CHAPTER

8b

REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

SERVICEABILITY OF BEAMS AND ONE-WAY SLABS

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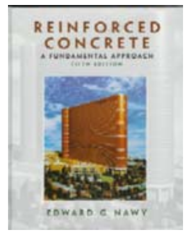

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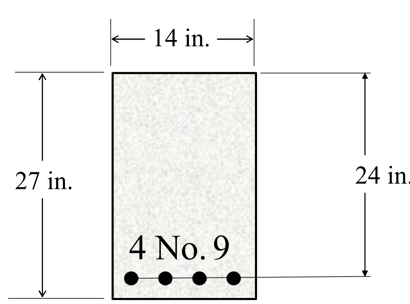
Long-Term Deflection

- Example 2

  Calculate I_g, I_{gt}, M_{cr}, M_{crt}, and I_{cr} for the cross section shown. Assume f_y = 60,000 psi, f_c' =4000 psi, and E_s = 29 × 10[6] psi.



To be discussed in class

Long-Term Deflection

- Example 3

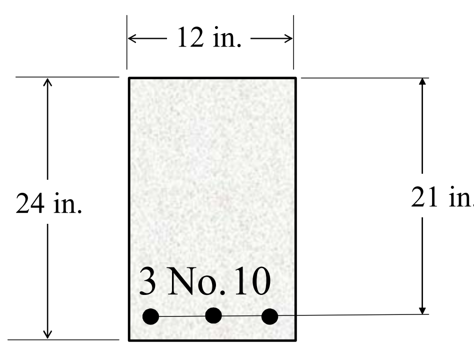
  The beam cross section shown in on simple span of 20 ft and carries service loads of 1.5 kips/ft dead load (includes beam weight) and 1.0 kip/ft live load. Use f_c'=3000 psi and f_y = 60,000 psi, and assume n = 9.

  – (a) compute the immediate deflection due to dead load and live load.

  – Calculate the long-term deflection due to the dead load. Assume the time period for sustained loads to be in excess of 5 years.

Long-Term Deflection

- Example 3 (cont'd)



To be discussed in class

Long-Term Deflection

- The total long-term deflection is given by

$$\Delta_{LT} = \Delta_L + \lambda_\infty \Delta_D + \lambda_i \Delta_{LS} \qquad (12)$$

Δ_{LT} = total long-term deflection
Δ_D = initial dead-load deflection
Δ_{LS} = initial sustained live-load deflection (a percentage of the immediate Δ_L determined by expected duration of sustained load)
λ_∞ = time-dependent multiplier for infinite duration of sustained load
λ_i = time-dependent multiplier for limited load duration

Permissible Deflections in Beams and One-Way Slabs

- Empirical Methods of Minimum Thickness Evaluation for Deflection Control
  - The ACI Code recommends in Table 2 minimum thickness for beams as a function of the span length.
  - No deflection computation is needed if the member is not supporting or attached to construction likely to be damaged by large deflection.
  - Other deflections would be computed and controlled as in Table 3.

Permissible Deflections in Beams and One-Way Slabs

Table 2. Minimum Thickness of Beams and One-Way Slabs Unless Deflections are Computed[a]

Member[b]	Minimum Thickness, *h*			
	Simply Supported	**One End Continuous**	**Both Ends Continuous**	**Cantilever**
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

Permissible Deflections in Beams and One-Way Slabs

- Table 2 Notations ***a*** and ***b***
  - ***a*:** Clear span length l is inches. Values given should be used directly for members with normal-weight concrete (w_c = 145 pcf) and grade-60 reinforcement. For other conditions, the values should be modified as follows: (1) For structural lightweight concrete having unit weights in the range of 90 to 120 lb/ft^3, the values should be multiplied by $(1.65 - 0.005w_c)$, but not less than 1.09, (2) For f_y other than 60,000 psi, the values should be multiplied by $(0.4 + f_y/100{,}000)$.
  - ***b*:** Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.

Permissible Deflections in Beams and One-Way Slabs

- Empirical Methods of Minimum Thickness Evaluation for Deflection Control (cont'd)
  - If the total beam thickness is less than required by the Table 3, the designer should verify the deflection serviceability performance of the beam through detailed computations of the immediate and long-term deflections.

Permissible Deflections in Beams and One-Way Slabs

Table 3. Minimum Permissible Ratios of Span l to deflection Δ (l = longer span)

Type of Member	Deflection Δ to be considered	$(l/\Delta)_{min}$
Flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	180^a
Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	360
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of total deflection occurring after attachment of nonstructural elements: sum of long-term deflection due to all sustained loads (dead load plus any sustained portion of live load) and immediate deflection due to any additional live load^b	480^c
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		240^c

Permissible Deflections in Beams and One-Way Slabs

- Permissible Limits of Calculated Deflection
  - If Table 2 is not used, the ACI Code requires that the calculated deflection for a beam or one-way slab has to satisfy the serviceability requirement of minimum permissible deflection for the various structural conditions listed in Table 3.

Permissible Deflections in Beams and One-Way Slabs

- Computation of Deflection
  - Deflection of structural members is a function of
    - Span length
    - Support
    - End conditions
    - Restrain due continuity
    - Type of loading (e.g., point or distributed loads)
    - Flexural stiffness EI of the member

Permissible Deflections in Beams and One-Way Slabs

- Computation of Deflection (cont'd)
  - The general expression for maximum deflection Δ_{max} in an elastic member can be expressed as

$$\Delta_{\max} = K\frac{Wl_n^3}{48E_cI_c} \tag{13}$$

W = total load on the span
l_n = clear span length
E_c = modulus of concrete
I_c = moment of inertia of the concrete section
K = a factor depending on the degree of fixity of the support

Permissible Deflections in Beams and One-Way Slabs

- Computation of Deflection (cont'd)
  - Eq.13 can also be written in terms of moment such that the deflection at any point in a beam is

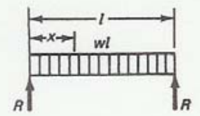
$$\Delta_{\max} = k\frac{ML^2}{48E_cI_e} \tag{14}$$

k = a factor depending on support fixity and load conditions
M = moment acting on the section
I_e = effective moment of inertia

Table 4 provides maximum elastic deflection values in terms of the gravity Load for typical beams loaded with uniform or concentrated load.

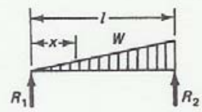
Permissible Deflections in Beams and One-Way Slabs

Table 4



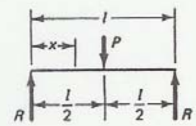
$M_x = \frac{wx}{2}(l - x)$

Δ_{max} (at center) $= \frac{5wl^4}{384EI}$



$\Delta_{max}\left(\text{at } x = l\sqrt{1-\sqrt{\tfrac{8}{15}}} = 0.5193l\right) = 0.01304\,\frac{Wl^3}{EI}$

$\Delta x = \frac{Wx}{180EI\,l^2}(3x^4 - 10l^2x^2 + 7l^4)$

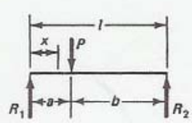


Δ_{max} (at point of load) $= \frac{Pl^3}{48EI}$

$\Delta x\left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{48EI}(3l^2 - 4x^2)$

Permissible Deflections in Beams and One-Way Slabs

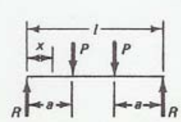
Table 4 (cont'd)



$\Delta_{max}\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\,l}$

Δa (at point of load) $= \frac{Pa^2b^2}{3EI\,l}$

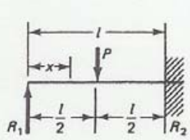
Δx (when $x < a$) $= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$



Δ_{max} (at center) $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δx (when $x < a$) $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δx (when $x > a$ and $< (l - a)$) $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$



$\Delta_{max}\left(\text{at } x = l\sqrt{\tfrac{1}{5}} = 0.4472l\right) = \frac{Pl^3}{48EI\sqrt{5}} = 0.009317\,\frac{Pl^3}{EI}$

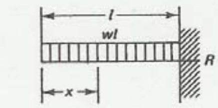
Δx (at point of load) $= \frac{7Pl^3}{768EI}$

$\Delta x\left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{96EI}(3l^2 - 5x^2)$

$\Delta x\left(\text{when } x > \frac{l}{2}\right) = \frac{P}{96EI}(x - l)^2(11x - 2l)$

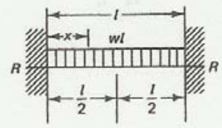
Permissible Deflections in Beams and One-Way Slabs

Table 4 (cont'd)



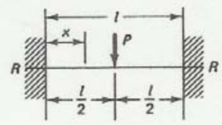
Δ_{max} (at free end) $= \dfrac{wl^4}{8EI}$

$\Delta x = \dfrac{w}{24EI}(x^4 - 4l^3x + 3l^4)$



Δ_{max} (at center) $= \dfrac{wl^4}{384EI}$

$\Delta x = \dfrac{wx^2}{24EI}(l - x)^2$

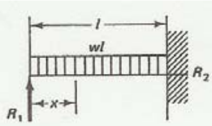


Δ_{max} (at center) $= \dfrac{Pl^3}{192EI}$

$\Delta x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{Px^2}{48EI}(3l - 4x)$

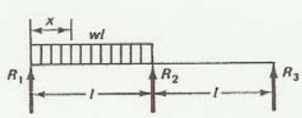
Permissible Deflections in Beams and One-Way Slabs

Table 4 (cont'd)

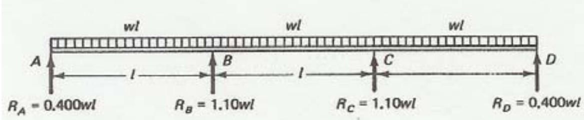


$\Delta_{max} \left(\text{at } x = \dfrac{l}{16}(1 + \sqrt{33}) = 0.4215l\right) = \dfrac{wl^4}{185EI}$

$\Delta x = \dfrac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$



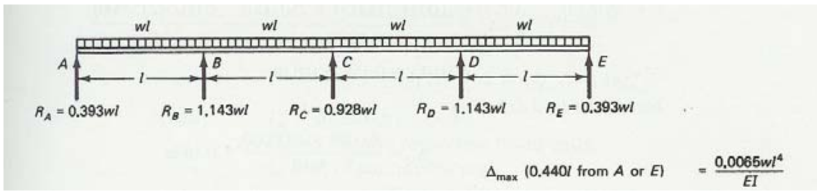
Δ_{max} (0.472l from R_1) $= \dfrac{0.0092wl^4}{EI}$



Δ_{max} (0.446l from A or D) $= \dfrac{0.0069wl^4}{EI}$

Permissible Deflections in Beams and One-Way Slabs

Table 4 (cont'd)



Deflection of Continuous Beams

- A simple procedure is to use the weighted average section properties as required by ACI code provisions:
  1. Beams with both ends continuous:

$$\text{average } I_e = 0.70\, I_m + 0.15(I_{e1} + I_{e2}) \qquad (15)$$

  2. Beams with one end continuous:

$$\text{average } I_e = 0.85\, I_m + 0.15(I_{ec}) \qquad (16)$$

I_m = midspan section I_e
I_{e1}, I_{e2} = I_e for the respective beam ends
I_{ec} = I_e of continuous end

Deflection of Continuous Beams

- Deflection of T-Beams
  - The gross moment of inertia I_g for T-beam is given by

$$I_g = \frac{bh_f^3}{12} + bh_f\left(\bar{y} - \frac{h_f}{2}\right)^2 + \frac{b_w\left(h-h_f\right)^3}{12} + b_w\left(h-h_f\right)\left(y_t - \frac{h-h_f}{2}\right)^2 \qquad (17)$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$y_t = h - \bar{y}$$

Deflection of Continuous Beams

- Deflection of T-Beams (cont'd)

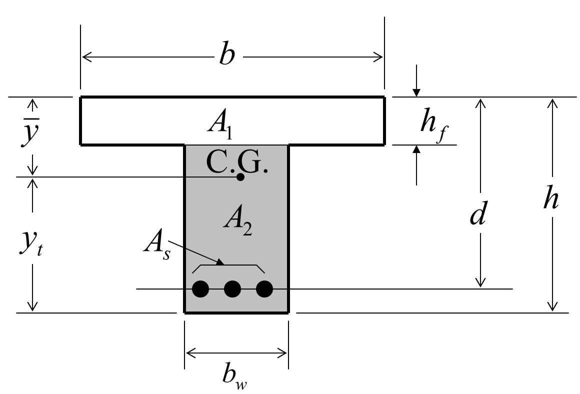


Figure 10

Deflection of Continuous Beams

- Deflection of T-Beams (cont'd)
  - The moment of inertia of I_{cr} of the cracked section can be calculated using the following expression:

$$I_{cr} = \frac{1}{3}b_w\left(c-h_f\right)^3 + \frac{1}{12}bh_f^3 + bh_f\left(c-\frac{h_f}{2}\right)^2 + nA_s\left(d-c\right)^2 \qquad (18)$$

  - Where c is found by solving the following quadratic equation:

$$b_w\left(c-h_f\right)^2 - 2nA_s\left(d-c\right) + bh_f\left(2c-h_f\right) = 0 \qquad (19)$$

Deflection of Continuous Beams

- Deflection of Beams with Compressive Steel
  - The moment of inertia of I_{cr} of the cracked section can be calculated using the following expression:

$$I_{cr} = \frac{bc^3}{3} + nA_s\left(d-c\right)^2 + \left(n-1\right)A_s'\left(c-d'\right)^2 \qquad (20)$$

  - Where c is found by solving the following quadratic equation:

$$\frac{bc^2}{2} + \left[nA_s + \left(n-1\right)A_s'\right]c - nA_sd - \left(n-1\right)A_s'd' = 0 \qquad (21)$$

Deflection of Continuous Beams

- Deflection of Beams with Compressive Steel

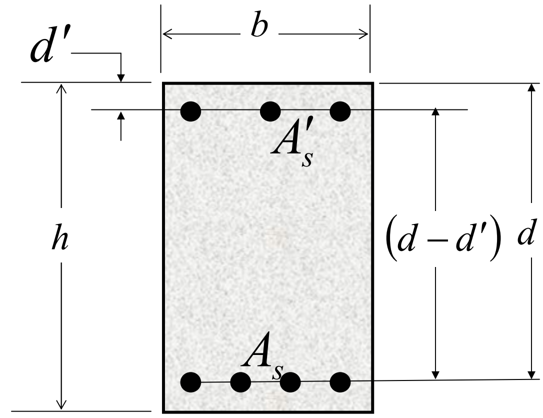


Figure 11

Deflection of Continuous Beams

- Bending Moment Deflections in Continuous Beams

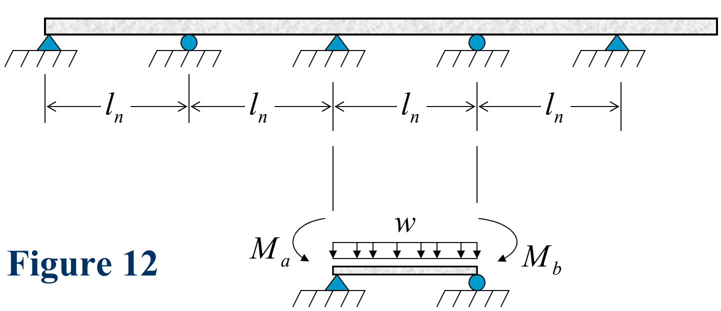


Figure 12

Deflection of Continuous Beams

- Bending Moment Deflections in Continuous Beams
  - The midspan deflection is

$$\Delta_c = \frac{5l^2}{48EI}\left[M_m + 0.1\left(M_a + M_b\right)\right] \qquad (22)$$

M_a, M_b = negative service load bending moment
M_m = midspan moment

Deflection of Continuous Beams

- Bending Moment Deflections in Continuous Beams
  - An approximate approach for deflection can be used as follows:

$$\Delta_c = \frac{5wl_n^4}{384E_cI_e} - \frac{Ml_n^2}{8E_cI_e} \qquad (23)$$

M = negative moment at supports (if values are different use average moment)
l_n = clear span
w = uniformly distributed service load

Crack Control

- Cracking could have an effect on corrosion of the reinforcement.
- There is no clear correlation between corrosion and surface crack widths in the usual range found in structures with reinforcement stresses at service load levels.
- Also, there is no clear experimental evidence available regarding the crack width beyond which a corrosion exists.

Crack Control

- Exposure tests indicate that concrete quality, adequate consolidation, and ample concrete cover may be more important in corrosion considerations than is crack width.
- Rather than a small number of large cracks, it is more desirable to have only hairline cracks and to accept more numerous cracks if necessary.

Crack Control

- To achieve this, the current ACI Code (Section 10.6.3) states that flexural tension reinforcement be well distributed in the maximum tension zones of a member.
- Section 10.6.4 contains a provision for maximum spacing s that is intended to control surface cracks to a width that is generally acceptable in practice.

Crack Control

- The maximum spacing is limited to

$$s = \frac{540}{f_s} - 2.5c_c \leq 12\left(\frac{36}{f_s}\right) \qquad (24)$$

s = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face

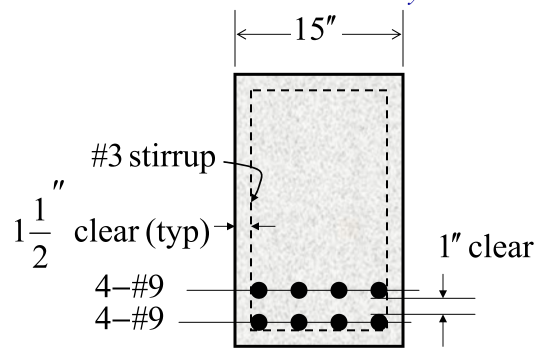
f_s = calculated stress, ksi. This may be taken as 60% of specified yield strength.

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in.

Crack Control

- Example 4

Check the steel distribution for the beam shown to establish whether reasonable control of flexural cracking is accomplished in accordance with the ACI Code. Use f_y = 60,000 psi.



Crack Control

- Example 4 (cont'd)
  - Calculate the center-to-center spacing between No. 9 bars:

$$s = \frac{15 - 2(1.5) - 2(0.375) - 2\left(\frac{1.128}{2}\right)}{3} = 3.37 \text{ in.}$$

  - Assume positive moment and calculate the concrete clear cover from the bottom (tension) face of the beam to the surface of the nearest tension reinforcement:

$$c_c = 1.5 + 0.375 = 1.875 \text{ in.}$$

Crack Control

- Example 4 (cont'd)

Table 5. Reinforced Steel Properties

Bar number	3	4	5	6	7	8	9	10	11	14	18
Unit weight per foot (lb)	0.376	0.668	1.043	1.502	2.044	2.670	3.400	4.303	5.313	7.650	13.60
Diameter (in.)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

Crack Control

- Example 4 (cont'd)
  - Calculate f_s using 60% of f_y:

$$f_s = 0.60 f_y = 0.60(60\text{ ksi}) = 36.0\text{ ksi}$$

  - Calculate maximum spacing allowed usibg ACI equation (Eq. 24):

$$s = \frac{540}{f_s} - 2.5c_c = \frac{540}{36.0} - 2.5(1.875) = 10.31\text{ in.}$$

Crack Control

- Example 4 (cont'd)
  - Check the upper limit for ACI equation 24:

$$12\left(\frac{36}{f_s}\right) = 12\left(\frac{36}{36}\right) = 12\text{ in.} > 10.31\text{ in.}$$

O.K.

  - And lastly:

$$3.37\text{ in.} < 10.31\text{ in.}$$

O.K.