

ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #11

Problem 7.5/7.9

Given the state of stress shown

To determine (a) the principal planes (b) the principal stresses.

7.9 (a) the orientation of the planes of maximum in-plane shearing stress. (b) the maximum in-plane shearing stress (c) the corresponding normal stress.

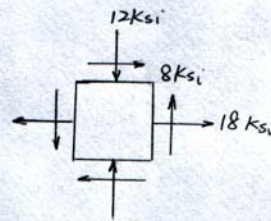
FBD: as shown

FPU: stress transformation

Solution:

stress σ_x , σ_y , τ_{xy}

$$\sigma_x = 18 \text{ Ksi} \quad \sigma_y = -12 \text{ Ksi} \quad \tau_{xy} = 8 \text{ Ksi}$$



7.5(a) principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(8 \text{ Ksi})}{(18 \text{ Ksi}) - (-12 \text{ Ksi})} = 0.5333$$

$$\therefore 2\theta_p = 28.07^\circ \quad \text{and} \quad 28.07^\circ + 180^\circ = 208.07^\circ$$

$$\therefore \theta_p = 14.04^\circ \quad \text{and} \quad 104.04^\circ$$

7.5(b) principal stresses

$$\begin{aligned} \sigma_{\max, \min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{(18 \text{ Ksi}) + (-12 \text{ Ksi})}{2} \pm \sqrt{\left(\frac{(18 \text{ Ksi}) - (-12 \text{ Ksi})}{2}\right)^2 + (8 \text{ Ksi})^2} \\ &= 3 \pm 17 \text{ Ksi} \end{aligned}$$

$$\therefore \sigma_{\max} = 20 \text{ Ksi} \quad \sigma_{\min} = -14 \text{ Ksi}$$

7.9(a) Orientation of the planes of maximum in-plane shearing stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{(18 \text{ Ksi}) - (-12 \text{ Ksi})}{2(8 \text{ Ksi})} = -1.875$$

$$2\theta_s = -61.93^\circ \quad \text{and} \quad -61.93^\circ + 180^\circ = 118.07^\circ$$

$$\theta_s = -30.96^\circ \quad \text{and} \quad 59.04^\circ$$

7.9(b) Maximum in-plane shearing stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{(18 \text{ Ksi}) - (-12 \text{ Ksi})}{2}\right)^2 + (8 \text{ Ksi})^2} = 17 \text{ Ksi}$$

7.9(c) Corresponding normal stress

$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{(18 \text{ Ksi}) + (-12 \text{ Ksi})}{2} = 3 \text{ Ksi}$$

Problem 7.7/7.11

Given the state of stress shown

To determine 7.7 (a) the principal planes (b) the principal stresses
7.9 (a) the orientation of the planes of maximum in-plane shearing stress (b) the maximum in-plane shearing stress (c) the corresponding normal stress

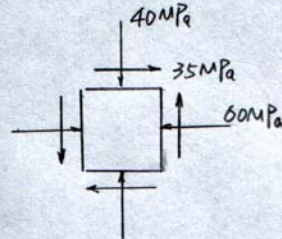
FBD as shown

FPU stress transformation

Solution

stress σ_x σ_y τ_{xy}

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$



7.7 (a) Principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35 \text{ MPa})}{(-60 \text{ MPa}) - (-40 \text{ MPa})} = -3.50$$

$$2\theta_p = -74.05^\circ \quad \text{and} \quad -74.05^\circ + 180^\circ = 105.95^\circ$$

$$\therefore \theta_p = -37.02^\circ \quad \text{and} \quad 52.98^\circ$$

7.7 (b) principal stresses

$$\begin{aligned} \sigma_{\max, \min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{(-60 \text{ MPa}) + (-40 \text{ MPa})}{2} \pm \sqrt{\left(\frac{(-60 \text{ MPa}) - (-40 \text{ MPa})}{2}\right)^2 + (35 \text{ MPa})^2} \\ &= -50 \pm 36.4 \text{ MPa} \end{aligned}$$

$$\sigma_{\max} = -13.6 \text{ MPa} \quad \sigma_{\min} = -86.4 \text{ MPa}$$

7.11 (a) the orientation of the planes of maximum in-plane shearing stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{(-60 \text{ MPa}) - (-40 \text{ MPa})}{2(35 \text{ MPa})} = 0.2857$$

$$2\theta_s = 15.95^\circ \quad \text{and} \quad 15.95^\circ + 180^\circ = 195.95^\circ$$

$$\theta_s = 7.98^\circ \quad \text{and} \quad 97.98^\circ$$

7.11 (b) the maximum in-plane shearing stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{(-60 \text{ MPa}) - (-40 \text{ MPa})}{2}\right)^2 + (35 \text{ MPa})^2} = 36.4 \text{ MPa}$$

7.11 (c) corresponding normal stress

$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{(-60 \text{ MPa}) + (-40 \text{ MPa})}{2} = -50 \text{ MPa}$$

Problem 7.16

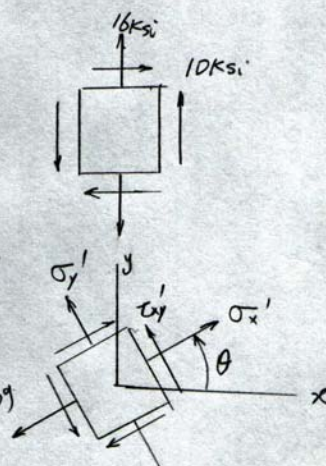
Given the state of stress shown
 To determine the normal and shearing stress after the element has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

FBD as shown
 FPU stress transformation

Solution:

Stress σ_x σ_y τ_{xy}

$$\sigma_x = 0, \sigma_y = 16 \text{ Ksi}, \tau_{xy} = 10 \text{ Ksi}$$



(a) stress $\sigma_{x'}$ $\sigma_{y'}$ $\tau_{x'y'}$ when $\theta = -25^\circ$ $2\theta = -50^\circ$

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{0 + 16 \text{ Ksi}}{2} + \frac{0 - 16 \text{ Ksi}}{2} \cos(-50^\circ) + (10 \text{ Ksi}) \sin(-50^\circ) \\ &= -4.80 \text{ Ksi} \end{aligned}$$

$$\begin{aligned} \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{0 - (16 \text{ Ksi})}{2} \sin(-50^\circ) + (10 \text{ Ksi}) \cos(-50^\circ) \\ &= 0.30 \text{ Ksi} \end{aligned}$$

$$\begin{aligned} \sigma_{y'} &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\ &= \frac{0 + (16 \text{ Ksi})}{2} - \frac{0 - (16 \text{ Ksi})}{2} \cos(-50^\circ) - (10 \text{ Ksi}) \sin(-50^\circ) \\ &= 20.80 \text{ Ksi} \end{aligned}$$

(b) stress $\sigma_{x'}$ $\sigma_{y'}$ $\tau_{x'y'}$ when $\theta = 10^\circ$ $2\theta = 20^\circ$

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{0 + 16 \text{ Ksi}}{2} + \frac{0 - 16 \text{ Ksi}}{2} \cos(20^\circ) + (10 \text{ Ksi}) \sin(20^\circ) \\ &= 3.90 \text{ Ksi} \end{aligned}$$

$$\begin{aligned} \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{0 - (16 \text{ Ksi})}{2} \sin(20^\circ) + (10 \text{ Ksi}) \cos(20^\circ) = 12.13 \text{ Ksi} \end{aligned}$$

$$\begin{aligned} \sigma_{y'} &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\ &= \frac{0 + (16 \text{ Ksi})}{2} - \frac{0 - (16 \text{ Ksi})}{2} \cos(20^\circ) - (10 \text{ Ksi}) \sin(20^\circ) \\ &= 12.10 \text{ Ksi} \end{aligned}$$

Problem 7.17

Given the wooden member with 15° with the vertical of the grain.

To determine (a) the in-plane shearing stress parallel to the grain (b) the normal stress perpendicular to the grain.

FBD as shown

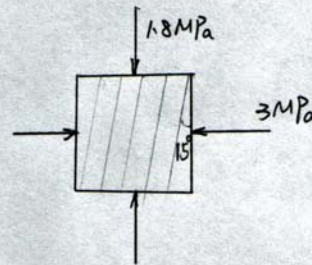
FPU stress transformation

Solution:

stress σ_x σ_y τ_{xy}

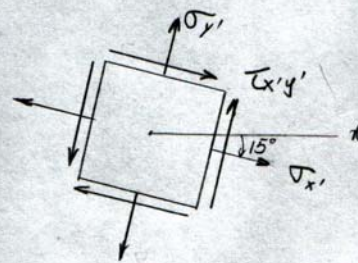
$$\sigma_x = -3 \text{ MPa} \quad \sigma_y = -1.8 \text{ MPa} \quad \tau_{xy} = 0$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$



(a) In-plane shearing stress

$$\begin{aligned} \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{(-3 \text{ MPa}) - (-1.8 \text{ MPa})}{2} \sin(-30^\circ) + 0 \\ &= -0.300 \text{ MPa} \end{aligned}$$



(b) The normal stress

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{(-3 \text{ MPa}) + (-1.8 \text{ MPa})}{2} + \frac{(-3 \text{ MPa}) - (-1.8 \text{ MPa})}{2} \cos(-30^\circ) + 0 \\ &= -2.92 \text{ MPa} \end{aligned}$$