CHAPTER

9b

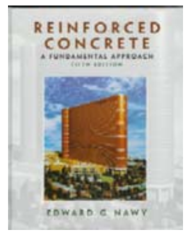
REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

COMBINED COMPRESSION AND BENDING: COLUMNS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

SPRING 2004



By
Dr . Ibrahim. Assakkaf

ENCE 454 – Design of Concrete Structures

Department of Civil and Environmental Engineering
University of Maryland, College Park

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Analysis of Short Columns: Small Eccentricity

- The analysis of short columns carrying axial loads that have small eccentricities involves:
  - Checking the maximum design axial load strength, and
  - Checking the various details of the reinforcing.

Analysis of Short Columns: Small Eccentricity

- Procedure for Analysis of Short Columns with Small Eccentricities

1. Check ρ_g within acceptable limits as stipulated by the ACI Code:

$$0.01 \le \rho_g \le 0.08$$

2. Check the number of bars within acceptable limits for the clear space (see Table 2, Table 9, Handout). The minimum number is four for bars with rectangular or circular ties and six for bars enclosed by spirals.

Analysis of Short Columns: Small Eccentricity

3. Calculate the maximum design axial load strength $\phi P_{n(\max)}$.
4. Check the lateral reinforcing. For ties, check size, spacing, and arrangement. For spirals, check ρ_s, and clear distance.

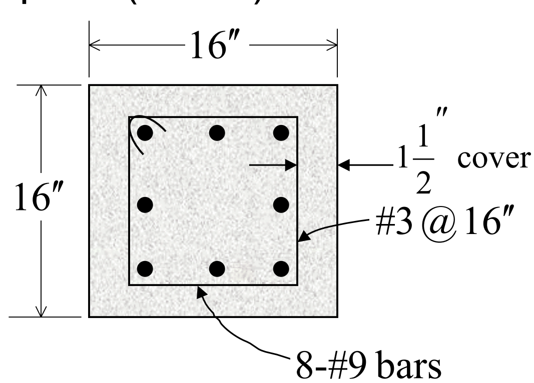
Analysis of Short Columns: Small Eccentricity

- Example 4

  Find the maximum design axial load strength for the tied column of cross section shown in the figure. Check the ties. Assume a short column. The materials strength specified are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi for both longitudinally steel and ties.

Analysis of Short Columns: Small Eccentricity

- Example 4 (cont'd)



Analysis of Short Columns: Small Eccentricity

- Example 4 (cont'd)
  1. Check the steel ratio for longitudinal steel:

$$\rho_g = \frac{A_{st}}{A_g} = \frac{8.00}{(16)^2} = 0.0313$$

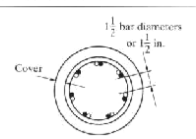
$$0.01 < 0.0313 < 0.08 \qquad \text{OK}$$

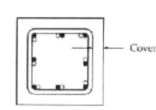
  2. From Table 1 (Table 9, Handout), using a 13-in. core (column size less cover on each side), the maximum number of No. 9 bars is eight. This is OK.

Analysis of Short Columns: Small Eccentricity

Table 9, Handout

Table 2. Preferred Maximum Number of Column Bars in One Row





Recommended spiral or tie bar number	Core size (in.) = column size − 2 × cover	Circular area (in.²)	Bar number (circular) #5	#6	#7	#8	#9	#10	#11	Square area (in.²)	Bar number (square) #5	#6	#7	#8	#9	#10	#11
3[a]	9	63.6	8	7	7	6	—	—	—	81	8	8	8	8	4	4	4
	10	78.5	10	9	8	7	6	—	—	100	12	8	8	8	8	4	4
	11	95.0	11	10	9	8	7	6	—	121	12	12	8	8	8	8	4
	12	113.1	12	11	10	9	8	7	6	144	12	12	12	8	8	8	8
	13	132.7	13	12	11	10	8	7	6	169	16	12	12	12	8	8	8
	14	153.9	14	13	12	11	9	8	7	196	16	16	12	12	12	8	8
	15	176.7	15	14	13	12	10	9	8	225	16	16	16	12	12	12	8
4	16	201.1	16	15	14	12	11	9	8	256	20	16	16	16	12	12	8
	17	227.0	18	16	15	13	12	10	9	289	20	20	16	16	12	12	8
	18	254.5	19	17	15	14	12	11	10	324	20	20	16	16	16	12	12
	19	283.5	20	18	16	15	13	11	10	361	24	20	20	16	16	12	12
	20	314.2	21	19	17	16	14	12	11	400	24	24	20	20	16	12	12
	21	346.4	22	20	18	17	15	13	11	441	28	24	20	20	16	16	12
	22	380.1	23	21	19	18	15	14	12	484	28	24	24	20	20	16	12
5	23	415.5	24	22	21	19	16	14	13	529	28	28	24	24	20	16	16
	24	452.4	25	23	21	20	17	15	13	576	32	28	24	24	20	16	16
	25	490.9	26	24	22	20	18	16	14	625	32	28	28	24	20	20	16
	26	530.9	28	25	23	21	19	16	14	676	32	32	28	24	24	20	16
	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

[a] No. 4 tie for No. 11 or larger longitudinal reinforcement.

Analysis of Short Columns: Small Eccentricity

- Example 4 (cont'd)

3. The maximum design axial load strength may now be computed as follows:

$$\phi P_{n(\max)} = 0.80\phi\left[0.85 f'\left(A_g - A_{st}\right)_c + f_y A_{st}\right]$$
$$= 0.80(0.65)[0.85(4)(256-8)+(60)(8)]$$
$$= 688 \text{ kips}$$

4. Check the ties. The size of No. 3 is OK for longitudinal bar size up to No. 10. The spacing of the ties must not exceed the smaller of

Analysis of Short Columns: Small Eccentricity

- Example 4 (cont'd)

48 tie-bar diameter = 38 (3/8) = 18 in.

16 longitudinal-bar diameter = 16 (1.128) = 18 in.

Least column dimension = 16 in. ← smallest

Therefore, the tie spacing is OK.

Check clear distance:

$$\text{clear distance} = \frac{16-2(1.5)-2(3/8)-3(1.128)}{2}$$
$$= 4.4 \text{ in.} < 6 \text{ in.} \qquad \text{OK}$$

Analysis of Short Columns: Small Eccentricity

- Example 5

A short circular spiral column having a diameter of 18 in. is reinforced with eight No. 9 bars. The cover is 1 ½ in., and the spiral is 3/8 in. in diameter spaced 2 in. o.c. Find the maximum design axial load strength and check the spiral. Use f_c' = 3000 psi and f_y = 40,000 psi

Analysis of Short Columns: Small Eccentricity

- Example 5 (cont'd)

1. Check the steel ratio for longitudinal steel:

$$\rho_g = \frac{A_{st}}{A_g} = \frac{8.00}{\pi(9)^2} = 0.0314$$

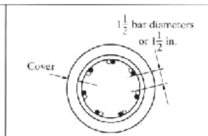
$$0.01 < 0.0314 < 0.08 \qquad \text{OK}$$

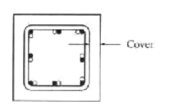
2. From Table 2 (Table 9 Handout), and for circular column, the maximum number of # 9 bars is 10. This is OK

Analysis of Short Columns: Small Eccentricity

Table 9, Handout

Table 2. Preferred Maximum Number of Column Bars in One Row





Recommended spiral or tie bar number	Core size (in.) = column size − 2 × cover	Circular area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11	Square area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11
3[a]	9	63.6	8	7	7	6	—	—	—	81	8	8	8	8	4	4	4
	10	78.5	10	9	8	7	6	—	—	100	12	8	8	8	8	4	4
	11	95.0	11	10	9	8	7	6	—	121	12	12	8	8	8	8	4
	12	113.1	12	11	10	9	8	7	6	144	12	12	12	8	8	8	8
	13	132.7	13	12	11	10	8	7	6	169	16	12	12	12	8	8	8
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	15	176.7	15	14	13	12	10	9	8	225	16	16	16	12	12	12	8
4	16	201.1	16	15	14	12	11	9	8	256	20	16	16	16	12	12	8
	17	227.0	18	16	15	13	12	10	9	289	20	20	16	16	12	12	8
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	26	530.9	28	25	23	21	19	16	14	676	32	32	28	24	24	20	16
	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

[a] No. 4 tie for No. 11 or larger longitudinal reinforcement.

Analysis of Short Columns: Small Eccentricity

- Example 5 (cont'd)

$$A_g = \frac{\pi D^2}{4} = \frac{\pi (18)^2}{4} = 254.5 \text{ in}^2$$

3. The maximum design axial load strength may now be computed as follows:

$$\phi P_{n(\max)} = 0.85\phi\left[0.85 f'\left(A_g - A_{st}\right)_c + f_y A_{st}\right]$$
$$= 0.85(0.70)[0.85(3)(254.5-8)+(40)(8)]$$
$$= 564 \text{ kips}$$

4. Check spirals: 3/8 in. spiral

$$\text{actual } \rho_s = \frac{4A_{sp}}{D_c s} = \frac{4(0.11)}{15(2)} = 0.0147$$

Analysis of Short Columns: Small Eccentricity

- Example 5 (cont'd)

$$\text{actual } \rho_s = \frac{A_{sp}\pi D_s}{\left(\pi D_c^2/4\right)(s)}$$

Approximate:

$$\text{actual } \rho_s = \frac{4A_{sp}}{D_c s}$$

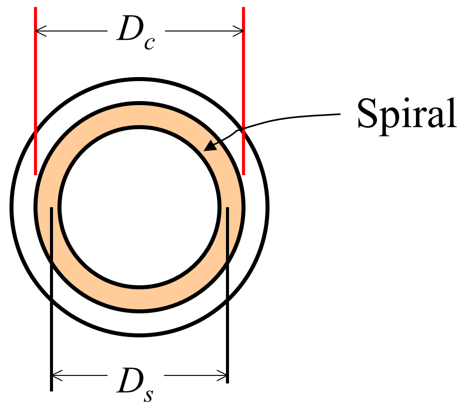


Figure 5. Definition of D_c and D_s

Analysis of Short Columns: Small Eccentricity

- Example 5 (cont'd)

$$\rho_{s(\min)} = 0.45\left(\frac{A_g}{A_c}-1\right)\frac{f_c'}{f_y} = 0.45\left(\frac{254.5}{176.7}-1\right)\frac{3}{40} = 0.0149$$

$$\rho_{s(\min)} = 0.0149 \approx 0.0147 \text{ OK (slightly underreinforced)}$$

Clear distance between spiral loops:

$$\text{clear distance} = 2 - \frac{3}{8} = 1.63 \text{ in}$$

$$1'' < 1.63'' < 3''$$

Therefore, 3/8 in dia. spiral @ 2-in. is OK

Design of Short Columns: Small Eccentricity

- The design of reinforced concrete columns involves the following:
  - Proportioning of the steel and concrete areas.
  - Selection of properly sized and spaced ties or spirals.
- Since the ratio of steel to concrete area must fall within a given range:

$$0.01 \le \rho_g \le 0.08$$

Design of Short Columns: Small Eccentricity

The main strength equation is modified as follows to include this term:

For tied column:

$$\phi P_{n(\max)} = 0.80\phi\left[0.85 f'\left(A_g - A_{st}\right)_c + f_y A_{st}\right] \quad (13)$$

$$\rho_g = \frac{A_{st}}{A_g} \Rightarrow A_{st} = \rho_g A_g \quad (14)$$

Therefore,

$$\begin{aligned}\phi P_{n(\max)} &= 0.80\phi\left[0.85 f'\left(A_g - \rho A_g\right)_c + f_y \rho_g A_g\right] \\ &= 0.80\phi A_g\left[0.85 f'(1-\rho)_c + f_y \rho_g\right]\end{aligned} \quad (15)$$

Design of Short Columns: Small Eccentricity

- Since

$$P_u \leq \phi P_{n(\max)} \qquad (16)$$

an expression can be written for required A_g in terms of the material strength, P_u and ρ_g.

Design of Short Columns: Small Eccentricity

- For Tied Columns:

$$\text{required } A_g = \frac{P_u}{0.80\phi\left[0.85 f_c'\left(1-\rho_g\right)+f_y\rho_g\right]} \qquad (17)$$

- For Spiral Columns:

$$\text{required } A_g = \frac{P_u}{0.85\phi\left[0.85 f_c'\left(1-\rho_g\right)+f_y\rho_g\right]} \qquad (18)$$

Design of Short Columns: Small Eccentricity

- Procedure for Design of Short Columns with Small Eccentricities

1. Establish the material strengths. Establish the desired ρ_g (if any).
2. Establish the factored axial load P_u.
3. Determine the required gross column area A_g.
4. Select the column dimensions. Use full-inch increments.

Design of Short Columns: Small Eccentricity

5. Find the load carried by the concrete and the load required to be carried by the longitudinal steel. Determine the required longitudinal steel area. Select the longitudinal steel.
6. Design the lateral reinforcing (ties or spiral).
7. Sketch the design.

Design of Short Columns: Small Eccentricity

- Example 6

Design a square-tied column to carry axial service loads of 320 kips dead load and 190 kips live load. There is no identified applied moment. Assume that the column is short. Use ρ_g about 0.03, f_c' = 4000 psi, and f_y = 60,000 psi.

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

1. Given values are as follows:

   ρ_g =0.03, f_c' = 4000 psi, and f_y = 60,000 psi

2. The factored axial load is

$$P_u = 1.2(320) + 1.6(190) = 688 \text{ kips}$$

3. The required gross column area is (from Eq. 17)

$$\text{required } A_g = \frac{P_u}{0.80\phi\left[0.85 f_c'\left(1-\rho_g\right) + f_y \rho_g\right]}$$

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

$$\text{required } A_g = \frac{688}{0.80(0.65)[0.85(4)(1-0.03)+(60)(0.03)]} = 260 \text{ in}^2$$

4. The required size of a square column will be

$$\sqrt{260} = 16.12 \text{ in.}$$

Use a 16-in.-square column. This choice will require that the actual ρ_g be slightly in excess of 0.03

$$\text{actual } A_g = (16)^2 = 256 \text{ in}^2$$

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

5. The load on the concrete is

$$\begin{aligned}\text{load on concrete} &= 0.80\phi(0.85 f_c')A_g(1-\rho_g) \\ &= 0.80(0.65)(0.85)(4)(256)(1-0.03) \\ &= 439 \text{ kips}\end{aligned}$$

Therefore, the load to be carried by the steel is

$$\text{load by steel} = 688 - 439 = 249 \text{ kips}$$

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

The required steel area can be calculated from

$$\text{required } A_{st} = \frac{249}{0.80(0.65)(60)} = 7.98 \text{ in}^2$$

Use eight No. 10 bars (A_{st} =10.16 in^2), see Table 3.

Table 2 (Table 9 Handout) indicates a maximum of eight No. 10 bars for a 13-in. core. OK

Design of Short Columns: Small Eccentricity

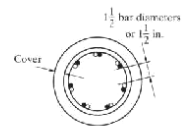
Table 3. Areas of Multiple of Reinforcing Bars (in^2)

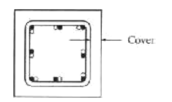
Number of bars	Bar number #3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Design of Short Columns: Small Eccentricity

Table 9, Handout

Table 2. Preferred Maximum Number of Column Bars in One Row





Recommended spiral or tie bar number	Core size (in.) = column size − 2 × cover	Circular area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11	Square area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11
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	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

[a] No. 4 tie for No. 11 or larger longitudinal reinforcement.

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

6. Design the ties. From Table 2 (Table 9, Handout), select a No. 3 tie. The spacing must be greater than the smaller of

$$48 \text{ tie-bar diameter} = 48(3/8) = 18 \text{ in.}$$

$$16 \text{ longitudinal-bar diamater} = 16(1.27) = 20.3 \text{ in.}$$

$$\text{least column dimension} = 16 \text{ in.} \longleftarrow \text{Controls}$$

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

Use No. 3 ties spaced 16 in. o.c. Check the arrangement with reference to the figure. The clear space between adjacent bars in the same face is

2 (1.5 in.) Cover; 2 × dia. of No. 3 tie; dia. of No. 10 bar

$$\frac{16-3-0.75-3(1.27)}{2}=4.22\text{ in.}<6.0\text{ in.}$$

Therefore, no additional ties are required by the ACI Code.

Design of Short Columns: Small Eccentricity

- Example 3 (cont'd)

Table 4. Reinforced Steel Properties

Bar number	3	4	5	6	7	8	9	10	11	14	18
Unit weight per foot (lb)	0.376	0.668	1.043	1.502	2.044	2.670	3.400	4.303	5.313	7.650	13.60
Diameter (in.)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

Design of Short Columns: Small Eccentricity

- Example 6 (cont'd)

  7. The design sketch is as follows: