Solution to Homework Set #3

ENCE 454 – Design of Concrete Structures - SPRING 2004

Assigned T, 2/24 Due T, 3/2

Problem 1:

Textbook: 5.7 (Part b only)

***** SOLUTION *****

b) Given

$b = 12$ in $\quad A_s = 4\ \#9 = 4.0\ in^2 \quad f'_c = 6{,}000$ psi

$h = 24$ in $\quad A'_s = 2\ \#8 = 1.58\ in^2 \quad f_y = 60{,}000$ psi

$d_t = 21.5$ in $\quad \beta_1 = 0.75$

$d' = 2.5$ in.

Assume compression steel has yielded.

$$a = \frac{(4 - 1.58)(60{,}000)}{0.85(6{,}000)(12)} = 2.37 \text{ in.}$$

$$c = \frac{a}{0.75} = \frac{2.37}{0.75} = 3.16 \text{ in.}$$

$$\varepsilon'_s = 0.003\left(\frac{3.16 - 2.5}{3.16}\right) = 0.00063 \text{ in/in.} < 0.002 \text{ in/in.}$$

$\therefore$ Compression steel has not yielded.

First Trial

$$f'_s = \varepsilon'_s E_s = (0.00063)(29 \times 10^6) = 18{,}245 \text{ psi}$$

$$a = \frac{(4)(60{,}000) - (1.58)(18{,}245)}{0.85(6{,}000)(12)} = 3.45 \text{ in.}$$

$$c = \frac{3.45}{0.75} = 4.60 \text{ in.}$$

$$\varepsilon'_s = 0.003\left(\frac{4.60 - 2.5}{4.60}\right) = 0.00137 \text{ in/in.}$$

Second Trial

$$f'_s = (0.00137)(29 \times 10^6) = 39{,}725 \text{ psi}$$

$$a = \frac{(4)(60{,}000) - (1.58)(39{,}725)}{0.85(6{,}000)(12)} = 2.90 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.90}{0.75} = 3.86 \text{ in.}$$

19

$$\varepsilon_s' = 0.003\left(\frac{3.86-2.5}{3.86}\right) = 0.00106 \text{ in/in}.$$

After several trials: $f_s' = 34{,}878$ psi $\quad a = 3.02$ in $\quad c = 4.03$ in.

$$\varepsilon_s' = 0.003\left(\frac{4.03-2.5}{4.03}\right) = 0.0011 \text{ in/in}.$$

$$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{21.5-4.03}{4.03}\right) = 0.0130 \text{ in/in} > 0.002 \text{ in/in}$$

$\therefore$ Tension steel has yielded.

$0.0130 \text{ in/in} > 0.005 \text{ in/in}$ $\quad \therefore$ Tension-controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{4}{(12)(21.5)} = 0.016$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0039 < 0.016 \quad \therefore \text{ satisfies ACI Code}$$

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s' (d - d')$$

$$= [(4)(60{,}000) - (1.58)(34{,}878)]\left(21.5 - \frac{3.02}{2}\right) + (1.58)(34{,}878)(21.5-2.5)$$

$$M_n = 4{,}742{,}940 \text{ in-lb}$$

$$\phi M_n = (0.90)(4{,}742{,}940) = 4{,}268{,}646 \text{ in-lb}.$$

Problem 2:

Textbook: 5.9

***** SOLUTION *****

5.9. Solve Problem 5.3 if two No. 6 bars are added as compression reinforcement.

Solution :

Given

$b = 10$ in. $\quad A_s = 3\#0.9 = 3.0\ in^2 \quad f'_c = 4{,}000$ psi

$h = 20$ in. $\quad A'_s = 2\#0.6 = 0.88\ in^2 \quad f_y = 60{,}000$ psi

$d_t = 17$ in. $\quad \beta_1 = 0.85$

$d' = 2.5$ in.

assume compression steel has yielded :

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b} = \frac{(3 - 0.88)(60{,}000)}{0.85(4{,}000)(10)} = 3.74 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.74}{0.85} = 4.40 \text{ in.}$$

$$\varepsilon'_s = 0.003\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{4.40 - 2.5}{4.40}\right) = 0.00130 \text{ in/in.} < 0.002 \text{ in/in.}$$

∴ Compression steel has not yielded.

First Trial

$$f'_s = \varepsilon'_s E_s = (0.00133)(29 \times 10^6) = 37{,}584 \text{ psi}$$

$$a = \frac{(3)(60{,}000) - (0.88)(37{,}584)}{0.85(4{,}000)(10)} = 4.32 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{4.32}{0.85} = 5.08 \text{ in.}$$

$$\varepsilon'_s = 0.003\left(\frac{5.08 - 2.5}{5.08}\right) = 0.00152 \text{ in/in.}$$

Second Trial

$$f_s' = (0.00152)(29\times10^6) = 44,218 \text{ psi}$$

$$a = \frac{(3)(60,000) - (0.88)(44,218)}{0.85(4,000)(10)} = 4.15 \text{ in.}$$

$$c = \frac{4.15}{0.85} = 4.88 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.88-2.5}{4.88}\right) = 0.00146 \text{ in/in.}$$

After several trials: $f_s' = 42,935$ psi $a = 4.18$ in. $c = 4.92$ in.

$$\epsilon_s' = 0.003\left(\frac{4.92-2.5}{4.92}\right) = 0.0015 \text{ in/in.}$$

$$\epsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{17-4.92}{4.92}\right) = 0.0074 \text{ in/in.} > 0.002 \text{ in/in}$$

$\therefore$ Tension steel has yielded

$0.0074 > 0.005$ $\therefore$ Tension-controlled $\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(10)(17)} = 0.018$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0033 < 0.018 \quad \therefore \text{ Satisfies ACI code}$$

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s' (d - d')$$
$$= [(3)(60,000) - (0.88)(42,935)]\left(17 - \frac{4.18}{2}\right) + (0.88)(42,935)(17-2.5)$$

$$M_n = 2,668,105 \text{ in-lb.}$$

$$M_u = \frac{M_n}{\phi=0.90} = 2,401,295 \text{ in-lb.} = \frac{w_u \ell^2}{8}$$

$$w_u = \frac{(2,401,295)(8)}{(26*12)^2} = 197.3 \text{ lb/in} = 2368 \text{ lb/ft.}$$

From Problem 5.3, Self-weight = 208.3 lb/ft.

$$w_u = 1.2\,DL + 1.6\,LL$$

$$2368 = 1.2(208.3) + 1.6\,LL$$

$$LL = 1323 \text{ lb/ft.}$$

Problem 3:

Textbook: 5.10 (Part a only)

***** SOLUTION *****

Given:

$b_w = 12$ in. $\quad A_s = 3$ No. 9 $= 3.0$ in^2

$b = 30$ in. $\quad f'_c = 4{,}000$ psi

$h_f = 3$ in. $\quad f_y = 60{,}000$ psi

$d_t = 18$ in. $\quad \beta_1 = 0.85$

Assume as a rectangular section:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3(60{,}000)}{0.85(4000)(30)} = 1.76 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.76}{0.85} = 2.08 \text{ in.} < h_f = 3 \text{ in.} \quad \textbf{∴ Treat as rectangular section}$$

Problem 4:

Calculate the nominal moment strength M_n and the design moment M_u of the section shown in part (b) for Problem 5.10 of your textbook.

***** SOLUTION *****

$a = 2.94$ in.

$A_s = 5.0$ in^2

$d_t = 27$ in.

$$M_n = 5(60{,}000)\left(27 - \frac{2.94}{2}\right) = 7{,}659{,}000 \text{ in - lb}$$

$$M_u = 0.90 M_n = 6{,}893{,}100 \text{ in - lb}$$