ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #14

Problem 10.10

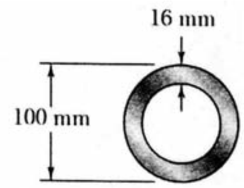


Fig. P10.10

[Given] $L = 5.0\,m$, $d_o = 100\,mm$, $t = 16\,mm$, $E = 200\,GPa$

[To Find] P_{cr}

[FBD] N/A

[FPU] $P_{cr} = \frac{\pi^2 EI}{L^2}$

[Solution]

$$C_o = \frac{1}{2} d_o = 50\,mm = 0.05\,m$$

$$C_i = C_o - t = 50\,mm - 16\,mm = 34\,mm = 0.034\,m$$

$$I = \frac{\pi}{4}\left(C_o^4 - C_i^4\right) = \frac{\pi}{4}\left[(0.05\,m)^4 - (0.034\,m)^4\right] = 3.8592 \times 10^{-6}\,m^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \cdot (200 \times 10^9\,Pa)(3.8592 \times 10^{-6}\,m^4)}{(5.0\,m)^2} = 304708\,N = \boxed{304.7\,kN}$$

Problem 10.11

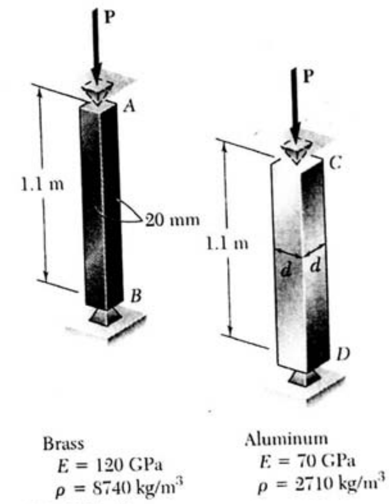


Fig. P10.11

[Given] →

[To Find] (a) P_{cr} for the brass ; (b) d for aluminum with same P_{cr} ; (c) $\frac{m_a}{m_b}$

[FBD] N/A

[FPU] $P_{cr} = \frac{\pi^2 E I}{L^2}$

[Solution]

(a) Brass strut :

$$I_b = \frac{1}{12} b_b h_b^3 = \frac{1}{12} d_b^4 = \frac{1}{12}(0.02\,m)^4 = 1.33333 \times 10^{-8}\ m^4$$

$$P_{cr} = \frac{\pi^2 E_b I_b}{L_b^2} = \frac{\pi^2 (120 \times 10^9\,Pa)(1.33333 \times 10^{-8}\,m^4)}{(1.1\,m)^2} = 13050.7\,N = \boxed{13.05\ kN}$$

(b) Aluminum strut

$$P_{cr} = \frac{\pi^2 E_a I_a}{L_a^2} = \frac{\pi^2 E_a \left(\frac{1}{12}\right)(d_a^4)}{L_a^2}$$

$$\therefore d_a = \sqrt[4]{\frac{12 P_{cr} L_a^2}{\pi^2 E_a}} = \sqrt[4]{\frac{(12)(13050.7N)(1.1m)^2}{\pi^2 (70 \times 10^9\,Pa)}} = 0.022885\ m = \boxed{22.9\ mm}$$

(c) $m_a = \rho_a V_a = \rho_a L_a d_a^2$

$m_b = \rho_b V_b = \rho_b L_b d_b^2$

$$\therefore \frac{m_a}{m_b} = \frac{\rho_a L_a d_a^2}{\rho_b L_b d_b^2} = \left(\frac{\rho_a}{\rho_b}\right)\left(\frac{d_a}{d_b}\right)^2 = \left(\frac{2710\ kg/m^3}{8740\ kg/m^3}\right)\left(\frac{22.9\,mm}{20\,mm}\right)^2 = 0.406 = \boxed{40.6\%}$$

Problem 10.14

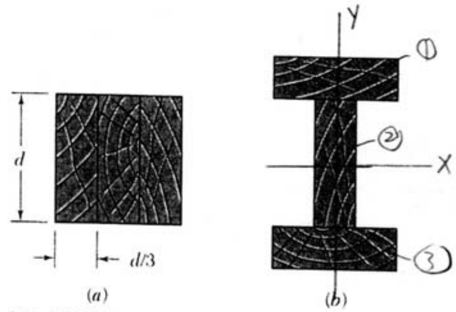


Fig. P10.14

[Given] $\longrightarrow$

[To Find] $\frac{P_{cr}^{(a)}}{P_{cr}^{(b)}}$

[FBD] N/A

[FPU] $P_{cr} = \frac{\pi^2 E I}{L^2}$

[Solution]

(a) $I_a = \frac{1}{12} b_a h_a^3 = \frac{1}{12} d^4$

$$P_{cr}^{(a)} = \frac{\pi^2 E_a I_a}{L_a^2} = \frac{\pi^2 E\left(\frac{1}{12} d^4\right)}{L^2} = \frac{\pi^2 E d^4}{12 L^2}$$

(b) $I_b = I_{min} = I_y = I_1 + I_2 + I_3 = \frac{1}{12}\left(\frac{d}{3}\right)d^3 + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)d^3 = \frac{19}{324} d^4$

$$P_{cr}^{(b)} = \frac{\pi^2 E_b I_b}{L_b^2} = \frac{\pi^2 E\left(\frac{19}{324} d^4\right)}{L^2} = \frac{19\pi^2 E d^4}{324 L^2}$$

$$\frac{P_{cr}^{(a)}}{P_{cr}^{(b)}} = \frac{\frac{\pi^2 E d^4}{12 L^2}}{\frac{19\pi^2 E d^4}{324 L^2}} = \boxed{1.421}$$

Problem 10.17

[Given] $L = 22 \text{ ft}$, $F.S. = 2.3$, $E = 29 \times 10^6 \text{ Psi}$

[To Find] P_{all}

[FBD] N/A

[FPU] $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

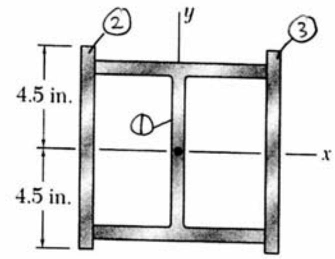


Fig. P10.17

[Solution]

① W8X35: $I_x^{①} = 127 \text{ in}^4$, $I_y^{①} = 42.6 \text{ in}^4$, $b_f = 8.02 \text{ in}$

② and ③ For each plate $A = (0.5 \text{ in})(9.0 \text{ in}) = 4.5 \text{ in}^2$

$$I_x^{②} = \frac{1}{12}(0.5 \text{ in})(9.0 \text{ in})^3 = 30.375 \text{ in}^4$$

$$I_y^{②} = \frac{1}{12}(9.0 \text{ in})(0.5 \text{ in})^3 + A\left(\frac{b_f}{2} + \frac{0.5 \text{ in}}{2}\right)^2$$

$$= \frac{1}{12}(9.0 \text{ in})(0.5 \text{ in})^3 + (4.5 \text{ in}^2)\left(\frac{8.02 \text{ in}}{2} + \frac{0.5 \text{ in}}{2}\right)^2$$

$$= 81.75795 \text{ in}^4$$

Total:

$$I_x = I_x^{①} + I_x^{②} + I_x^{③} = 127 \text{ in}^4 + 30.375 \text{ in}^4 + 30.375 \text{ in}^4 = 187.75 \text{ in}^4$$

$$I_y = I_y^{①} + I_y^{②} + I_y^{③} = 42.6 \text{ in}^4 + 81.75795 \text{ in}^4 + 81.75795 \text{ in}^4 = 206.1159 \text{ in}^4$$

$$I_{min} = I_x = 187.75 \text{ in}^4$$

$$L = 22 \text{ ft} = 264 \text{ in}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29 \times 10^6 \text{ psi})(187.75 \text{ in}^4)}{(264 \text{ in})^2} = 771027.44 \text{ lb}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{771027.44 \text{ lb}}{2.3} = 335229.3 \text{ lb} = \boxed{335 \text{ kips}}$$

Problem 10.19

Given $P = 5.2\,kN$ $E = 200\,GPa$

To determine the factor of safety

FBD as shown

FPU stability of structure. $P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$

Solution:

<u>Equilibrium</u>

Using joint B as a free body, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad P\cos 70^\circ - F_{BD}\sin 45^\circ = 0$

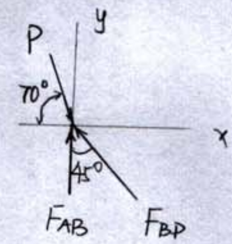
$+\uparrow \Sigma F_y = 0 \quad F_{AB} + F_{BD}\cos 45^\circ - P\sin 70^\circ = 0$

$\therefore F_{AB} = 3.1079\,kN$ (Compression)

$F_{BC} = 2.5152\,kN$ (Compression)

<u>Buckling in member AB</u>

$I_{AB} = \frac{\pi}{4}(\frac{d}{2})^4 = \frac{\pi}{4}(\frac{18\,mm}{2})^4 = 5.153 \times 10^3\,mm^4 = 5.153 \times 10^{-9}\,m^4$



$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9 Pa)(5.153 \times 10^{-9} m^4)}{(1.2m)^2}$$

$$= 7.0636 \times 10^3 N = 7.0636\,kN$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636\,kN}{3.1079\,kN} = 2.27$$

<u>Buckling in member BC</u>

$I_{BC} = \frac{\pi}{4}(\frac{d}{2})^4 = \frac{\pi}{4}(\frac{22\,mm}{2})^4 = 11.499 \times 10^3\,mm^4 = 11.499 \times 10^{-9}\,m^4$

$L_{BC}^2 = (1.2m)^2 + (1.2m)^2 = 2.88\,m^2$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9 Pa)(11.499 \times 10^{-9} m^4)}{2.88\,m^2} = 7.8813 \times 10^3 N$$

$$= 7.8813\,kN$$

$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813\,kN}{2.5152\,kN} = 3.13$$

Since member AC is in tension, min(2.27, 3.13) governs.

$\therefore F.S. = 2.27$

Problem 10.21

Given $d_o = 32mm$, $t = 4mm$, $E = 70GPa$, $F.S. = 2.3$

To determine the allowable load P_o

FBD not required

FPU stability of structure, $P_{cr} = \frac{\pi^2 EI}{L_e^2}$

Solution:

$$C_o = \frac{1}{2}d = \frac{1}{2}(32mm) = 16mm$$

$$C_i = C_o - t = (16mm) - (4mm) = 12mm$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}((16mm)^4 - (12mm)^4) = 35.1858 \times 10^3 mm^4$$

$$= 35.1858 \times 10^{-9} m^4$$

$$\pi^2 EI = \pi^2 (70 \times 10^9 Pa)(35.1858 \times 10^{-9} m^4) = 24309 \ N \cdot m^2$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309 \ N \cdot m^2}{L_e^2}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569 \ N \cdot m^2}{L_e^2}$$

(1) Both ends pinned $L_e = (1)(2.0m) = 2.0m$

$$P_{all} = \frac{10569 N \cdot m^2}{(2.0m)^2} = 2642 N = 2.642 kN$$

(2) One fixed end, one free end $L_e = (2)(2.0m) = 4.0m$

$$P_{all} = \frac{10569 N \cdot m^2}{(4.0m)^2} = 661 N = 0.661 kN$$

(3) Both ends fixed $L_e = (0.5)(2.0m) = 1.0m$

$$P_{all} = \frac{10569 N \cdot m^2}{(1.0m)^2} = 10569 N = 10.569 kN$$

(4) One fixed end, one pinned end $L_e = (0.7)(2.0m) = 1.4m$

$$P_{all} = \frac{10569 N \cdot m^2}{(1.4m)^2} = 5392 N = 5.392 kN$$

(5) Both ends pinned $L_e = (1.0)(2.0m) = 2.0m$

$$P_{all} = \frac{10569 N \cdot m^2}{(2m)^2} = 2642 N = 2.642 kN$$

Problem 10.25

Given Column ABC

To determine (a) the ratio b/d when $F.S._{xz} = FS_{yz}$

(b) design the cross section, when $F.S. = 2.7$, $P = 1.2\,\text{kips}$

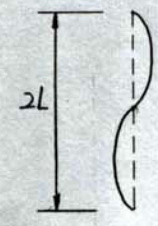
$L = 24\,\text{in}$ and $E = 10.6 \times 10^6\,\text{psi}$

FBD not required

FPU stability of structure, $P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$

Solution:

Buckling in xz-plane: $L_e = L = 24\,\text{in}$

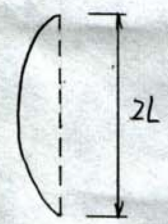


$$I = \frac{1}{12} d b^3$$

$$P = \frac{P_{cr}}{F.S.} = \frac{\pi^2 E I}{(F.S.) L_e^2} = \frac{\pi^2 E d b^3}{12 (F.S.) L_e^2}$$

$$(F.S.) = \frac{\pi^2 E d b^3}{12 P L^2}$$

Buckling in yz-plane $L_e = 2L = 48\,\text{in}$



$$I = \frac{1}{12} b d^3$$

$$P = \frac{P_{cr}}{F.S.} = \frac{\pi^2 E I}{(F.S.) L_e^2} = \frac{\pi^2 E b d^3}{12 (F.S.) L_e^2}$$

$$(F.S.) = \frac{\pi^2 E b d^3}{48 P L^2}$$

(a) $$\frac{\pi^2 E d b^3}{12 P L^2} = \frac{\pi^2 E b d^3}{48 P L^2}$$

$$\therefore \frac{b^2}{d^2} = \frac{1}{4} \qquad \therefore \frac{b}{d} = \frac{1}{2}$$

(b) $$d b^3 = \frac{12 P L^2 (F.S.)}{\pi^2 E} = \frac{(12)(1.2 \times 10^3\,\text{lb})(24\,\text{in})^2}{\pi^2 (10.6 \times 10^6\,\text{psi})}$$

$$= 0.21406$$

$$\therefore d = 1.144\,\text{in}$$

$$b = \frac{1}{2} d = \frac{1}{2}(1.144\,\text{in}) = 0.572\,\text{in}$$

Problem 10.27

Given the uniform brass bar AB.

To determine (a) the ratio b/d for which $(F.S.)_h = (F.S.)_v$
(b) F. S. when $P = 1.8 \times 10^3$ lb. $L = 84$ in, $d = 1.5$ in, $E = 1.5 \times 10^6$ psi

FBD not required

FPU stability of structure $P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$

Solution:

<u>Buckling in horizontal plane</u>

$$L_e = \frac{1}{2}L, \quad I = \frac{1}{12} d b^3$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{4\pi^2 E d b^3}{12 L^2}$$

$$(F.S.)_h = \frac{P_{cr}}{P} = \frac{4\pi^2 E d b^3}{12 P L^2}$$

<u>Buckling in vertical plane</u>

$$L_e = L, \quad I = \frac{1}{12} b d^3$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E b d^3}{12 L^2}$$

$$(F.S.)_v = \frac{P_{cr}}{P} = \frac{\pi^2 E b d^3}{12 P L^2}$$

$$\therefore \frac{4\pi^2 E d b^3}{12 P L^2} = \frac{\pi^2 E b d^3}{12 P L^2}$$

$\therefore$ (a) $b = \frac{1}{2} d$

(b) $$F.S. = \frac{\pi^2 E b d^3}{12 P L^2} = \frac{\pi^2 (15 \times 10^6 \text{psi})(\frac{1}{2} \cdot 1.5 \text{in})(1.5 \text{in})^3}{12 \cdot (1.8 \times 10^3 \text{lb})(84 \text{in})^2}$$

$$= 2.46$$