ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #11

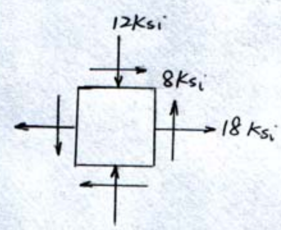
Problem 7.5/7.9

Given: the state of stress shown

To: determine 7.5 (a) the principal plans (b) the principal stresses.

7.9 (a) the orientation of the planes of maximum in-plane shearing stress. (b) the maximum in-plane shearing stress (c). the corresponding normal stress.

FBD: as shown



FPU: stress transformation

Solution:

stress σ_x, σ_y, τ_{xy}

$\sigma_x = 18Ksi \quad \sigma_y = -12Ksi \quad \tau_{xy} = 8Ksi$

7.5(a) principal plan

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(8Ksi)}{(18Ksi)-(-12Ksi)} = 0.5333$$

$\therefore 2\theta_p = 28.07^\circ$ and $28.07^\circ + 180 = 208.07^\circ$

$\therefore \theta_p = 14.04^\circ$ and 104.04°

7.5(b) principal stresses

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{(18Ksi)+(-12Ksi)}{2} \pm \sqrt{\left(\frac{(18Ksi)-(-12Ksi)}{2}\right)^2 + (8Ksi)^2}$$

$$= 3 \pm 17 Ksi$$

$\therefore \sigma_{max} = 20Ksi \quad \sigma_{min} = -14Ksi$

7.9(a) Orientation of the planes of maximum in-plane shearing stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{(18Ksi)-(-12Ksi)}{2(8Ksi)} = -1.875$$

$2\theta_s = -61.93^\circ$ and $-61.93^\circ + 180^\circ = 118.07^\circ$

$\theta_s = -30.96^\circ$ and 59.04°

7.9(b) Maximum in-plane shearing stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{(18Ksi)-(-12Ksi)}{2}\right)^2 + (8Ksi)^2} = 17Ksi$$

7.9(c) Corresponding normal stress

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(18Ksi)+(-12Ksi)}{2} = 3Ksi$$

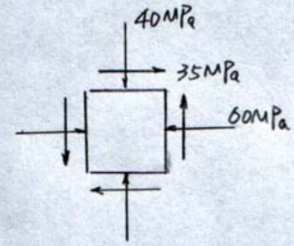
Problem 7.7/7.11

Given the state of stress shown

To determine 7.7 (a) the principal planes (b) the principal stresses
7.9 (a) the orientation of the planes of maximum in-plane shearing stress. (b) the maximum in-plane shearing stress (c) the corresponding normal stress

FBD as shown

FPU stress transformation



Solution

stress σ_x σ_y τ_{xy}

$\sigma_x = -60MPa \quad \sigma_y = -40MPa \quad \tau_{xy} = 35MPa$

7.7 (a) Principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35MPa)}{(-60MPa)-(-40MPa)} = -3.50$$

$$2\theta_p = -74.05^\circ \quad \text{and} \quad -74.05^\circ + 180^\circ = 105.95^\circ$$

$$\therefore \theta_p = -37.02^\circ \quad \text{and} \quad 52.98^\circ$$

7.7 (b) principal stresses

$$\sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{(-60MPa)+(-40MPa)}{2} \pm \sqrt{\left(\frac{(-60MPa)-(-40MPa)}{2}\right)^2+(35MPa)^2}$$

$$= -50 \pm 36.4MPa$$

$$\sigma_{max} = -13.6MPa \quad \sigma_{min} = -86.4MPa$$

7.11 (a) the orientation of the planes of maximum in-plane shearing stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{(-60MPa)-(-40MPa)}{2\cdot(35MPa)} = 0.2857$$

$$2\theta_s = 15.95^\circ \quad \text{and} \quad 15.95^\circ + 180^\circ = 195.95^\circ$$

$$\theta_s = 7.98^\circ \quad \text{and} \quad 97.98^\circ$$

7.11 (b) the maximum in-plane shearing stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \sqrt{\left(\frac{(-60MPa)-(-40MPa)}{2}\right)^2+(35MPa)^2} = 36.4MPa$$

7.11 (c) corresponding normal stress

$$\sigma' = \sigma_{ave} = \frac{\sigma_x+\sigma_y}{2} = \frac{(-60MPa)+(-40MPa)}{2} = -50MPa$$

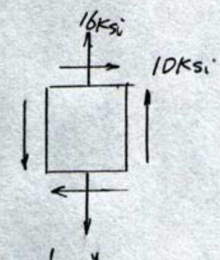
Problem 7.16

Given: The state of stress shown

To: determine the normal and shearing stress after the element has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

FBD: as shown

FPU: stress transformation

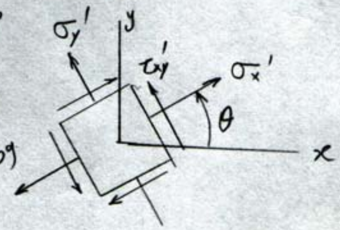


Solution:

Stress σ_x σ_y τ_{xy}

$\sigma_x = 0$, $\sigma_y = 16$ ksi $\tau_{xy} = 10$ ksi

(a) stress $\sigma_{x'}$ $\sigma_{y'}$ $\tau_{x'y'}$ when $\theta = -25°$ $2\theta = -50°$



$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + 16\text{ksi}}{2} + \frac{0 - 16\text{ksi}}{2}\cos(-50°) + (10\text{ksi})\sin(-50°)$$

$$= -4.80\text{ ksi}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{0 - (16\text{ksi})}{2}\sin(-50°) + (10\text{ksi})\cos(-50°)$$

$$= 0.30\text{ ksi}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0 + (16\text{ksi})}{2} - \frac{0 - (16\text{ksi})}{2}\cos(-50°) - (10\text{ksi})\sin(-50°)$$

$$= 20.80\text{ ksi}$$

(b) stress $\sigma_{x'}$ $\sigma_{y'}$ $\tau_{x'y'}$, when $\theta = 10°$ $2\theta = 20°$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + (16\text{ ksi})}{2} + \frac{0 - (16\text{ksi})}{2}\cos(20°) + (10\text{ksi})\sin(20°)$$

$$= 3.90\text{ ksi}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{0 - (16\text{ksi})}{2}\sin(20°) + (10\text{ksi})\cos(20°) = 12.13\text{ ksi}$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0 + (16\text{ksi})}{2} - \frac{0 - (16\text{ksi})}{2}\cos(20°) - (10\text{ksi})\sin(20°)$$

$$= 12.10\text{ ksi}$$

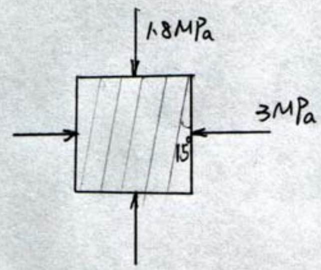
Problem 7.17

Given the wooden member with 15° with the vertical of the grain.

To determine (a) the in-plane shearing stress parallel to the grain (b) the normal stress perpendicular to the grain.

FBD as shown

FPU stress transformation



Solution:

stress σ_x σ_y τ_{xy}

$\sigma_x = -3MPa$ $\sigma_y = -1.8MPa$ $\tau_{xy} = 0$

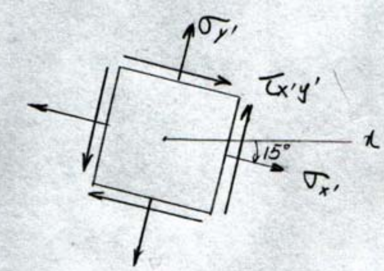
$\theta = -15°$ $2\theta = -30°$

(a) In-plane shearing stress

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\sin 2\theta$$

$$= -\frac{(-3MPa)-(-1.8MPa)}{2}\sin(-30°) + 0$$

$$= -0.300\ MPa$$



(b) The normal stress

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{(-3MPa)+(-1.8MPa)}{2} + \frac{(-3MPa)-(-1.8MPa)}{2}\cos(-30°) + 0$$

$$= -2.92\ MPa$$