CHAPTER

6a

REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

SHEAR AND DIAGONAL TENSION IN BEAMS

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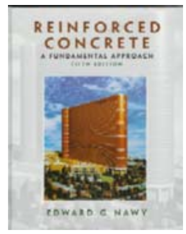

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ENCE 454 – Design of Concrete Structures

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Introduction

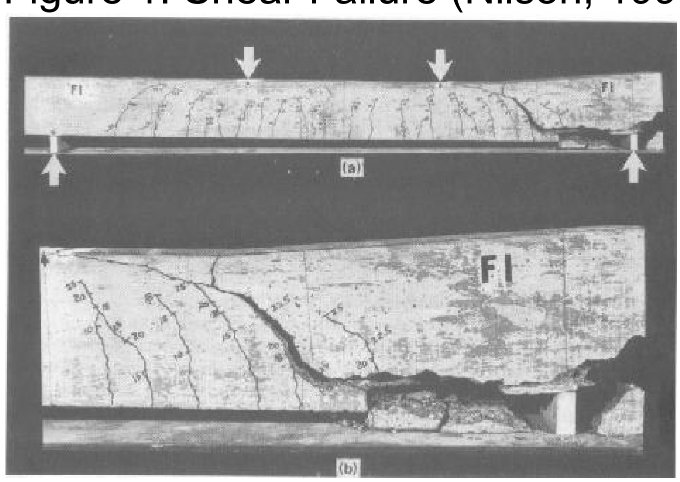
- The previous chapter dealt with the flexural strength of beams.
- Beams must also have an adequate safety margin against other types of failure such as shear, which may be more dangerous than flexural failure.
- The shear forces create additional tensile stresses that must be considered.

Introduction

- Shear Failure
  – Shear failure of reinforced concrete beam, more properly called "***diagonal tension failure***", is difficult to predict accurately.
  – In spite of many years of experimental research and the use of highly sophisticated computational tools, it is not fully understood.
  – If a beam without properly designed for shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly.

Introduction

Figure 1. Shear Failure (Nilson, 1997)



(a) Overall view, (b) detail near right support.

Introduction

- Shear Failure (cont'd)
  - Figure 1 shows a shear-critical beam tested under point loading.
  - With no shear reinforcement provided, the member failed immediately upon formation of the critical crack in the high-shear region near the right support.

Introduction

- Shear Failure (cont'd)

When are the shearing effects so large that they cannot be ignored as a design consideration?

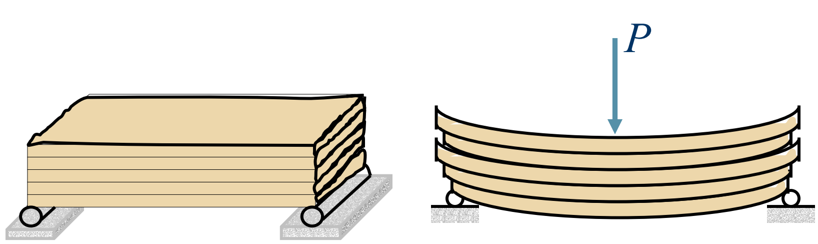
  - It is somehow difficult to answer this question.
  - Probably the best way to begin answering this question is to try to approximate the shear stresses on the cross section of the beam.

Introduction

- Shear Failure (cont'd)
  – Suppose that a beam is constructed by stacking several slabs or planks on top of another without fastening them together.
  – Also suppose this beam is loaded in a direction normal to the surface of these slabs.
  – When a bending load is applied, the stack will deform as shown in Figure 2a.
  – Since the slabs were free to slide on one another, the ends do not remain even but staggered.

Introduction

- Shear Failure (cont'd)



(a) Unloaded Stack of Slabs (b) Unglued Slabs loaded

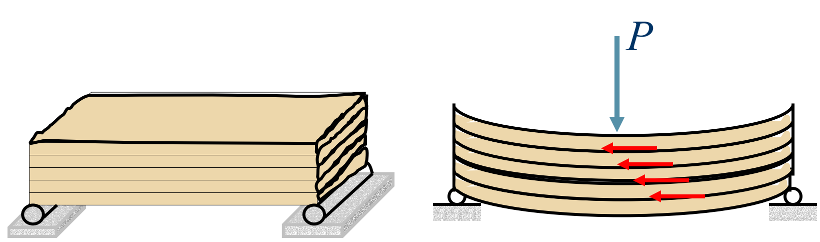
Figure 2a

Introduction

- Shear Failure (cont'd)
  – Each of the slabs behaves as independent beam, and the total resistance to bending of *n* slabs is approximately *n* times the resistance of one slab alone.
  – If the slabs of Figure 2b is fastened or glued, then the staggering or relative longitudinal movement of slabs would disappear under the action of the force. However, shear forces will develop between the slabs.
  – In this case, the stack of slabs will act as a solid beam.

Introduction

- Shear Failure (cont'd)



(c) Glued Slabs Unloaded

(d) Glued Slabs loaded

Figure 2b

Introduction

- Shear Failure (cont'd)
  - The fact that this solid beam does not exhibit this relative movement of longitudinal elements after the slabs are glued indicates the presence of *shearing stresses* on longitudinal planes.
  - Evaluation of these shearing stresses will be discussed in the next set of viewgraphs.

Introduction

- Theoretical Background
  - The concept of stresses acting in homogeneous beams are usually covered in various textbooks of mechanics of materials (strength of materials).
  - It can be shown that when the material is elastic, shear stresses can be computed by

$$v = \frac{VQ}{Ib} \qquad (1)$$

v = shear stress
V = external shear force
I = moment of inertia about neutral axis
Q = statical moment of area about N.A.
b = width of the cross section

Introduction

- Theoretical Background (cont'd)
  - Also, when the material is elastic, bending stresses can be computed from

$$f = \frac{Mc}{I} \qquad (2)$$

f = bending stress
M = external or applied moment
c = the distance from the neutral axis to out fiber of the cross section
I = moment of inertia of the cross section about N.A.

Introduction

- Theoretical Background (cont'd)
  - All points along the length of the beam, where the shear and bending moment are not zero, and at locations other than the extreme fiber or neutral axis, are subject to both shearing stresses and bending stresses.
  - The combination of these stresses produces maximum normal and shearing stresses in a specific plane inclined with respect to the axis of the beam.

Introduction

- Theoretical Background (cont'd)
  - The distributions of the bending and shear stresses acting individually are shown in Figs. 3, 4, 5, and 6.

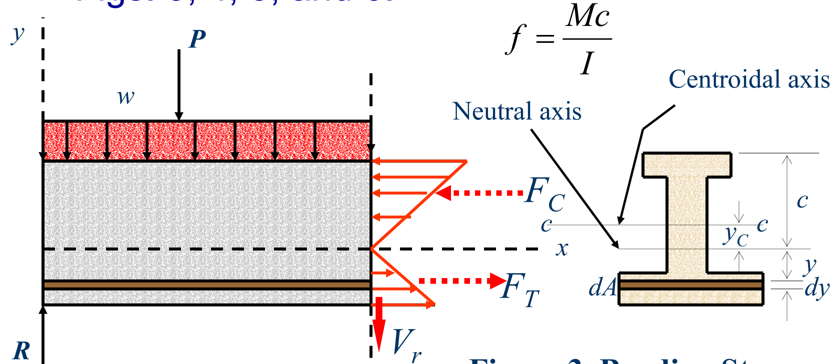
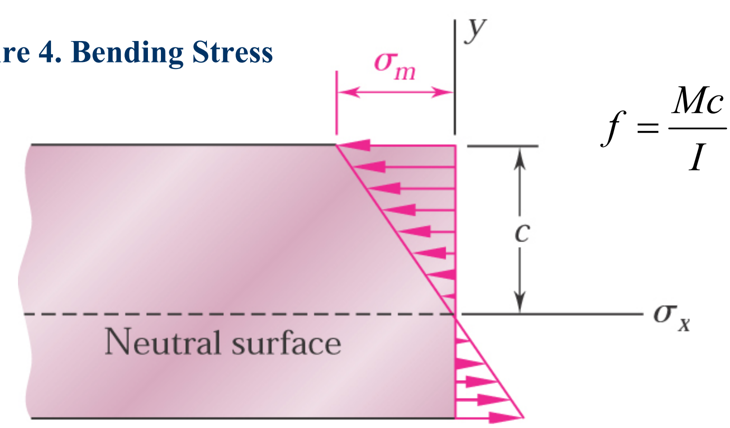


Figure 3. Bending Stress

Introduction

- Theoretical Background (cont'd)

Figure 4. Bending Stress

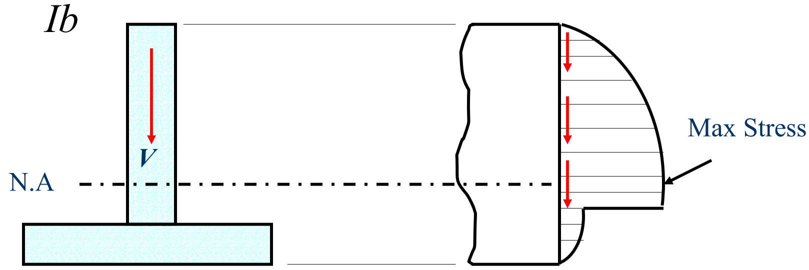


Introduction

- Theoretical Background (cont'd)

Figure 5. Vertical Shearing Stress

$$v = \frac{VQ}{Ib}$$

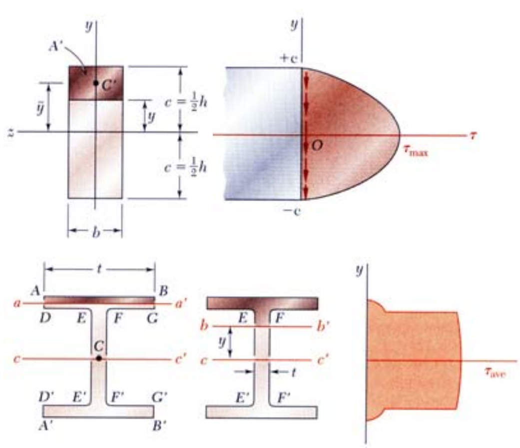


Introduction

- Theoretical Background (cont'd)

Figure 6. Vertical Shearing Stress

$$v = \frac{VQ}{Ib}$$

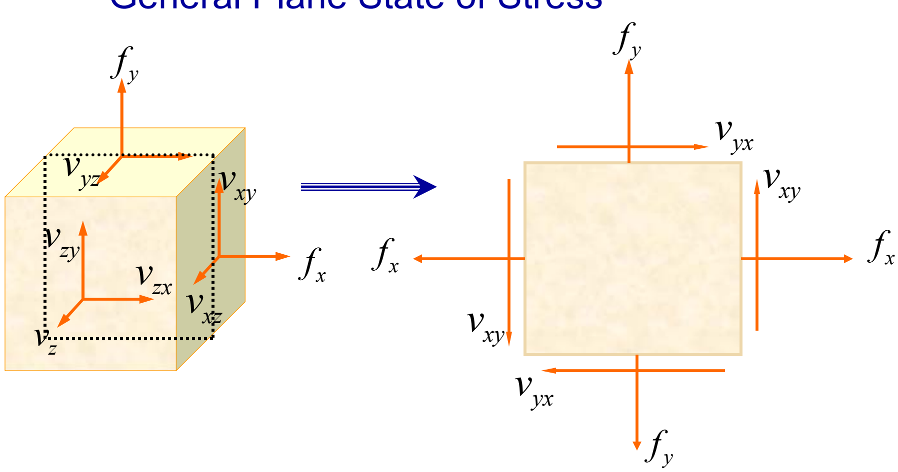


Introduction

- Principal Planes
  – The combination of bending moment and shearing stresses is of such a nature that maximum normal and shearing shearing stresses at a point in a beam exist on planes that are inclined with the axis of the beam.
  – These planes are commonly called ***principal planes***, and the stresses that act on them are referred to as ***principal stresses***.

Introduction

- Principal Planes
  – General Plane State of Stress



Introduction

- Principal Planes
  - General Plane State of Stress (cont'd)

Components:

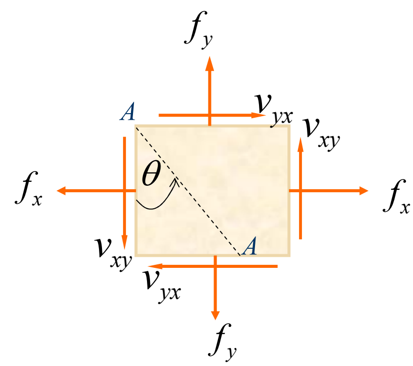
Normal Stress f_x

Normal Stress f_y

Shearing Stress v_{xy}

Shearing Stress v_{yx}

$$v_{xy} = v_{yx}$$



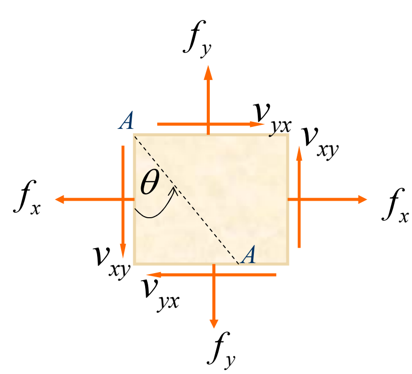
Introduction

- Principal Planes
  - General Plane State of Stress (cont'd)
    - Principal Stresses

$$f_{pr} = \frac{f_x - f_y}{2} \pm \sqrt{\frac{(f_x - f_y)^2}{4} + v_{xy}^2}$$

$$\text{when } f_y = 0,\ f_x = f,\ v_{xy} = v$$

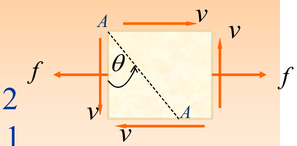
$$f_{pr} = \frac{f_x - f_y}{2} \pm \sqrt{\frac{(f_x - f_y)^2}{4} + v_{xy}^2}$$



Introduction

- Principal Stresses
  - The principal stresses in a beam subjected to shear and bending may be computed using the following equation:

$$f_{pr} = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \qquad (3)$$



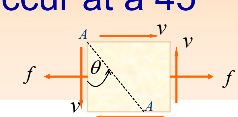
f_{pr} = principal stress
f = bending stress computed from Eq. 2
v = shearing stress computed from Eq. 1

Introduction

- Orientation Principal Planes
  - The orientation of the principal planes may be calculated using the following equation:

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2v}{f}\right) \qquad (4)$$

  - Note that at the neutral axis of the beam, the principal stresses will occur at a 45^0 angle.



Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam

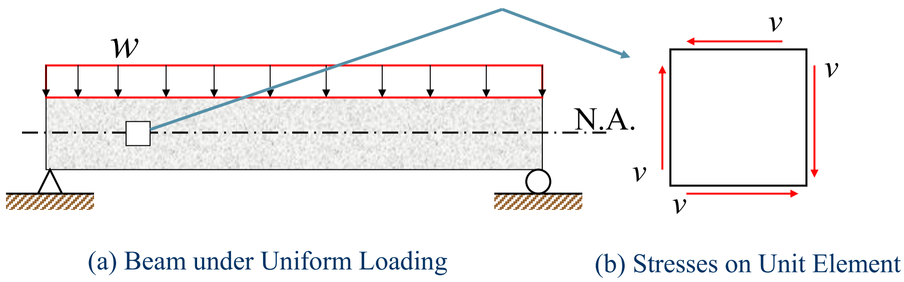
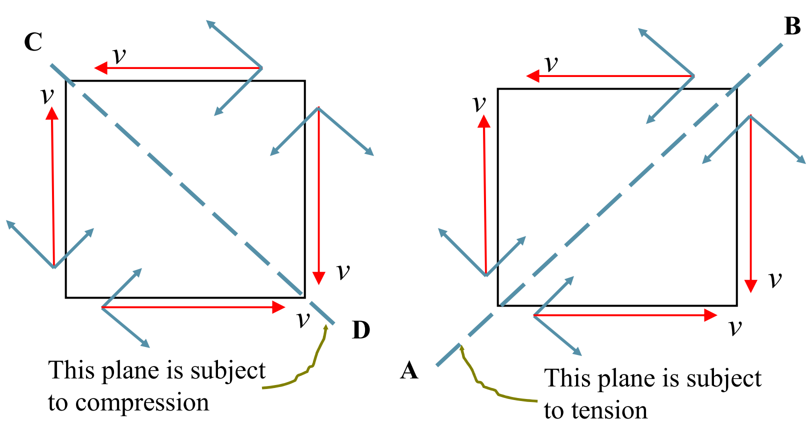


Figure 7. Shear Stress Relationship

Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam
  - Diagonal Tension

Figure 8



Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam
  - Diagonal Tension
    - Plane A-B is subjected to tension
    - While Plane C-D is subjected to compression.
    - The tension in Plane A-B is historically called "***diagonal tension***".
    - Note that concrete is strong in compression but weak in tension, and there is a tendency for concrete to crack on the plane subject to tension, Plane A-B.
    - When the tensile stresses are so high, it is necessary to provide reinforcement.

Introduction

- Diagonal Tension Failure
  - In the beams with which we are concerned, where the length over which a shear failure could occur (the shear span) is in excess of approximately three times the effective depth, the diagonal tension failure would be the mode of failure in shear.
  - Such a failure is shown in Figures 1 and 8.
  - For longer shear spans in plain concrete beams, cracks due to flexural tensile stresses would occur long before cracks due to diagonal tension.

Introduction

- Diagonal Tension Failure

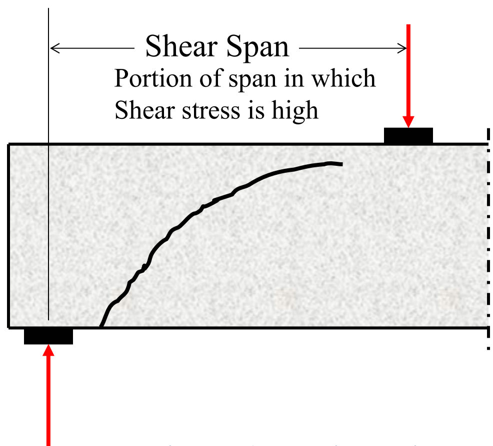
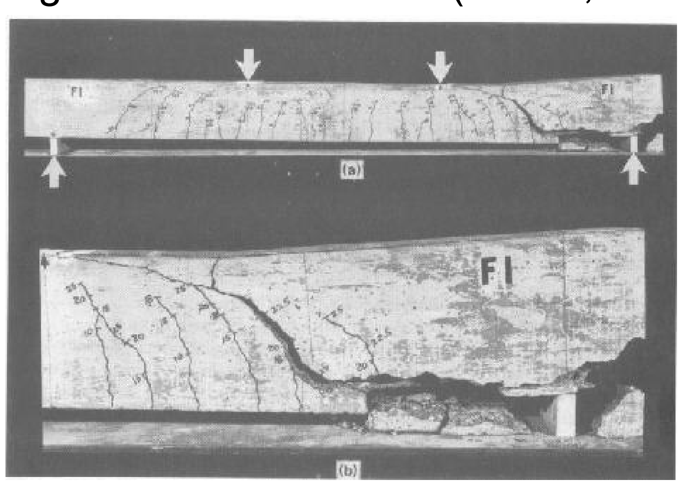


Figure 8. Typical Diagonal Tension Failure

Introduction

Figure 1. Shear Failure (Nilson, 1997)



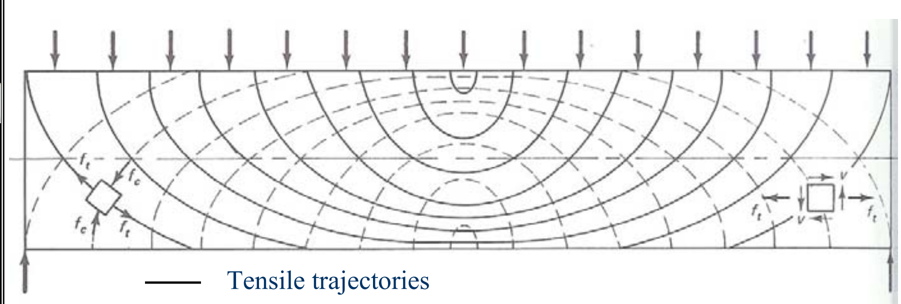
(a) Overall view, (b) detail near right support.

Introduction

- Diagonal Tension Failure
  – The principal stress in tension acts at an approximately 45^0 plane to the normal at sections *close to support*, as seen in Figure 9.
  – Because of the low tensile strength of concrete, diagonal cracking develops along planes perpendicular to the planes of principal tensile stress; hence the *term diagonal tension cracks*.
  – To prevent such cracks, special diagonal tension reinforcement has to be provided.

Introduction

- Diagonal Tension Failure



Tensile trajectories

compressive trajectories

Figure 9. Trajectories of principal stresses in a homogeneous Isotropic beam

Beams without Diagonal Tension Reinforcement

- Modes of Failure of Beams without Diagonal Tension Reinforcement
  - The slenderness of the beam or its shear span/depth determines the failure of the plain concrete beam.
  - Figure 10 demonstrates schematically the failure patters.
  - The shear span *a* for concentrated load is the distance between the point of application of the load and the face of the support.

Beams without Diagonal Tension Reinforcement

- Modes of Failure of Beams without Diagonal Tension Reinforcement (cont'd)

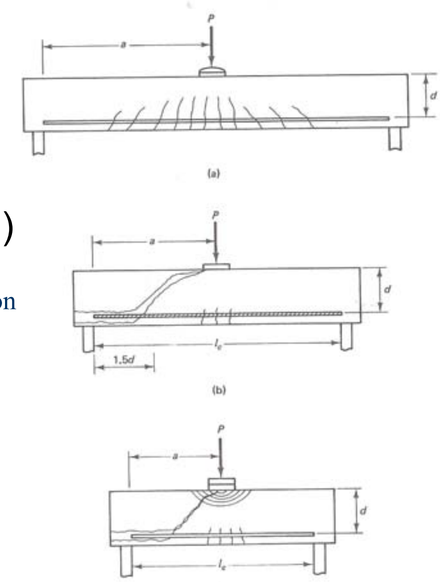


Figure 10. Failure Patterns as a function of beam slenderness;
(a) flexure failure;
(b) diagonal tension failure;
(c) shear compression failure.

Beams without Diagonal Tension Reinforcement

- Modes of Failure of Beams without Diagonal Tension Reinforcement (cont'd)
  - Basically, three modes of failure or their combinations are of interest:
    1. Flexure Failure
    2. Diagonal Tension Failure
    3. Shear Compression Failure
  - The more slender the beam, the stronger the tendency toward flexure behavior.

Beams without Diagonal Tension Reinforcement

- Modes of Failure of Beams without Diagonal Tension Reinforcement (cont'd)
  - The shear span l_c for distributed loads is the clear beam span.
  - Table 1 gives expected range values of the shear span/depth ratio for the different modes of failure and types of beams.

Beams without Diagonal Tension Reinforcement

Table 1. Beam Slenderness Effect on Mode of Failure

Beam Category	Failure Mode	Shear Span/Depth Ratio as a Measure of Slenderness	
		Concentrated load, a/d	Distributed Load, l_c/d
Slender	Flexure (F)	Exceeds 5.5	Exceeds 16
Intermediate	Diagonal Tension (DT)	2.5 to 5.5	11 to 16
Deep	Shear Compression (SC)	1 to 2.5	1 to 5

ACI Philosophy and Rationale

- Basis of ACI Design for Shear
  - The ACI provides design guidelines for shear reinforcement based on the vertical shear force V_u that develops at any given cross section of a member.
  - Although it is really the diagonal tension for which shear reinforcing must be provided, diagonal tensile forces (or stresses) are not calculated.
  - Traditionally, vertical shear force has been taken to be good indicator of diagonal tension present.

Shear Reinforcement Design Requirements

- Shear or Web Reinforcement
  - The basic rationale for the design of the shear reinforcement, or web reinforcement as it usually called in beams, is to provide steel to cross the diagonal tension cracks and subsequently keep them from opening.
  - In reference to Figure 8, it is seen that the web reinforcement may take several forms such as:

Shear Reinforcement Design Requirements

- Shear or Web Reinforcement (cont'd)
  1. Vertical stirrups (see Fig. 11)
  2. Inclined or diagonal stirrups
  3. The main reinforcement bent at ends to act as inclined stirrups (see Fig. 12).
  - The most common form of web reinforcement used is the vertical stirrup.
  - This reinforcement appreciably increases the ultimate shear capacity of a bending member.

Shear Reinforcement Design Requirements

- Shear or Web Reinforcement (cont'd)
  - Vertical Stirrups

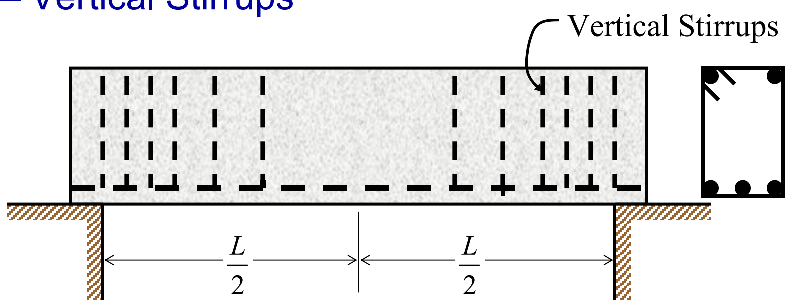


Figure 11. Types of Web Reinforcement

Shear Reinforcement Design Requirements

- Shear or Web Reinforcement (cont'd)
  - Bent-up Longitudinal Bars

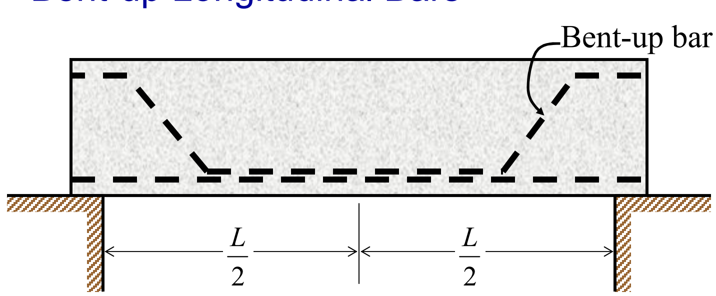


Figure 12. Type of Web Reinforcement

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

For member that are subject to shear and flexure only, the amount of shear force that the concrete (unreinforced for shear)can resist is

$$V_c = \lambda \times 2\sqrt{f_c'}b_w d \qquad (5)$$

Note, for rectangular beam $b_w = b$
λ= 1.0 for normal concrete
λ = 0.85 for sand-lightweight concrete
λ = 0.75 for all light weight concrete

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

When axial compression also exists, V_n in Eq. 5 becomes

$$V_c = 2\lambda\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f_c'}b_w d \qquad (6)$$

Note, for rectangular beam $b_w = b$, and N_u/A_g is expressed in psi
N_u = axial load (positive in compression)
A_g = gross area of the section

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

When significant axial tension also exists, V_n in Eq. 5 becomes

$$V_c = 2\lambda\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f_c'}b_w d \qquad (7)$$

Note, for rectangular beam $b_w = b$, and N_u/A_g is expressed in psi
N_u = axial load (negative in compression)
A_g = gross area of the section

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

The ACI also permits the use of more refined equation in replacement of Eq. 5, that is

$$V_c = 1.9\lambda\sqrt{f_c'}b_w d + 2500\rho_w \frac{V_u d}{M_u} b_w d \le 3.5 b_w d\sqrt{f_c'} \qquad (8)$$

Note, for rectangular beam $b_w = b$, and $V_u d / M_u \le 1$
$V_u = \phi V_n$ = factored shear force, taken d distance from FOS
$M_u = \phi M_n$ = factored bending moment, taken d distance from FOS

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - The design shear force V_u results from the application of factored loads.
  - Values of V_u are most conveniently determined using a typical shear force diagram.
  - Theoretically, no web reinforcement is required if

$$V_u \leq \varphi V_c \qquad (9)$$

Shear Reinforcement Design Requirements

Table 2. Resistance or Strength Reduction Factors

Structural Element	Factor ϕ
Beam or slab; bending or flexure	0.90
Columns with ties	0.65
Columns with spirals	0.75
Columns carrying very small axial load (refer to Chapter 9 for more details)	0.65 – 0.9 or 0.70 – 0.9
Beam: shear and torsion	0.75

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - However, the code requires that a minimum area of shear reinforcement be provided in all reinforced concrete flexural members when $V_u > ½ \phi V_c$, except as follows:
    - In slabs and footings
    - In concrete joist construction as defined in the code.
    - In beams with a total depth of less than 10 in., 2 ½ times the flange thickness, or one-half the width of the web, whichever is greater.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - In cases where shear reinforcement is required for strength or because $V_u > ½ \phi V_c$, the minimum area of shear reinforcement shall be computed from

$$A_v = \max\left[0.75\sqrt{f'_c}\,\frac{b_w s}{f_y}, \frac{50 b_w s}{f_y}\right] \qquad (10)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

Where

A_v = total cross-sectional area of web reinforcement within a distance s, for single loop stirrups, $A_v = 2A_s$
A_s = cross-sectional area of the stirrup bar (in^2)
b_w = web width = b for rectangular section (in.)
s = center-to-center spacing of shear reinforcement in a direction parallel to the longitudinal reinforcement (in.)
f_y = yield strength of web reinforcement steel (psi)
f'_c = compressive strength of concrete (psi)

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

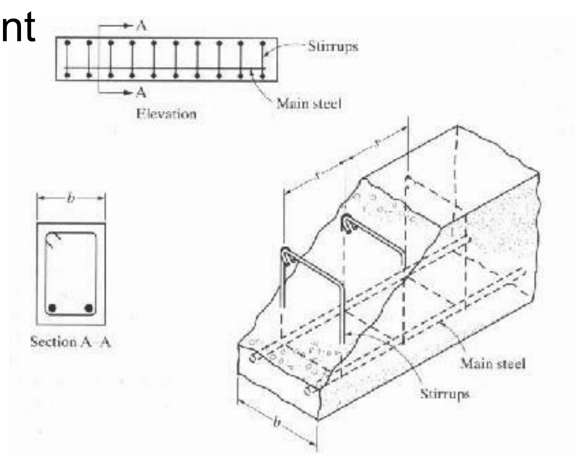


Figure 13. Isometric section showing stirrups partially exposed

Shear Reinforcement Design Requirements

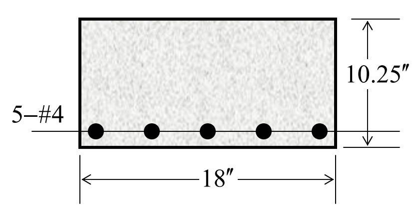
- Example 1

A reinforced concrete beam of rectangular cross section shown in the figure is reinforced for moment only (no shear reinforcement). Beam width *b* = 18 in., *d* = 10.25 in., and the reinforcing is five No. 4 bars. Calculate the maximum factored shear force V_u permitted on the member by the ACI Code. Use $f_c' = 4{,}000$ psi, and f_y = 60,000 psi, and normal concrete.

Shear Reinforcement Design Requirements

- Example 1 (cont'd)

Since no shear reinforcement is provided, the ACI Code requires that



$$\text{maximum } V_u = \frac{1}{2}\phi V_c$$

$$= \frac{1}{2}\phi\left(\lambda \times 2\sqrt{f_c'}b_w d\right) \qquad \text{Eq. 5}$$

$$= \frac{1}{2}(0.75)(2)\left(\sqrt{4000}\right)(18)(10.25) = 8{,}752 \text{ lb}$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - According to the ACI Code, the design of beams for shear is based on the following relation:

$$\phi V_n \geq V_u \qquad (11)$$

Where
ϕ = strength reduction factor (= 0.75 for shear)
$V_n = V_c + V_s$
V_s = nominal shear strength provided by reinforcement

$$= \frac{A_v f_y d}{s} \quad \text{for vertical stirrups} \qquad (12)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - For inclined stirrups, the expression for nominal shear strength provided by reinforcement is

$$V_s = \frac{A_v f_y d(\sin\alpha + \cos\alpha)}{s} \qquad (13)$$

  - For α = 450, the expression takes the form

$$V_s = \frac{1.414 A_v f_y d}{s} \qquad (14)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - The design for stirrup spacing can be determined from

$$\text{required } s = \frac{A_v f_y d}{V_s} \quad \text{(for vertical stirrups)} \tag{15}$$

and

$$\text{required } s = \frac{1.414 A_v f_y d}{V_s} \quad \text{(for } 45^0 \text{ stirrups)} \tag{16}$$

where

$$V_s = \frac{V_u - \phi V_c}{\phi} \tag{17}$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - According to the ACI Code, the maximum spacing of stirrups is the smallest value of

$$s_{\max} = \frac{A_v f_y}{50 b_w}$$

$$s_{\max} = \frac{d}{2} \tag{18}$$

$$s_{\max} = 24 \text{ in.}$$

If V_s exceeds $4\sqrt{f'_c}\, b_w d$, the maximum spacing must not exceed $d/4$ or 12 in.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - It is not usually good practice to space vertical stirrups closer than 4 in.
  - It is generally economical and practical to compute spacing required at several sections and to place stirrups accordingly in groups of varying spacing. Spacing values should be made to not less than 1-in. increments.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section
    - The maximum shear usually occurs in this section near the support.
    - For stirrup design, the section located a distance d from the face of the support is called the "***critical section***"
    - Sections located less than a distance d from the face of the support may be designed for the same V_u as that of the critical section.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section

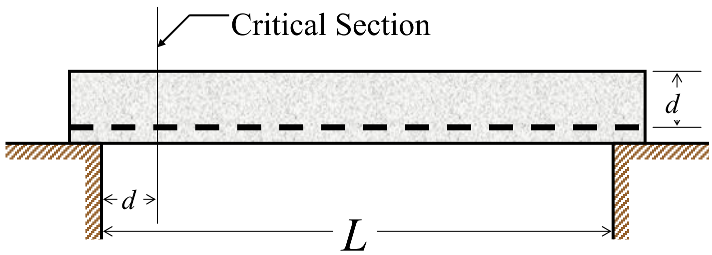


Figure 14

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section (cont'd)
    - The stirrup spacing should be constant from the critical section back to the face of the support based on the spacing requirements at the critical section.
    - The first stirrup should be placed at a maximum distance of *s*/2 from the face of the support, where *s* equals the immediately adjacent required spacing (a distance of 2 in. is commonly used.

Shear Analysis Procedure

- The shear analysis procedure involves the following:
  - Checking the shear strength in an existing member
  - Verifying that the various ACI code requirements have been satisfied and met.
- Note that the member may be reinforced concrete section or plain concrete section.

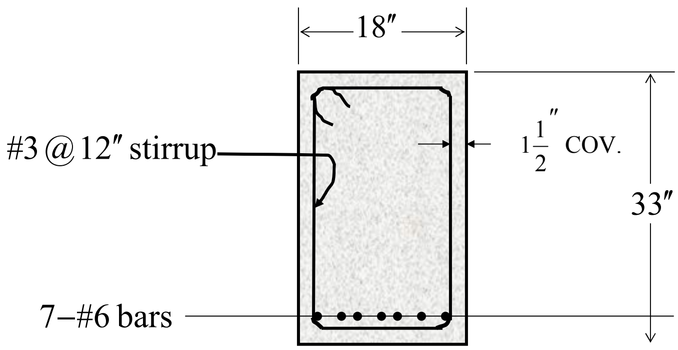
Shear Analysis Procedure

- Example 2

  A reinforced concrete beam of rectangular cross section shown is reinforced with seven No. 6 bars in a single layer. Beam width b = 18 in., d = 33 in., single-loop No. 3 stirrups are placed 12 in. on center, and typical cover is 1 ½ in. Find V_c, V_s, and the maximum factored shear force permitted on this member. Use f_c' = 4,000 psi and f_y = 60,000 psi. Assume normal concrete.

Shear Analysis Procedure

- Example 2 (cont'd)



Shear Analysis Procedure

- Example 2 (cont'd)
  - The force that can be resisted by concrete alone is

$$V_c = 2\lambda\sqrt{f_c'}b_w d = \frac{2(1)\sqrt{4{,}000}(18)(33)}{1000} = 75.1\text{ kips}$$

  - The nominal shear force provided by the steel is

$$V_s = \frac{A_v f_y d}{s} = \frac{(2\times 0.11)(60)(33)}{12} = 36.3\text{ kips}$$

  - The maximum factored shear force is

$$\text{maximum } V_u = \phi V_c + \phi V_s = 0.75(75.1+36.3)$$

$$= 83.6\text{ kips}$$