}{l_{BC}} = \frac{0.44\,m}{0.64\,m}$$

$$\Rightarrow \delta_E = \frac{0.44}{0.64}\delta_B + \frac{0.20}{0.64}\delta_C$$

$$= \frac{0.44}{0.64} \times 1.32 \times 10^{-4}\,m + \frac{0.20}{0.64} \times 6 \times 10^{-5}\,m$$

$$= 1.095 \times 10^{-4}\,m = \boxed{0.1095\,mm}$$