

ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #2

Problem 2.1

Given: a steel rod $L = 2.2 \text{ m}$; $E = 200 \text{ GPa}$
 $\delta \leq 1.2 \text{ mm}$ when $P = 8.5 \text{ kN}$

To Find: (a) the smallest diameter d
(b) the corresponding σ

FBD: not required

$$\text{FPU: } \delta = \frac{PL}{AE}, \quad \sigma = \frac{P}{A}$$

Solution:

$$(a) \quad \delta = \frac{PL}{AE} \quad \therefore A = \frac{PL}{E\delta} = \frac{(8.5 \times 10^3)(2.2)}{(200 \times 10^9)(1.2 \times 10^{-3})} = 77.92 \times 10^{-6} \text{ m}^2$$

$$A = \frac{\pi d^2}{4} \quad \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(77.92 \times 10^{-6})}{\pi}} = 9.96 \times 10^{-3} \text{ m} \\ = \underline{\underline{9.96 \text{ mm}}}$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{8.5 \times 10^3}{77.92 \times 10^{-6}} = 109.1 \times 10^6 \text{ Pa} = \underline{\underline{109.1 \text{ MPa}}}$$

Problem 2.63

Given: a rod with $d = \frac{7}{8}$ in. is subject to a tension force $P = 17$ kips

$$\nu = 0.3, \quad E = 29 \times 10^6 \text{ psi}, \quad L = 8 \text{ in.}$$

To Find: (a) the elongation of the rod δ_x

(b) the change in diameter δ_y

FBD: 

$$\text{FPU: } \sigma = \frac{P}{A}, \quad \epsilon_x = \frac{\sigma}{E}, \quad \epsilon_y = -\nu \epsilon_x$$

Solution:

$$P = 17 \text{ kips} = 17 \times 10^3 \text{ lb}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{0.60132} = 28.27 \times 10^3 \text{ psi}$$

$$\epsilon_x = \frac{\sigma}{E} = \frac{28.27 \times 10^3}{29 \times 10^6} = 974.9 \times 10^{-6}$$

$$\text{(a) } \delta_x = L \epsilon_x = (8.0)(974.9 \times 10^{-6}) = 7.80 \times 10^{-3} \text{ in} = \underline{\underline{0.00780 \text{ in.}}}$$

$$\text{(b) } \epsilon_y = -\nu \epsilon_x = -(0.3)(974.9 \times 10^{-6}) = -292.5 \times 10^{-6}$$

$$\delta_y = d \epsilon_y = \left(\frac{7}{8}\right)(-292.5 \times 10^{-6}) = -256 \times 10^{-6} \text{ in} = \underline{\underline{-0.000256 \text{ in.}}}$$

Problem 2.65

Given: an aluminum pipe, length $L = 2\text{ m}$, outer diameter $d_o = 240\text{ mm}$

Wall thickness $t = 10\text{ mm}$ $P = -640\text{ kN}$

$E = 73\text{ GPa}$, $\nu = 0.33$.

To Find: (a) the change in length δ .

(b) the change in its outer diameter Δd_o

(c) the change in its wall thickness.

FBD: not required

FPU: $\delta = \frac{PL}{AE}$, $\epsilon_x = \frac{\delta}{L}$, $\epsilon_y = -\nu \epsilon_x$

Solution: $d_o = 240\text{ mm}$ $t = 10\text{ mm}$ $d_i = d_o - 2t = 220\text{ mm}$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2 \\ = 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = -640 \times 10^3 \text{ N}$$

$$(a) \delta = \frac{PL}{AE} = \frac{(-640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m} \\ = \underline{\underline{-2.43 \text{ mm}}}$$

$$\epsilon_x = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

$$\epsilon_y = -\nu \epsilon_x = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$(b) \Delta d_o = d_o \epsilon_y = (240)(400.4 \times 10^{-6}) = \underline{\underline{0.0961 \text{ mm}}}$$

$$(c) \Delta t = t \epsilon_y = (10)(400.4 \times 10^{-6}) = \underline{\underline{0.00400 \text{ mm}}}$$

Problem 2.68

Given: $P = 600 \text{ lb}$ $t_0 = \frac{1}{16} \text{ in.}$ $w_0 = \frac{1}{2} \text{ in.}$

$E = 29 \times 10^6 \text{ psi}$, $\nu = 0.30$ $L_0 = 2.00$

- To Find: (a) change in length δ_x
 (b) change in width δ_y
 (c) change in thickness δ_z
 (d) change in cross-sectional area of portion AB

FBD: not required

FPU: $\sigma = \frac{P}{A}$, $\epsilon_x = \frac{\sigma}{E}$, $\epsilon_y = \epsilon_z = -\nu \epsilon_x$

Solution: $A_0 = (\frac{1}{2})(\frac{1}{16}) = 0.03125 \text{ in.}^2$

$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi}$

$\epsilon_x = \frac{\sigma}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$

(a) $\delta_x = L_0 \epsilon_x = (2.0)(662.07 \times 10^{-6}) = \underline{\underline{1.324 \times 10^{-3} \text{ in.}}}$

$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$

(b) $\delta_y = w_0 \epsilon_y = (\frac{1}{2})(-198.62 \times 10^{-6}) = \underline{\underline{-99.3 \times 10^{-6} \text{ in.}}}$

(c) $\delta_z = t_0 \epsilon_z = (\frac{1}{16})(-198.62 \times 10^{-6}) = -12.41 \times 10^{-6} \text{ in.}$

(d) $A = wt = w_0(1 + \epsilon_y) t_0(1 + \epsilon_z)$
 $= w_0 t_0 (1 + \epsilon_y + \epsilon_z + \epsilon_y \epsilon_z)$

$\Delta A = A - A_0 = w_0 t_0 (\epsilon_y + \epsilon_z + \epsilon_y \epsilon_z)$

$= (\frac{1}{2})(\frac{1}{16})(-198.62 \times 10^{-6} - 198.62 \times 10^{-6} + (-198.62 \times 10^{-6})^2)$

$= -12.41 \times 10^{-6} \text{ in.}^2$

Problem 2.15

Given: $l_{AB} = 2 \text{ in}$, $d_{AB} = 1.5 \text{ in}$, $l_{BC} = 3 \text{ in}$, $d_{BC} = 1 \text{ in}$, $l_{CD} = 2 \text{ in}$, $d_{CD} = 1.5 \text{ in}$

$$E = 29 \times 10^6 \text{ psi}$$

To Find: (a) $\delta_{AD} = 0.002 \text{ in}$, $P = ?$

(b) $\delta_{BC} = ?$

FBD: Not required

$$\text{FPU: } \delta = \frac{PL}{AE}$$

Solution:

$$(a) \delta_{AB} = \frac{P_{AB} l_{AB}}{A_{AB} E} = \frac{(P)(2 \text{ in})}{\frac{\pi}{4} (1.5 \text{ in})^2 (29 \times 10^6 \text{ psi})} = 3.90265 \times 10^{-8} P \text{ (in)}$$

$$\delta_{BC} = \frac{P_{BC} l_{BC}}{A_{BC} E} = \frac{(P)(3 \text{ in})}{\frac{\pi}{4} (1 \text{ in})^2 (29 \times 10^6 \text{ psi})} = 1.317144 \times 10^{-7} P \text{ (in)}$$

$$\delta_{CD} = \frac{P_{CD} l_{CD}}{A_{CD} E} = \frac{(P)(2 \text{ in})}{\frac{\pi}{4} (1.5 \text{ in})^2 (29 \times 10^6 \text{ psi})} = 3.90265 \times 10^{-8} P \text{ (in)}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$0.002 \text{ (in)} = 3.90265 \times 10^{-8} P + 1.317144 \times 10^{-7} P + 3.90265 \times 10^{-8} P \text{ (in)}$$

$$\therefore P = 9.534 \times 10^3 \text{ lb} = \boxed{9.534 \text{ kips}}$$

$$(b) \delta_{BC} = 1.317144 \times 10^{-7} \times 9.534 \times 10^3 \text{ in} = \boxed{1.256 \times 10^{-3} \text{ in}}$$

Problem 2.18

Given: steel rod ABC : $E = 200 \text{ GPa}$, $d = 36 \text{ mm}$, $L_{AB} = 2 \text{ m}$, $L_{BC} = 3 \text{ m}$, $F_B = 50 \text{ kN}$
brass rod CD : $E = 105 \text{ GPa}$, $d = 36 \text{ mm}$, $L_{CD} = 2.5 \text{ m}$, $F_D = 100 \text{ kN}$

To Find: (a) δ_C ; (b) δ_D

FBD : N/A

$$\text{FPU : } \delta = \frac{PL}{AE}$$

Solution :

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.036 \text{ m})^2 = 1.01787 \times 10^{-3} \text{ m}^2$$

Portion	P_i	L_i	E_i	$\delta_i = P_i L_i / AE_i$
AB	150 kN	2 m	200 GPa	$1.474 \times 10^{-3} \text{ m}$
BC	100 kN	3 m	200 GPa	$1.474 \times 10^{-3} \text{ m}$
CD	100 kN	2.5 m	105 GPa	$2.339 \times 10^{-3} \text{ m}$

$$(a) \delta_C = \delta_{AB} + \delta_{BC} = 1.474 \times 10^{-3} \text{ m} + 1.474 \times 10^{-3} \text{ m} = 2.948 \times 10^{-3} \text{ m} \\ = \boxed{2.95 \text{ mm}}$$

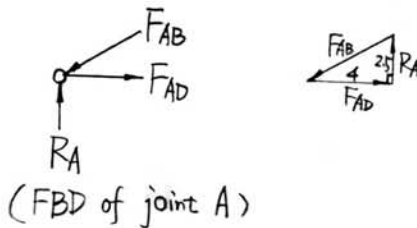
$$(b) \delta_D = \delta_C + \delta_{CD} = 2.948 \times 10^{-3} \text{ m} + 2.339 \times 10^{-3} \text{ m} = 5.287 \times 10^{-3} \text{ m} \\ = \boxed{5.29 \text{ mm}}$$

Problem 2.22

Given: steel truss, $E = 200 \text{ GPa}$, $A_{AB} = 2400 \text{ mm}^2$, $A_{AD} = 1800 \text{ mm}^2$, $F_B = 228 \text{ kN}$

To Find: δ_{AB} and δ_{AD}

FBD:



FPU: $\delta = \frac{PL}{AE}$

Solution:

Statics: Reactions are 114 kN upward at A and C.

Member BD is a zero force member.

Use joint A as a free body, draw the force triangle and solve for F_{AB} and F_{AD} by proportions:

$$L_{AB} = \sqrt{(4.0\text{m})^2 + (2.5\text{m})^2} = 4.717 \text{ m}$$

$$F_{AB} = \frac{4.717}{2.5} R_A = \frac{4.717}{2.5} \times 114 \text{ kN} = 215.10 \text{ kN}$$

$$F_{AD} = \frac{4}{2.5} \times R_A = \frac{4}{2.5} \times 114 \text{ kN} = 182.4 \text{ kN}$$

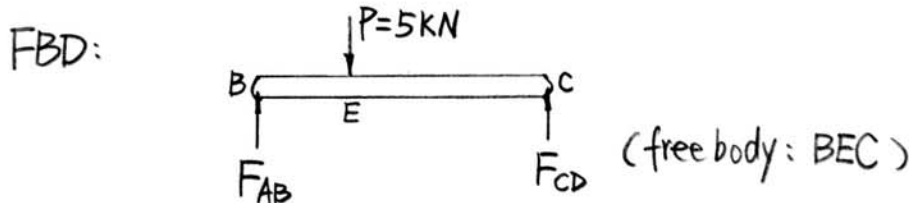
$$\begin{aligned} \text{(a)} \quad \delta_{AB} &= \frac{F_{AB} L_{AB}}{A_{AB} E} = \frac{(215.10 \text{ kN})(4.717 \text{ m})}{(2400 \text{ mm}^2)(200 \text{ GPa})} = \frac{(215.10 \times 10^3 \text{ N})(4.717 \text{ m})}{(2400 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})} \\ &= 2.11 \times 10^{-3} \text{ m} = \boxed{2.11 \text{ mm}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \delta_{AD} &= \frac{F_{AD} L_{AD}}{A_{AD} E} = \frac{(182.4 \text{ kN})(4.0 \text{ m})}{(1800 \text{ mm}^2)(200 \text{ GPa})} = \frac{(182.4 \times 10^3 \text{ N})(4.0 \text{ m})}{(1800 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})} \\ &= 2.03 \times 10^{-3} \text{ m} = \boxed{2.03 \text{ mm}} \end{aligned}$$

Problem 2.27

Given: $l_{AB} = 0.36\text{ m}$, $l_{CD} = 0.36\text{ m}$, $l_{BE} = 0.20\text{ m}$, $l_{EC} = 0.44\text{ m}$
 $P = 5\text{ kN}$, $E = 75\text{ GPa}$, $A_{AB} = A_{CD} = 125\text{ mm}^2$

To Find: δ_E



FPU: $\delta = \frac{PL}{AE}$

Solution:

Use member BEC as a free body

$$+\uparrow \sum M_C = 0 \quad (0.44\text{ m})(P) - (0.64\text{ m})(F_{AB}) = 0$$

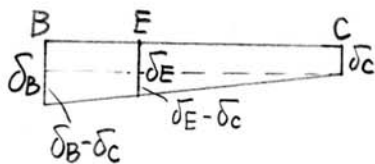
$$\therefore F_{AB} = \frac{0.44}{0.64} P = \frac{0.44}{0.64} \times 5\text{ kN} = 3.4375\text{ kN}$$

$$+\uparrow \sum M_B = 0 \quad (0.64\text{ m})(F_{CD}) - (0.20\text{ m})(P) = 0$$

$$\therefore F_{CD} = \frac{0.20}{0.64} P = \frac{0.20}{0.64} \times 5\text{ kN} = 1.5625\text{ kN}$$

$$\delta_B = \delta_{AB} = \frac{F_{AB} l_{AB}}{A_{AB} E} = \frac{(3.4375 \times 10^3\text{ N})(0.36\text{ m})}{(125 \times 10^{-6}\text{ m}^2)(75 \times 10^9\text{ Pa})} = 1.32 \times 10^{-4}\text{ m}$$

$$\delta_C = \delta_{CD} = \frac{F_{CD} l_{CD}}{A_{CD} E} = \frac{(1.5625 \times 10^3\text{ N})(0.36\text{ m})}{(125 \times 10^{-6}\text{ m}^2)(75 \times 10^9\text{ Pa})} = 6 \times 10^{-5}\text{ m}$$



Deformation Diagram

$$\frac{\delta_E - \delta_C}{\delta_B - \delta_C} = \frac{l_{EC}}{l_{BC}} = \frac{0.44\text{ m}}{0.64\text{ m}}$$

$$\Rightarrow \delta_E = \frac{0.44}{0.64} \delta_B + \frac{0.20}{0.64} \delta_C$$

$$= \frac{0.44}{0.64} \times 1.32 \times 10^{-4}\text{ m} + \frac{0.20}{0.64} \times 6 \times 10^{-5}\text{ m}$$

$$= 1.095 \times 10^{-4}\text{ m} = \boxed{0.1095\text{ mm}}$$