CHAPTER

5b

Prentice Hall

REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

FLEXURE IN BEAMS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

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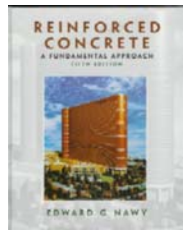

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ENCE 454 – Design of Concrete Structures

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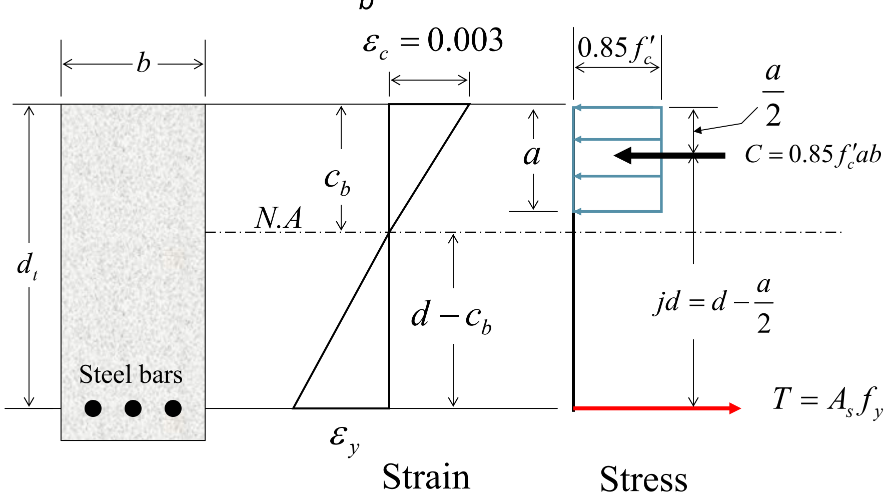
Reinforcement Ratio Limitations and Guidelines

Figure 12

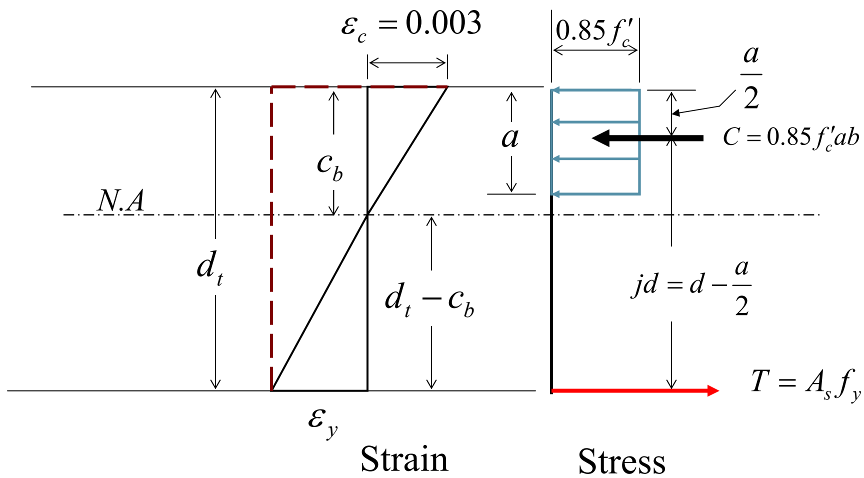
- Calculation of c_b For Balanced Beam



Reinforcement Ratio Limitations and Guidelines

Figure 13

- Calculation of c_b For Balanced Beam



Reinforcement Ratio Limitations and Guidelines

- Calculation of c_b For Balanced Beam

From similar triangles of the strain diagram of Figure 13:

$$\frac{c_b}{d_t} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} = \frac{0.003}{0.003 + \varepsilon_y} \qquad (15)$$

or

$$c_b = \frac{0.003}{0.003 + \varepsilon_y} d_t \qquad (16)$$

Reinforcement Ratio Limitations and Guidelines

- Calculation of c_b For Balanced Beam
  - Or

$$\frac{c_b}{d_t} = \frac{0.003}{0.003 + \varepsilon_y} \qquad (17)$$

  - Since $\varepsilon_y = f_y/E_s$ (where $E_s = 29 \times 10^6$ psi), hence,

$$\frac{c_b}{d_t} = \frac{0.003}{0.003 + \frac{f_y}{E_s}} = \frac{0.003}{0.003 + \frac{f_y}{29 \times 10^6}} \qquad (18)$$

Reinforcement Ratio Limitations and Guidelines

- Calculation of c_b For Balanced Beam

Eq. 18 can rewritten to give

$$\frac{c_b}{d_t} = \frac{87{,}000}{87{,}000 + f_y} \qquad (19)$$

c_b = balanced neutral axis depth
d_t = effective depth to extreme tensile reinforcement layer
f_y = yield point of steel grade

Reinforcement Ratio Limitations and Guidelines

- Reinforcement Ratio ρ_b at Balanced Condition

With Eq. 19, and $\rho_b = A_b/bd_t$

$$d_t = \frac{A_b}{b\rho_b} \qquad (20)$$

$$\frac{c_b}{d_t} = \frac{c_b}{\frac{A_b}{b\rho_b}} = \frac{87{,}000}{87{,}000 + f_y} \Rightarrow \rho_b = \frac{87{,}000}{87{,}000 + f_y}\left(\frac{A_b}{bc_b}\right) \qquad (21)$$

but $$T = C \Rightarrow A_b f_y = 0.85 f_c' ab = 0.85 f_c' \beta_1 c_b b \qquad (22)$$

Reinforcement Ratio Limitations and Guidelines

- Reinforcement Ratio ρ_b at Balanced Condition

From Eq. 22, the ratio A_b/b is given by

$$\frac{A_b}{b} = \frac{0.85 f_c' \beta_1 c_b}{f_y} \qquad (23)$$

Therefore, substituting into Eq. 21, gives

$$\rho_b = \frac{87{,}000}{87{,}000 + f_y}\left(\frac{A_b}{bc_b}\right) = \frac{87{,}000}{87{,}000 + f_y}\left(\frac{0.85 f_c' \beta_1 \cancel{c_b}}{f_y \cancel{c_b}}\right)$$

$$= \frac{87{,}000}{87{,}000 + f_y}\left(\frac{0.85 f_c' \beta_1}{f_y}\right) \qquad (24)$$

Reinforcement Ratio Limitations and Guidelines

- Steel Ratio Formula for Balanced Beam
  - Instead of using laborious techniques for determining the balanced steel of beam, the following formula can be used to determine the steel ratio ρ_b at the balance condition:

$$\rho_b = \frac{0.85 f'_c \beta_1}{f_y}\left(\frac{87{,}000}{f_y + 87{,}000}\right) \qquad (25)$$

where
f'_c = compressive strength of concrete (psi)
f_y = yield strength of steel (psi)
β_1 = factor that depends on f'_c as given by Eq. 3

Reinforcement Ratio Limitations and Guidelines

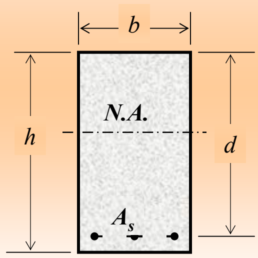
- Lower Limit for Steel Reinforcement
  - The ACI Code establishes a lower limit on the amount of tension reinforcement. The code states that where tensile reinforcement is required , the steel area A_s shall not be less than that given by

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad (26)$$

Note that for rectangular beam $b_w = b$

Reinforcement Ratio Limitations and Guidelines

- Steel Ratio
  - The steel ratio (sometimes called reinforcement ratio) is given by



$$\rho = \frac{A_s}{bd} \qquad (27)$$

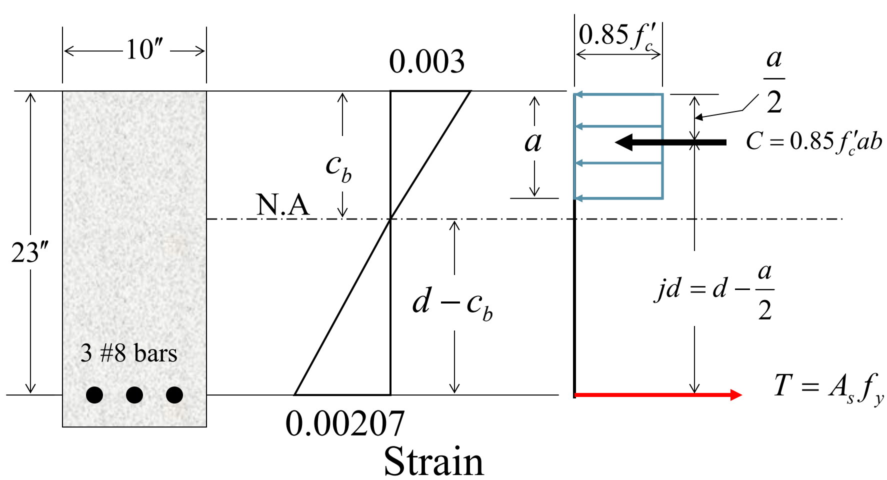
For ductile behavior, ACI Code recommends that

$\rho_{\max}$ ranges from 50 to 60% of ρ_b

Reinforcement Ratio Limitations and Guidelines

- Example 2 (cont'd)

Figure 14



Strain Limits Method for Analysis and Design

- The nominal flexural strength of a concrete member is reached when the net compressive strain in the extreme fibers reaches the ACI Code-assumed limit of 0.003 in./in.
- It also stipulates that when the tensile strain in the extreme tension steel ε_t is sufficiently large at a value equal or greater than 0.005 in./in.; the behavior is fully ***ductile***.

Strain Limits Method for Analysis and Design

- The reinforced concrete beam is characterized by ACI as
  - Tension-controlled
    - Ductile mode of failure, with ample warning of failure as denoted by excessive cracking and deflection (preferred)
  - Compression-controlled, or
    - Brittle mode of failure, with little warning of such impending failure
  - Transitional
    - Beams with small axial load and large moment.

Strain Limits Method for Analysis and Design

- ACI-318-02 Code Strain Limits
  - Compression-controlled limit:

$$\varepsilon_t = \frac{f_y}{E_s} = \frac{60{,}000}{29 \times 10^6} = 0.002$$

  - Tension-controlled limit:

$$\varepsilon_t = 0.005 \quad (\text{where } \rho/\rho_b = 0.63)$$

  - Transitional:

$$0.002 \le \varepsilon_t \le 0.005$$

Strain Limits Method for Analysis and Design

- ACI-318-02 Code Strain Limits

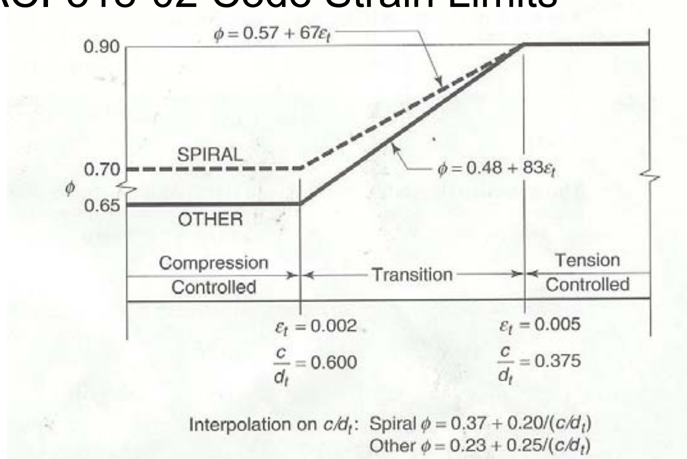


Figure 14. Strain Limit Zones and variation of Strength Reduction Factor ϕ

Strain Limits Method for Analysis and Design

- Strain Limits in Terms of Neutral Axis Depth

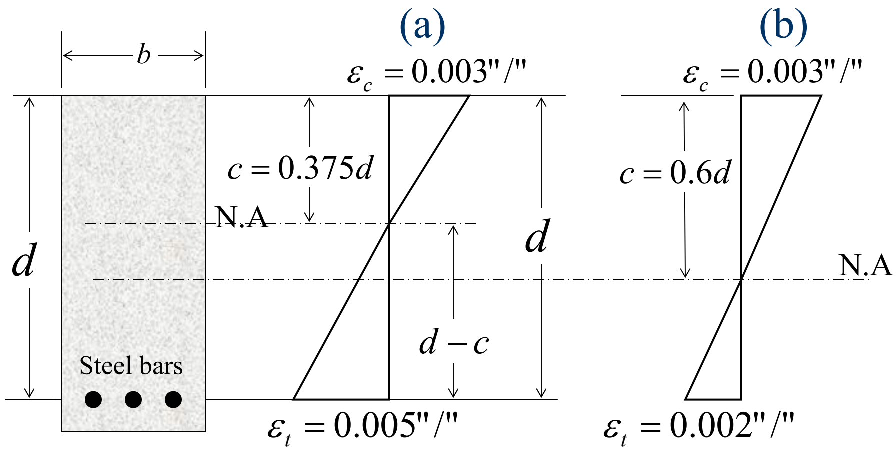


Figure 15. Strain Limits (a) Tension Controlled; (b) Compression Controlled

Strain Limits Method for Analysis and Design

- Variation of ϕ as a Function of Strain
  - Variation of the ϕ value for the range of strain between ε_t = 0.002 and ε_t = 0.005 can be linearly interpolated to give the following expression:

Tied Sections:

$$0.65 \le [\varphi = 0.48 + 83\varepsilon_t] \le 0.90 \qquad (28)$$

Spirally-Reinforced Sections:

$$0.70 \le [\varphi = 0.57 + 67\varepsilon_t] \le 0.90 \qquad (29)$$

Strain Limits Method for Analysis and Design

- Variation of ϕ as a Function of Neutral Axis Depth Ratio *c*/*d*
  - Eq. 18 can be expressed in terms c/d_t, where d_t is the effective depth of the layer of reinforcement closest to the tensile face of the section as follows:

Tied Sections:

$$0.65 \leq \left[\varphi = 0.23 + \frac{0.25}{c/d_t} \right] \leq 0.90 \qquad (30)$$

Spirally-Reinforced Sections:

$$0.70 \leq \left[\varphi = 0.37 + \frac{0.20}{c/d_t} \right] \leq 0.90 \qquad (31)$$

Strain Limits Method for Analysis and Design

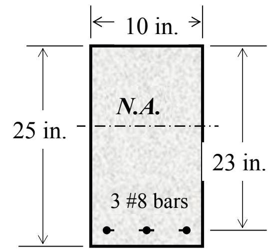
- Flexural Members with Axial Load
  - The ACI Code stipulates that for flexural members with axial load exceeding 0.10 $f'_c A_g$, the net tensile strain ε_t at nominal strength should not be less than

0.004

  - Therefore, in the transition zone of Figure 14, the minimum strain value in flexural members for determining the ϕ value is 0.004.

Strain Limits Method for Analysis and Design

- Example 4
  - Determine the amount of steel required to create a balanced condition for the beam shown, where f'_c = 4,000 psi. Assume A615 grade 60 steel that has a yield strength of 60 ksi and a modulus of elasticity = 29 × 10^6 psi. Also check the code requirement for ductile-type beam with the 3 #8 bars.



Strain Limits Method for Analysis and Design

- Example 4 (cont'd)

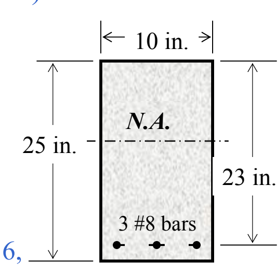
Area for No. 8 bar = 0.79 in^2 (see Table 1)

Therefore, $A_s = 3(0.79) = 2.37 \text{ in}^2$

The strain at which the steel yields is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29 \times 10^6} = 0.00207 \text{ in./in.}$$

In reference to the strain diagram of Fig. 16, and from similar triangles,



$$\frac{c_b}{0.003} = \frac{d - c_b}{0.00207}$$

Strain Limits Method for Analysis and Design

Table 1. ASTM Standard - English Reinforcing Bars

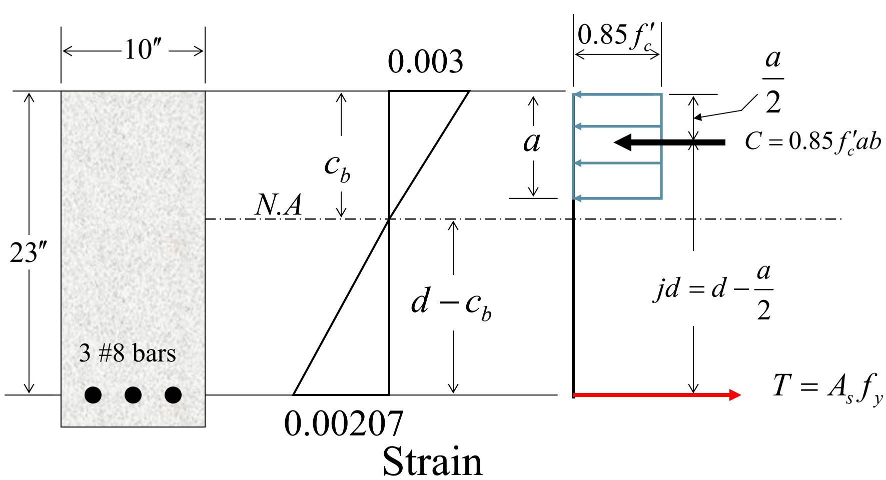
Bar Designation	Diameter in	Area in²	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Strain Limits Method for Analysis and Design

- Example 4 (cont'd)

Figure 16



Strain Limits Method for Analysis and Design

- Example 4 (cont'd)

$$\frac{c_b}{0.003} = \frac{23 - c_b}{0.00207}$$

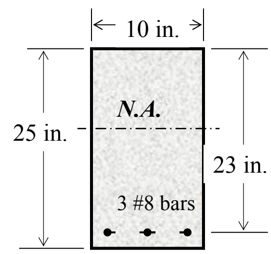
From which,

$$c_b = 13.6 \text{ in.}$$

Using Eqs. 3 and 4:

$$\beta_1 = 0.85 \text{ because } f_c' = 4{,}000 \text{ psi}$$

$$a = \beta_1 c = 0.85(13.6) = 11.6 \text{ in.}$$



Strain Limits Method for Analysis and Design

- Example 4 (cont'd)

$$C_b = 0.85 f_c' a_b b = 0.85(4)(11.6)(10) = 394.4 \text{ kips}$$

$$C_b = T_b = A_{sb} f_y$$

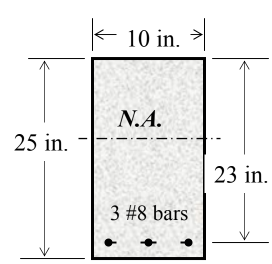
Therefore,

$$A_{sb} = \frac{T_b}{f_y} = \frac{394.4}{60} = 6.57 \text{ in}^2$$

Hence, required steel for balanced condition = 6.57 in^2

Or from Eq. 25,

$$\rho_b = \frac{0.85 f_c' \beta_1}{f_y}\left(\frac{87{,}000}{f_y + 87{,}000}\right) = \frac{0.85(4000)(0.85)}{60{,}000}\left(\frac{87{,}000}{60{,}000 + 87{,}000}\right) = 0.02851$$



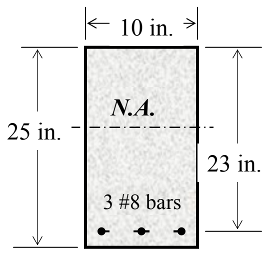
Strain Limits Method for Analysis and Design

- Example 4 (cont'd)

$$\rho_b = \frac{A_{sb}}{db} \Rightarrow A_{sb} = \rho_b db = 0.02851(23)(10) = 6.56 \text{ in}^2 \leftarrow \text{Same answer}$$

$$\rho = \frac{A_s}{db} = \frac{2.37}{23(10)} = 0.0103$$

$$\frac{\rho}{\rho_b} = \frac{0.0103}{0.02851} = 0.3614 < 0.63 \quad \underline{\text{OK}}$$



Also, we can check ε_t to make sure it is equal or greater than 0.005, or c/d_t is equal or less than 0.375 (see Figure 14)

Analysis of Singly Reinforced Rectangular Beams for Flexure

- Variables that need to be found or answered include the following:
  1. Find the strength ϕM_n.
  2. Check the adequacy of a given beam, or
  3. Find an allowable load that the beam can carry.
- The flow chart of Figure 17 can be used for the analysis of a reinforced concrete beam.

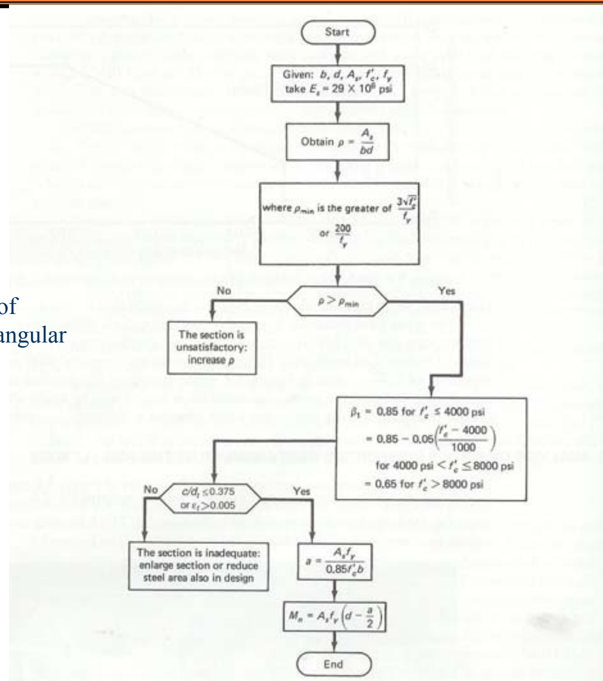


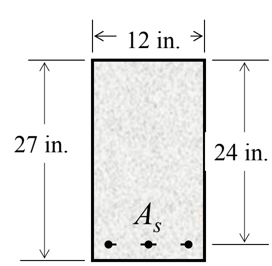
Figure 17.
Flowchart for Analysis of Singly Reinforced Rectangular Beams in Bending

Analysis of Singly Reinforced Rectangular Beams for Flexure

- Example 5 (to be discussed in class)

  For the beam cross-section shown in the figure, determine whether the failure of the beam will be initiated by crushing of concrete or yielding of steel:

  f'_c = 4000 psi, f_y = 60,000 psi, and A_s = 10 in^2



Analysis of Singly Reinforced Rectangular Beams for Flexure

- Example 6 (to be discussed in class)

Compute the nominal moment strength of the beam section shown in the figure:

f'_c = 6000 psi, f_y = 60,000 psi.