Solution to Homework Set #4

ENCE 454 – Design of Concrete Structures - SPRING 2004

Assigned T, 3/2   Due T, 3/9

Problem 1:

A reinforced concrete beam of rectangular cross section is reinforced for moment only and subjected to a shear V_u of 9000 lb. Beam width b = 12 in., d = 7.25 in., f_c' = 3000 psi, and f_y = 60,000 psi. Is the beam satisfactory for shear? Why? Justify your answer.

***** SOLUTION *****

Approximate h:

$h = d + 1.5 + 0.5 = 7.25 + 1.5 + 0.5 = 9.25$ in. < 10.0 in. $\Rightarrow$ beam is considered shallow.

$$\max V_u = \phi V_c = \phi \lambda 2\sqrt{f_c'} b_w d = 0.75(1)(2)\sqrt{3000}(12)(7.25) = 7{,}148 \text{ lb}$$

7,148 lb < 9000 lb $\Rightarrow$ Beam is N.G. in shear

Problem 2:

Assume that the beam of Problem 1 has an effective depth of 18 in. and is reinforced with No. 3 single-loop stirrups spaced at 10 in. on center. Determine the maximum shear V_u permissible.

***** SOLUTION *****

$$V_u \le \phi(V_c + V_s)$$

$$V_c = \lambda 2\sqrt{f_c'} b_w d = (1)(2)\sqrt{3000}(12)(18) = 23{,}661.61 \text{ lb} = 23.66 \text{ kips}$$

$$V_s = \frac{A_v f_y d}{s} = \frac{0.22(60)(18)}{10} = 23.76 \text{ kips}$$

Therefore,

$\max\ V_u = \phi(V_c + V_s) = 0.75\,(23.66 + 23.76) =$ **35.57 kips**

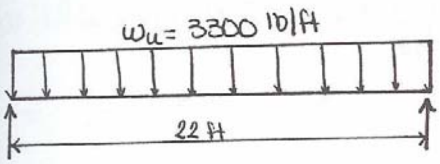
Problem 3:

Textbook: 6.1

***** SOLUTION *****

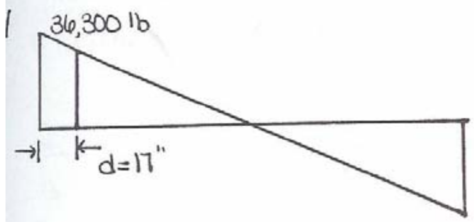
b_w = 12 in. (305 mm)
d = 17 in. (432 mm)
h = 20 in. (508 mm)
A_s = 6.0 in.^{2} (3780 mm^2)
f_c' = 4000 psi (27.6 MPa), normal-weight concrete
f_y = 60.000 psi (413.7 MPa)

Assume that no torsion exists.



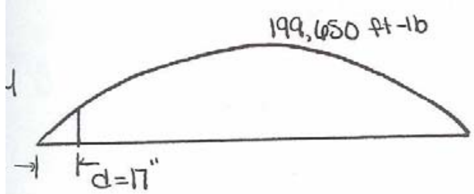
$$\text{self-weight} = \frac{12 \times 20}{144} \times 150 = 250 \text{ lb/ft}$$

$$w_u = 1.2(250+900) + 1.6(1200) = 3300 \text{ lb/ft}$$



$$V_u \text{ (at support)} = \frac{(3300)(22)}{2} = 36{,}300 \text{ lb.}$$

$$V_u \text{ (at d)} = 36{,}300 - 3300(17/12) = 31{,}625 \text{ lb.}$$



$$M_u \text{ (at d)} = 36{,}300(17/12) - \frac{3300(17/12)^2}{2} = 48{,}113 \text{ ft-lb} = 577{,}363 \text{ in-lb.}$$

Simplified Method :

$V_c = 2\lambda\sqrt{f'_c}\, b_w d = 2(1.0)\sqrt{4{,}000}\,(12)(17) = 25{,}804$ lb.

$V_n = \dfrac{V_u}{\phi = 0.75} = \dfrac{31{,}625}{0.75} = 42{,}167$ lb.

$V_n > \frac{1}{2} V_c$ ∴ shear reinforcement is necessary

$V_s = V_n - V_c = 42{,}167 - 25{,}804 = 16{,}363$ lb.

$\dfrac{A_v}{s} = \dfrac{V_s}{f_y d} = \dfrac{16{,}363}{(60{,}000)(17)} = 0.016\ \text{in}^2/\text{in}$

$\min \dfrac{A_v}{s} = \dfrac{50 b_w}{f_y} = \dfrac{50(12)}{60{,}000} = 0.01 < 0.016$ O.K

$\dfrac{A_v}{s} = 0.016$ in control

Try No. 3 stirrup , $A_v = 2(0.11) = 0.22\ \text{in}^2$

$s = \dfrac{A_v}{(A_v/s)} = \dfrac{0.22}{0.016} = 13.8$ in

$V_s = 16{,}363 < 4\sqrt{f'_c}\, bd = 4\sqrt{4000}\,(12)(17) = 51{,}608$ lb

∴ $s_{max} = \min\left\{ \dfrac{d}{2} = \dfrac{17}{2} = 8.5,\ 24 \right\} = 8.5\text{in} < 13.8\text{ in}$

⇒ Use No. 3 stirrups @ 8.5 in c-c .

Refined Method:

$$V_c = 1.9 b_w d \sqrt{f_c'} + 2500 \rho_w \frac{V_u d}{M_u} b_w d \not> 3.5 b_w d \sqrt{f_c'}$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{6}{12 \times 17} = 0.0294$$

$$\frac{V_u d}{M_u} = \frac{31,625 (17)}{577,363} = 0.93 \not> 1.0 \qquad \underline{\text{o.k}}$$

$$V_c = 1.9(12)(17)\sqrt{4000} + 2500(0.0294)(0.93)(12)(17) \not> 3.5(12)(17)\sqrt{4000}$$

$$= 38,458 \text{ lb} \not> 45,157 \text{ lb} \qquad \therefore V_c = 38,458 \text{ lb.}$$

$V_n > \frac{1}{2} V_c$ $\therefore$ shear reinforcement is required

$$V_s = V_n - V_c = 42,167 - 38,458 = 3,709 \text{ lb}$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{3709}{(60,000)(17)} = 0.0036 \text{ in}^2/\text{in}$$

$$\min \frac{A_v}{s} = 0.01 \text{ in}^2/\text{in} \longleftarrow \text{controls}$$

Try No. 3 stirrups, $A_v = 2(0.11) = 0.22 \text{ in}^2$

$$s = \frac{A_v}{(A_v/s)} = \frac{0.22}{0.01} = 22 \text{ in}$$

$$V_s = 3,709 \text{ lb} < 4\sqrt{f_c'}\, bd = 51,608 \text{ lb}$$

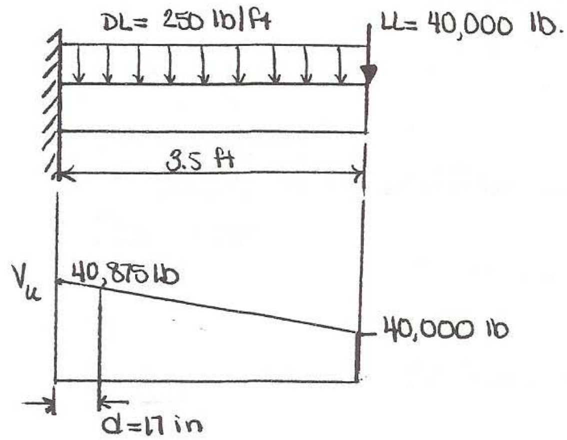
$$\therefore s_{max} = 8.5 \text{ in}$$

$\Rightarrow$ Use No. 3 stirrups @ 8.5 in c-c.

Problem 4:

Textbook: 6.3

***** SOLUTION *****



$$\text{Self-weight} = \frac{10 \times 20}{144} \times 150 = 208 \text{ lb/ft}$$

$$DL = 1.2\,(208) = 250 \text{ lb/ft}$$

$$LL = 1.6\,(25{,}000) = 40{,}000 \text{ lb.}$$

$$V_u \text{ at support} = 250 \times 3.5 + 40{,}000 = 40{,}875 \text{ lb}$$

$$V_u \text{ at } d = (40{,}875 - 40{,}000)\left(\frac{3.5 \times 12 - 17}{3.5 \times 12}\right) + 40{,}000 = 40{,}521 \text{ lb.}$$

$$V_n = \frac{V_u}{\phi = 0.75} = \frac{40{,}521}{0.75} = 54{,}028 \text{ lb.}$$

$$V_c = 2\lambda\sqrt{f'_c}\,b_w d = 2(1.0)\sqrt{3000}\,(10)(17) = 18{,}622 \text{ lb.}$$

$V_n > V_c/2$ $\therefore$ stirrups are required

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{54{,}028 - 18{,}622}{(60{,}000)(17)} = 0.035 \text{ in}^2/\text{in}$$

$$\min \frac{A_v}{s} = \frac{50 b_w}{f_y} = \frac{50(12)}{60{,}000} = 0.01 \text{ in}^2 < 0.035 \quad \therefore \frac{A_v}{s} = 0.035 \text{ in}^2/\text{in}$$

Try No. 4 stirrup, $A_v = 2(0.2) = 0.40\ in^2$

$$s = \frac{A_v}{(A_v/s)} = \frac{0.40}{0.035} = 11.4\ in.$$

$V_s = 54{,}028 - 18{,}622 = 35{,}406\ lb. < 4\sqrt{3000}(10)(17) = 37{,}245\ lb.$

$$\therefore\ s_{max} = \min\left\{\frac{d}{2} = 8.5,\ 24\ in\right\} = 8.5\ in.$$

$\Rightarrow$ Use No. 4 stirrups @ 8.5 in c-c.