REINFORCED CONCRETE
A Fundamental Approach - Fifth Edition

FLEXURE IN BEAMS

5d

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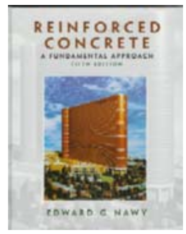

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Doubly Reinforced Sections

- Introduction
  - If a beam cross section is limited because of architectural or other considerations, it may happen that concrete cannot develop the compression force required to resist the given bending moment.
  - In this case, reinforcing steel bars are added in the compression zone, resulting in a so-called doubly reinforced beam, that is one with compression as well as tension reinforcement (Figure 21)

Doubly Reinforced Sections

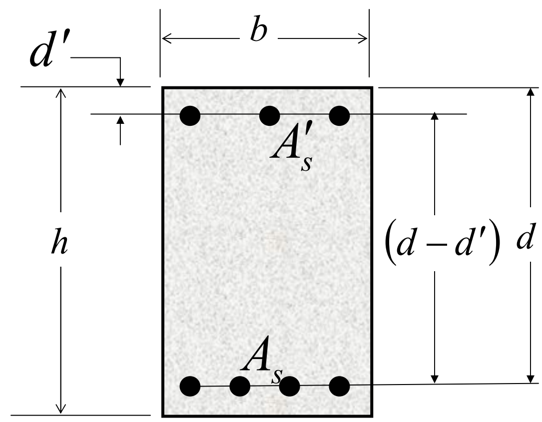


Figure 22. Doubly Reinforced Beam Sections

Doubly Reinforced Sections

- Introduction (cont'd)
  - The use of compression reinforcement has decreased markedly with the use of strength design methods, which account for the full strength potential of the concrete on the compressive side of the neutral axis.
  - However, there are situations in which compressive reinforcement is used for reasons other than strength.

Doubly Reinforced Sections

- Introduction (cont'd)
  - It has been found that the inclusion of some compression steel has the following advantages:
    - It will reduce the long-term deflections of members.
    - It will set a minimum limit on bending loading
    - It act as stirrup-support bars continuous through out the beam span

Doubly Reinforced Sections

- Introduction (cont'd)
  - Another reason for placing reinforcement in the compression zone is that when beams span more than two supports (continuous construction), both positive and negative moments will exist as shown in Figure 23.
  - In Figure 23, positive moments exist at A and C; therefore, the main tensile reinforcement would be placed in the bottom of the beam.
  - At B, however, a negative moment exists and the bottom of the beam is in compression. The tensile reinforcement, therefore, must be placed near the top of the beam.

Doubly Reinforced Sections

- Introduction (cont'd)

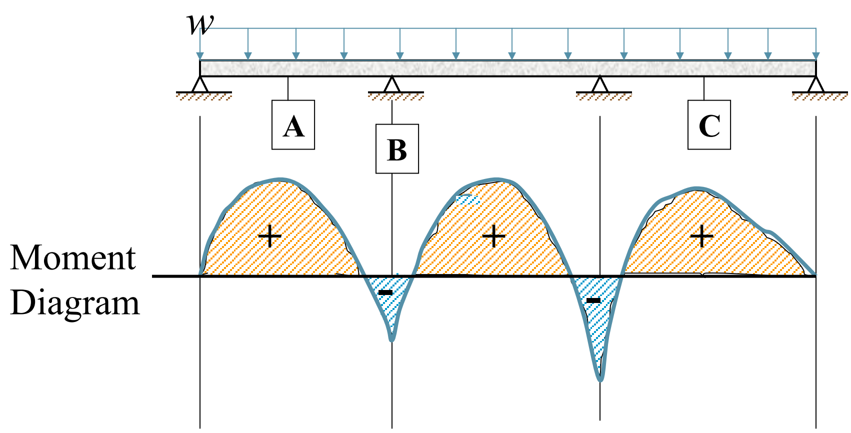


Figure 23. Continuous Beam

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress
  - The basic assumption for the analysis of doubly reinforced beams are similar to those for tensile reinforced beams.
  - The steel will behave elastically up to the point where the strain exceeds the yield strain ε_y. As a limit $f_s' = f_y$ when the compression strain $\varepsilon_s' \geq \varepsilon_y$.
  - If $\varepsilon_s' < \varepsilon_y$, the compression steel stress will be $f_s' = \varepsilon_s' E_s$.

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - If, in a doubly reinforced beam, the tensile steel ratio ρ is equal to or less than ρ_b, the strength of the beam may be approximated within acceptable limits by disregarding the compression bars.
  - The strength of such a beam will be controlled be tensile yielding, and the lever arm of the resisting moment will be little affected by the presence of comp. bars.

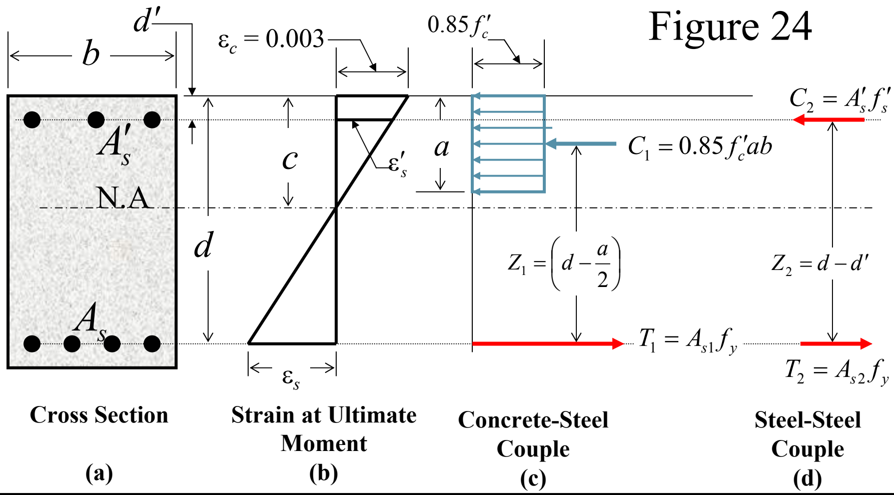
Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - If the tensile steel ratio ρ is larger than ρ_b, a somewhat elaborate analysis is required.
  - In Fig. 24a, a rectangular beam cross section is shown with compression steel A_s' placed at distance d' from the compression face and with tensile steel A_s at the effective depth d.

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

Figure 24



Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Notation for Doubly Reinforced Beam:

A'_s = total compression steel cross-sectional area
d = effective depth of tension steel
d' = depth to centroid of compressive steel from compression fiber
A_{s1} = amount of tension steel used by the concrete-steel couple
A_{s2} = amount of tension steel used by the steel-steel couple
A_s = total tension steel cross-sectional area ($A_s = A_{s1} + A_{s2}$)
M_{n1} = nominal moment strength of the concrete-steel couple
M_{n2} = nominal moment strength of the steel-steel couple
M_n = nominal moment strength of the beam
ε_s = unit strain at the centroid of the tension steel
ε'_s = unit strain at the centroid of the compressive steel

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Method of Analysis:
    - The total compression will now consist of two forces:

      C_1, the compression resisted by the concrete

      C_2, the compression resisted by the steel
    - For analysis, the total resisting moment of the beam will be assumed to consist of two parts or two internal couples: The part due to the resistance of the compressive concrete and tensile steel and the part due to the compressive steel and additional tensile steel.

Doubly Reinforced Sections

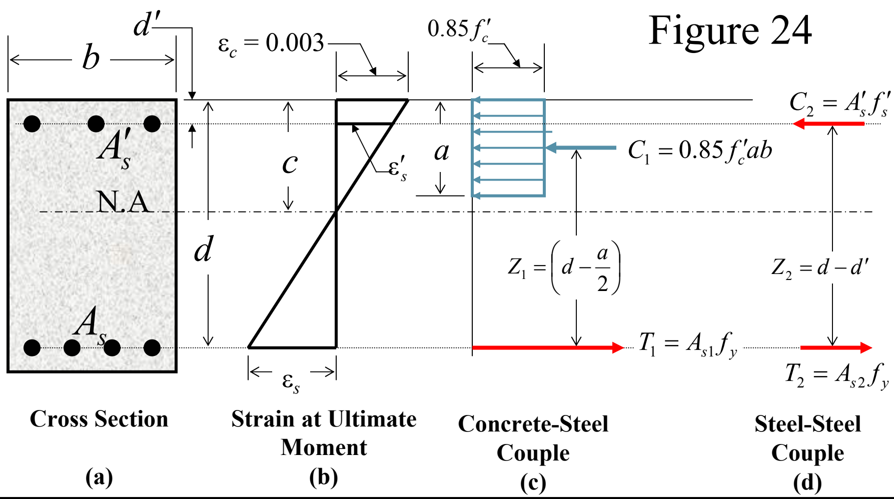
- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - The total nominal capacity may be derived as the sum of the two internal couples, neglecting the concrete that is displaced by the compression steel.
  - The strength of the steel-steel couple is given by (see Figure 24)

$$M_{n2} = T_2 Z_2 \qquad (36)$$

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

Figure 24



Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

$$M_{n2} = A_{s2} f_y (d - d') \qquad \text{assuming } f_s = f_y$$

$$C_2 = T_2 \Rightarrow A'_s f'_s = A_{s2} f_y \Rightarrow A'_s = A_{s2}$$

Therefore,

$$M_{n2} = A'_s f_y (d - d') \qquad (37)$$

– The strength of the concrete-steel couple is given by

$$M_{n1} = T_1 Z_1 \qquad (38)$$

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

$$M_{n1} = A_{s1} f_y \left(d - \frac{a}{2} \right) \qquad \text{assuming } f_s = f_y$$

$$A_s = A_{s1} + A_{s2} \Rightarrow A_{s1} = A_s - A_{s2}$$

since $A_{s2} = A'_s$, then

$$A_{s1} = A_s - A'_s$$

Therefore

$$M_{n1} = \left(A_s - A'_s \right) f_y \left[d - \frac{a}{2} \right] \qquad (39)$$

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Nominal Moment Capacity

    From Eqs. 37 and 39, the nominal moment capacity can be evaluated as

$$\begin{aligned} M_n &= M_{n1} + M_{n2} \\ &= \left(A_s - A'_s \right) f_y \left[d - \frac{a}{2} \right] + A'_s f_y \left(d - d' \right) \end{aligned} \qquad (40)$$

This equation is valid *only* if A'_s yields

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Eq. 40 is valid only if A_s' yields. Otherwise, the beam has to be treated as a singly reinforced beam neglecting the compression steel, or one has to find the actual stress f_s' in the compression reinforcement A_s' and use the actual force in the moment equilibrium equation.

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Determination of the Location of Neutral Axis:

$$c = \frac{a}{\beta_1} \qquad \rho = \frac{A_s}{bd} \quad \text{and} \quad \rho' = \frac{A_s'}{bd}$$

$$T = C_1 + C_2$$

$$A_s f_y = (0.85 f_c')ab + A_s' f_y$$

Therefore,

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{(\rho - \rho')f_y d}{0.85 f_c'} = \frac{A_{s1} f_y}{0.85 f_c' b}$$

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Location of Neutral Axis c

$$a = \frac{(A_s - A'_s)f_y}{0.85 f'_c b} = \frac{(\rho - \rho')f_y d}{0.85 f'_c} \quad (41)$$

$$c = \frac{a}{\beta_1} = \frac{(A_s - A'_s)f_y}{0.85\beta_1 f'_c b} = \frac{(\rho - \rho')f_y d}{0.85\beta_1 f'_c} \quad (42)$$

NOTE: if $f'_c \leq 4{,}000$ psi, then $\beta_1 = 0.85$, otherwise see next slide

Doubly Reinforced Sections

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - The value of β_1 may determined by

$$\beta_1 = \begin{cases} 0.85 & \text{for } f'_c \leq 4{,}000 \text{ psi} \\ 1.05 - 5\times10^{-5} f'_c & \text{for } 4{,}000 \text{ psi} < f'_c \leq 8{,}000 \text{ psi} \\ 0.65 & \text{for } f'_c > 8{,}000 \text{ psi} \end{cases} \quad (43)$$

Doubly Reinforced Sections

- Strain-Compatibility Check
  - For A'_s to yield, the strain ε'_s in the compression steel should be greater than or equal to the yield strain of reinforcing steel, which is

$$\varepsilon'_s = \frac{f_y}{E_s} \qquad (44)$$

  - The strain ε'_s can be calculated from similar triangles. Referring to Figure 24,

$$\varepsilon'_s = 0.003\left(\frac{c-d'}{c}\right) = 0.003\left(1-\frac{d'}{c}\right) \qquad (45)$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)
  - Substituting c of Eq. 42 into Eq. 45, gives

$$\varepsilon'_s = 0.003\left(1-\frac{d'}{c}\right) = 0.003\left[1-\frac{0.85\beta_1 f'_c d'}{(\rho-\rho')d f_y}\right] \qquad (46)$$

  - For compression steel to yield, the following condition must be satisfied:

$$\varepsilon'_s \geq \frac{f_y}{E_s} \quad \text{or} \quad \varepsilon'_s \geq \frac{f_y}{29\times10^6}$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)
  - The compression steel yields if

$$\varepsilon'_s \geq \frac{f_y}{29\times 10^6}$$

or

$$0.003\left[1 - \frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d}\right] \geq \frac{f_y}{29\times 10^6}$$

or

$$1 - \frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \geq \frac{f_y}{87{,}000}$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)

$$1 - \frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \geq \frac{f_y}{87{,}000}$$

or

$$-\frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \geq \frac{f_y}{87{,}000} - 1$$

or

$$-\frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \geq \frac{f_y - 87{,}000}{87{,}000}$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)

or

$$-\frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \geq \frac{f_y - 87{,}000}{87{,}000}$$

or

$$\frac{0.85\beta_1 f'_c d'}{(\rho - \rho')f_y d} \leq -\frac{f_y - 87{,}000}{87{,}000}$$

or

$$(\rho - \rho') \geq \frac{0.85\beta_1 f'_c d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)

If compression steel is to yield, then the following condition must be satisfied:

$$(\rho - \rho') \geq \frac{0.85\beta_1 f'_c d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right) \qquad (47)$$

Doubly Reinforced Sections

- Strain-Compatibility Check (cont'd)
  - If ε'_s is less than ε_y the stress in the compression steel, f'_s , can be computed as

$$f'_s = E_s \varepsilon'_s = 29\times10^6 \varepsilon'_s$$

or

$$f'_s = 29\times10^6 \times 0.003\left[1-\frac{0.85\beta_1 f'_c d'}{(\rho-\rho')f_y d}\right] \tag{48}$$

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress
  - The preceding equations are valid only if the compression steel has yielded when the beam reaches its ultimate strength.
  - In many cases, however, such as for wide, shallow beams reinforced with higher-strength steels, the yielding of compression steel may not occur when the beam reaches its ultimate capacity.

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress
  - It is therefore necessary to to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.
  - The development of these equations will be based on

$$\varepsilon_s' < \varepsilon_y \qquad (49)$$

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress
  - Development of the Equations for Condition II
    - Referring to Fig. 24,

$$T = C_1 + C_2$$
$$A_s f_y = (0.85 f_c')ba + f_s' A_s' \qquad (50)$$

- But

$$a = \beta_1 c \qquad (51)$$

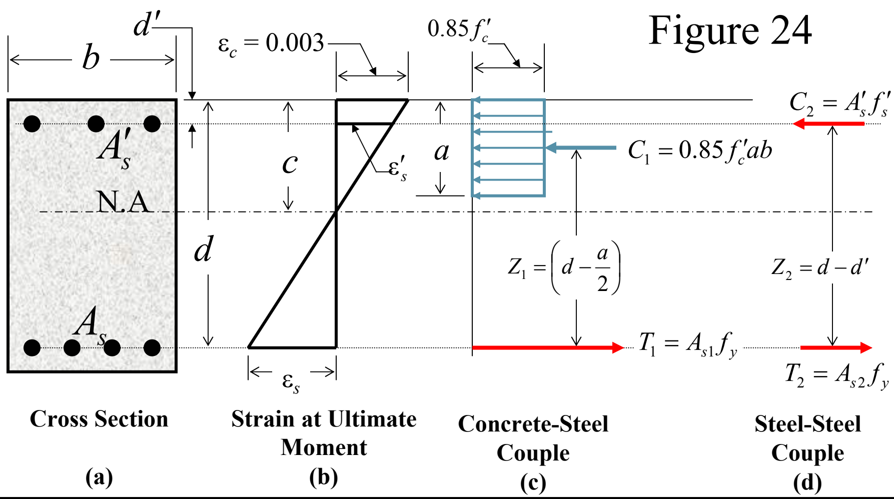
- and

$$f_s' = \varepsilon_s' E_s = \left[\frac{0.003(c - d')}{c}\right] E_s \qquad (52)$$

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress

Figure 24



Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress
  - Substituting Eqs 51 and 52 into Eq. 50, yields

$$A_s f_y = (0.85 f_c')b\beta_1 c + \left[\frac{0.003(c-d')}{c}\right]E_s A_s' \qquad (53)$$

  - Multiplying by *c*, expanding, and rearranging, yield

$$(0.85 f_c' b\beta_1)c^2 + (0.003E_s A_s' - A_s f_y)c - 0.003d'E_s A_s' = 0 \qquad (54)$$

  - If E_s is taken as 29 × 10[3] ksi, Eq. 54 will take the following form:

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress

  The following quadratic equation can be used to find *c* when $\varepsilon_s' < \varepsilon_y$:

$$\underbrace{\left(0.85 f_c' b \beta_1\right)}_{a} c^2 + \underbrace{\left(87 A_s' - A_s f_y\right)}_{b} c \underbrace{- 87 d' A_s'}_{c} = 0 \qquad (55)$$

Analogous to:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note:
The basic units are kips and inches in the equation.

Doubly Reinforced Sections

- Condition II: Compression Steel Below Yield Stress
  - In this case when $\varepsilon_s' < \varepsilon_y$, the nominal moment capacity of Eq. 40 becomes

$$M_n = M_{n1} + M_{n2}$$

$$= \left(A_s f_y - A_s' f_s'\right)\left[d - \frac{a}{2}\right] + A_s' f_s'\left(d - d'\right) \qquad (56)$$

where

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} \quad \text{and} \quad f_s' \text{ as given by Eq. 52}$$

Doubly Reinforced Sections

- ACI Code Ductility Requirements
  - The ACI Code limitation on ρ applies to doubly reinforced beams as well as to singly reinforced beams.
  - Steel ratio ρ shall not be less than given by

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} \qquad (57)$$

Doubly Reinforced Sections

- ACI Code Ductility Requirements
  - In order to ensure *tension-controlled* behavior, the ratio c/d_t should less than 0.375, that is

$$\frac{c}{d_t} \leq 0.375 \;\; \text{(preferably 0.30)} \qquad (58)$$

  - In this case, the strain ε_t in the tensile reinforcement is greater than 0.005, which can be computed by

$$\varepsilon_t = 0.003\left(\frac{d}{c} - 1\right) \qquad (59)$$

Doubly Reinforced Sections

- ACI-318-02 Code Strain Limits

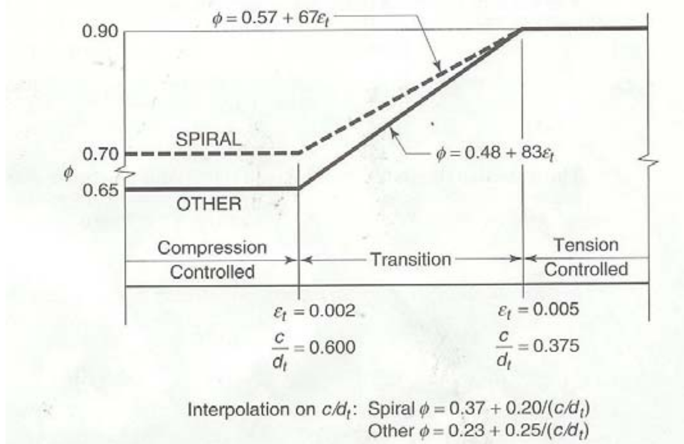
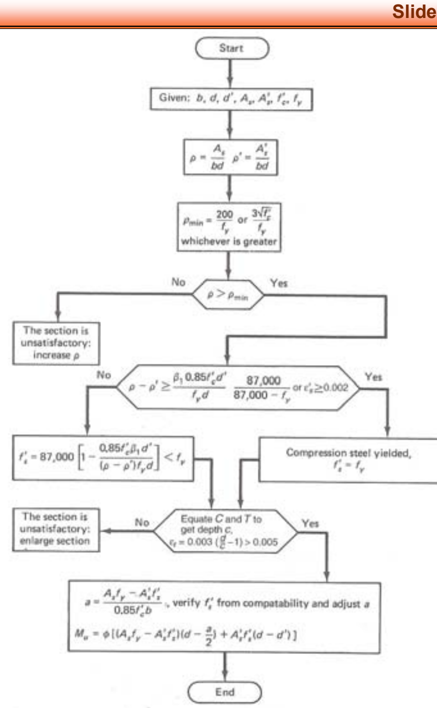


Figure 14. Strain Limit Zones and variation of Strength Reduction Factor ϕ

- **Figure 25.** Flow Chart For the Analysis of Doubly Reinforced Rectangular Beams



Trial-and-Adjustment Procedure for the Design of Doubly Reinforced Sections

1. *Midspan section*. The trial-and-adjustment procedure described for singly reinforced beam can be used if the section is rectangular.
2. *Support section*. The width b and the effective depth d are already known from part 1 together with the value of the external negative factored moment M_n.

Trial-and-Adjustment Procedure for the Design of Doubly Reinforced Sections

a) Find the strength M_{n1} singly reinforced section using the already established b and d dimensions of the section at midspan and a reinforcement area to give ε_t > 0.005.

b) From step (a), find $M_{n2} = M_n - M_{n1}$ and determine the resulting $A_{s2} = A'_s$. The total steel area at the tension side would be

$$A_s = A_{s1} + A'_s$$

c) Alternatively, determine how many bars are extended from the midspan to the support

Trial-and-Adjustment Procedure for the Design of Doubly Reinforced Sections

to give the A'_s to be used in calculating M_{n2}.

d) From step (c), find the value of $M_{n1} = M_n - M_{n2}$. Calculate A_{s1} for singly reinforced beam as the first part of the solution. Then determine total $A_s = A_{s1} + A'_s$. Verify that A_{s1} does not give $\varepsilon_t < 0.005$ if it is revised in the solution.

e) Check for the compatibility of strain in both alternatives to verify whether the compression steel yielded or not and use

Trial-and-Adjustment Procedure for the Design of Doubly Reinforced Sections

the corresponding stress in the steel for calculating the forces and moments.

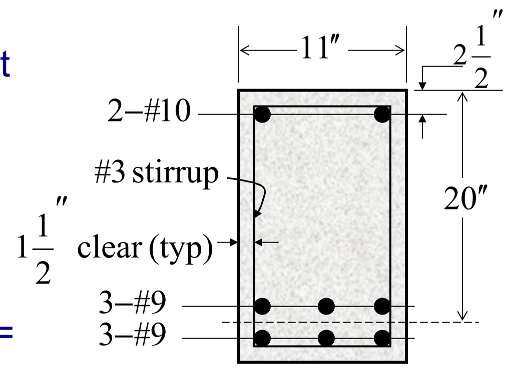
f) Check for satisfactory minimum reinforcement requirements.

g) Select the appropriate bar sizes.

Doubly Reinforced Beam Analysis

- Example 11: Compression steel yielded

Compute the practical moment capacity ϕM_n for the beam having a cross section as shown in the figure. Use f_c'= 3,000 psi and f_y = 60,000 psi.



Doubly Reinforced Beam Analysis

- Example 11 (cont'd)

Determine the values for A_s' and A_s:

From Table 6,

$$A_s' = \text{area of } 2\ \#10 = 2.54\ \text{in}^2$$

$$A_s = \text{area of } 6\ \#9 = 6.0\ \text{in}^2$$

Compute the steel ratio ρ:

$$\rho' = \frac{A_s'}{bd} = \frac{2.54}{11(20)} = 0.0115$$

$$\rho = \frac{A_s}{bd} = \frac{6}{11(20)} = 0.0273$$

Doubly Reinforced Beam Analysis

- Example 11 (cont'd)

Table 6. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number #3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Doubly Reinforced Beam Analysis

- Example 11 (cont'd)

Therefore,

$$A_{s2} = A'_s = 2.54 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 6.0 - 2.54 = 3.46 \text{ in}^2$$

$$(\rho - \rho') = 0.0273 - 0.0115 = 0.0158$$

Check whether compression steel yielded using Eq. 47

$$(\rho - \rho') \geq \frac{0.85\beta_1 f'_c d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

$$(\rho - \rho') \geq \frac{0.85(0.85)(3000)(2.5)}{60{,}000(20)}\left(\frac{87{,}000}{87{,}000 - 60{,}000}\right) = 0.0146$$

Doubly Reinforced Beam Analysis

- Example 11 (cont'd)

Therefore,

$$[(\rho - \rho') = 0.0158] > 0.0146$$

ductility is OK

The compression steel has yielded, and Eq. 40 for determining M_n can be used:

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{A_{s1} f_y}{0.85 f_c' b} = \frac{3.46(60)}{0.85(3)(11)} = 7.40 \text{ in.} \qquad \text{(Eq. 41)}$$

$$M_n = M_{n1} + M_{n2}$$

$$= (A_s - A_s')f_y \left[d - \frac{a}{2} \right] + A_s' f_y (d - d')$$

$$= 3.46(60)\left[20 - \frac{7.4}{2} \right] + 2.54(60)(20 - 2.5) = 6{,}050.9 \text{ in - k}$$

Doubly Reinforced Beam Analysis

- Example 11 (cont'd)

$$M_n = \frac{6{,}050.9}{12} \text{ ft - kips} = 504.2 \text{ ft - kips}$$

The practical moment capacity is evaluated as follows:

$$\phi M_u = 0.9(504.2) = 454 \text{ ft - kips}$$

Doubly Reinforced Beam Analysis

- <u>Example 12:</u> Compression steel not yielded

  Calculate the nominal moment strength M_n of the doubly reinforced section shown in the figure. Given:

$f'_c = 5000$ psi (normal - weight concrete)

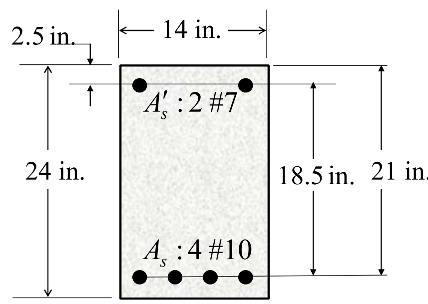
$f_y = 60{,}000$ psi

$d' = 2.5$ in.

$d_t = 21$ in.

$A_s = 4$ No. 10 bars

$A'_s = 2$ No. 7 bars



Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

  Determine the values for A'_s and A_s:

  From Table 6,

$$A_s = \text{area of } 4\ \#10 = 5.08\ \text{in}^2, \quad \rho = \frac{A_s}{bd} = \frac{5.08}{14(21)} = 0.0173$$

$$A'_s = \text{area of } 2\ \#7 = 1.20\ \text{in}^2, \quad \rho' = \frac{A'_s}{bd} = \frac{1.20}{14(21)} = 0.0041$$

Therefore,

$$A_s - A'_s = A_{s1} = 5.08 - 1.20 = 3.88\ \text{in}^2$$

$$(\rho - \rho') = 0.0173 - 0.0041 = 0.0132$$

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

Table 6. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number #3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

Check whether compression steel yielded using Eq. 47

$$(\rho - \rho') \geq \frac{0.85\beta_1 f'_c d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

$$(\rho - \rho') \geq \frac{0.85(0.80)(5000)(2.5)}{60{,}000(21)}\left(\frac{87{,}000}{87{,}000 - 60{,}000}\right) = 0.0217$$

Therefore,

$$[(\rho - \rho') = 0.0132] < 0.0217$$

and the compression steel did not yield and f'_s is less than f_y. Therefore use Eqs 55 and 56 to find M_n.

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

Using Eq. 55 to find c and consequently a:

$$\left(0.85 f'_c b \beta_1\right) c^2 + \left(87 A'_s - A_s f_y\right) c - 87 d' A'_s = 0$$

$$\left(0.85 f'_c b \beta_1\right) = 0.85 \times 5 \times 14 \times 0.80 = \boxed{47.6}$$

$$\left(87 A'_s - A_s f_y\right) = 87 \times 1.2 - 5.08 \times 60 = \boxed{-200.4}$$

$$87 d' A'_s = 87 \times 2.5 \times 1.2 = \boxed{261}$$

Therefore,

$$\boxed{47.6 c^2 - 200.4 c - 261 = 0}$$

⟵ Find c from quadratic Eq.

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

The solution to the quadratic equation is as follows:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$47.6 x^2 - 200.4 x - 261 = 0$$

$$x = \frac{-(-200.4) \pm \sqrt{(-200)^2 - 4(47.6)(-261)}}{2(47.6)} = \frac{200.4 \pm 299.49}{95.2}$$

$$x = -1.0409, \ 5.251$$

Therefore, take $c = 5.25$ in $\Rightarrow a = 5.25(0.80) = 4.2$ in.

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

Check ACI Code Requirements for minimum steel and strain limits:

$$\left[\text{Actual}\left(\rho-\rho'\right)=0.0132\right]>\left[\max\left(\frac{3\sqrt{f_c'}}{f_y},\frac{200}{f_y}\right)=0.0035\right] \quad \text{OK}$$

Using either Eq. 58 or 59, gives

$$\left[\frac{c}{d_t}=\frac{5.25}{21}=0.25\right]<0.375\ \text{(preferably 0.30)} \quad \text{OK}$$

or

$$\left[\varepsilon_t=0.003\left(\frac{d}{c}-1\right)=0.003\left(\frac{21}{5.25}-1\right)\right]=0.009>0.005 \quad \text{OK}$$

Doubly Reinforced Beam Analysis

- Table 7. Design Constants

f_c'	$\left[\frac{3\sqrt{f_c'}}{f_y}\geq\frac{200}{f_y}\right]$	ρ_b	Recommended Design Values: ρ	Recommended Design Values: R (ksi)
		f_y = 40,000 psi		
3000	0.0050	0.03712	0.0135	482.82
4000	0.0050	0.04949	0.0180	643.76
5000	0.0053	0.05823	0.0225	804.71
6000	0.0058	0.06551	0.0270	965.65
		f_y = 50,000 psi		
3000	0.0040	0.02753	0.0108	482.80
4000	0.0040	0.03671	0.0144	643.80
5000	0.0042	0.04318	0.0180	804.70
6000	0.0046	0.04858	0.0216	965.70
		f_y = 60,000 psi		
3000	0.0033	0.0214	0.0090	482.82
4000	0.0033	0.0285	0.0120	643.76
5000	0.0035	0.0335	0.0150	804.71
6000	0.0039	0.0377	0.0180	965.65
		f_y = 75,000 psi		
3000	0.0027	0.0155	0.0072	482.80
4000	0.0027	0.0207	0.0096	643.80
5000	0.0028	0.0243	0.0120	804.70
6000	0.0031	0.0274	0.0144	965.70

Doubly Reinforced Beam Analysis

- ACI-318-02 Code Strain Limits

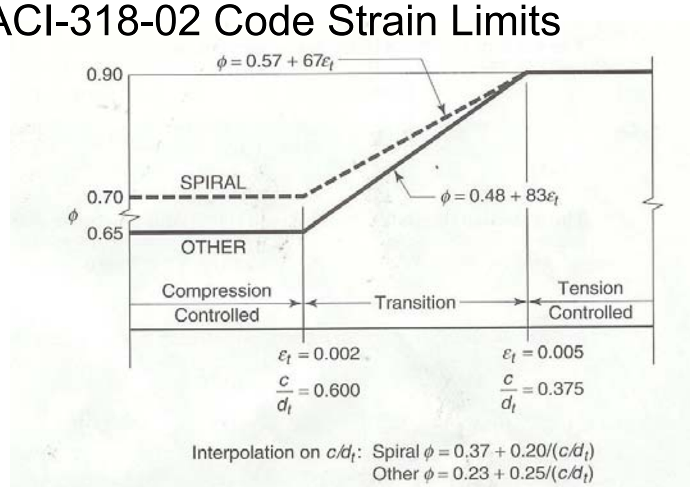


Figure 14. Strain Limit Zones and variation of Strength Reduction Factor ϕ

Doubly Reinforced Beam Analysis

- Example 12 (cont'd)

Since ε_t > 0.005, the strength reduction factor ϕ = 0.9. Therefore, the nominal moment strength M_n of the beam is computed using Eqs. 52 and 56 as follows:

$$f'_s = \left[\frac{0.003(c-d')}{c}\right]E_s = \left[\frac{0.003(5.25-2.5)}{5.25}\right]\times 29\times 10^3 = 45.57 \text{ ksi}$$

$$M_n = \left(A_s f_y - A'_s f'_s\right)\left[d - \frac{a}{2}\right] + A'_s f'_s (d - d')$$

$$= (5.08\times 60 - 1.2\times 45.57)\left[21 - \frac{4.2}{2}\right] + 1.2\times 45.57(21-2.5)$$

$$= 5{,}738.8 \text{ in-kips} = 478 \text{ ft-kips}$$ ANS.