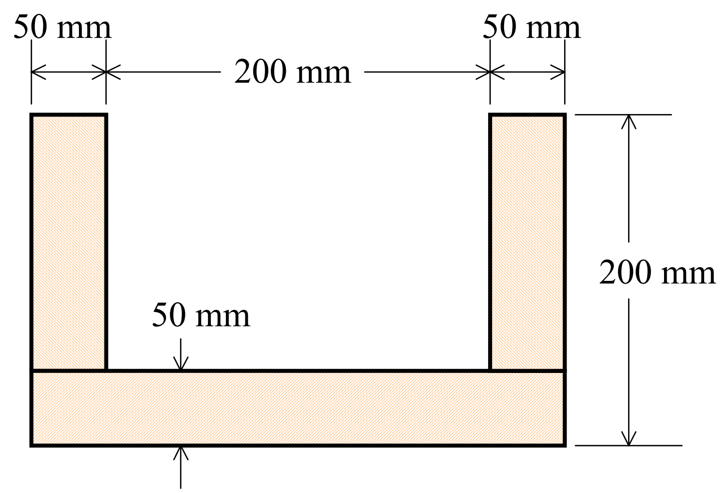
Solution to Homework Set #1

ENCE 454 – Design of Concrete Structures - SPRING 2004

Assigned T, 2/10 Due T, 2/17

Problem 1:

Determine the maximum flexural stress produced by the resisting moment M_r of -15 kN-m if the beam has the cross section shown in the figure.



****** **SOLUTION** ******

$$y_C = \frac{\sum yA}{\sum A} = \frac{50(200)(100) + 50(200)25 + 50(200)(100)}{3[50(100)]} = 75\text{ mm} \quad \text{(from bottom)}$$

$$I = \frac{1}{3}(100)(125)^3 + \frac{1}{3}(300)(75)^3 - \frac{1}{3}(200)(25)^3 = 106.25\times10^6\text{ mm}^4 = 106.25\times10^{-6}\text{ m}^4$$

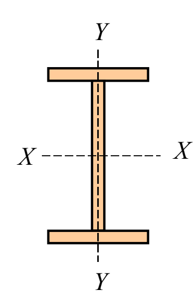
Therefore,

$$\sigma = -\frac{My}{I} = -\frac{-(15\times10^3)(125\times10^{-3})}{106.25\times10^{-6}} = 17.647\times10^6\ \frac{\text{N}}{\text{m}^2} = 17.65\text{ MPa} \quad \text{(Tension)}$$

Problem 2:

A wide-flange beam will be used to resist a bending moment M_r of 55,000 ft-lb. If the maximum flexural stress must not exceed 18,000 psi, select the most economical wide-flange section listed in Table B-1.

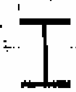
****** **SOLUTION** ******



$$\sigma = \frac{Mc}{I} = \frac{M}{S} \quad \Rightarrow \text{S} = \frac{M}{\sigma} = \frac{55{,}000(12)}{18{,}000} = 36.7\text{ in}^2$$

From the table provided:

Use a <u>W16 × 26 widw-flange beam</u>



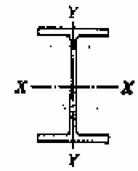
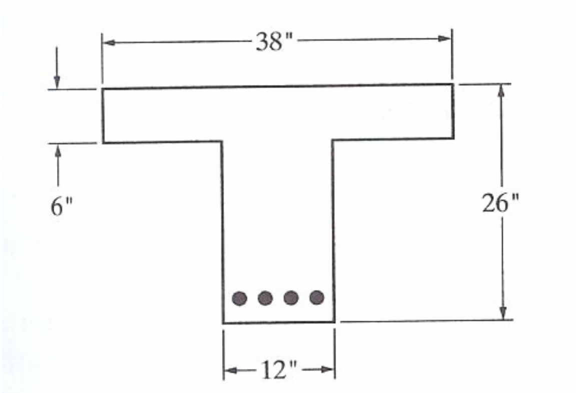


TABLE B-1 Wide-Flange Beams (U.S. Customary Units)

Designation	Area (in.²)	Depth (in.)	FLANGE Width (in.)	FLANGE Thickness (in.)	Web Thickness (in.)	AXIS X–X I (in.⁴)	AXIS X–X S (in.³)	AXIS X–X r (in.)	AXIS Y–Y I (in.⁴)	AXIS Y–Y S (in.³)	AXIS Y–Y r (in.)
W36 × 230	67.6	35.90	16.470	1.260	0.760	15000	837	14.9	940	114	3.7
× 160	47.0	36.01	12.000	1.020	0.650	9750	542	14.4	295	49.1	2.5
W33 × 201	59.1	33.68	15.745	1.150	0.715	11500	684	14.0	749	95.2	3.5
× 152	44.7	33.49	11.565	1.055	0.635	8160	487	13.5	273	47.2	2.4
× 130	38.3	33.09	11.510	0.855	0.580	6710	406	13.2	218	37.9	2.3
W30 × 132	38.9	30.31	10.545	1.000	0.615	5770	380	12.2	196	37.2	2.2
× 108	31.7	29.83	10.475	0.760	0.545	4470	299	11.9	146	27.9	2.1
W27 × 146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.2
× 94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.1
W24 × 104	30.6	24.06	12.750	0.750	0.500	3100	258	10.1	259	40.7	2.9
× 84	24.7	24.10	9.020	0.770	0.470	2370	196	9.79	94.4	20.9	1.9
× 62	18.2	23.74	7.040	0.590	0.430	1550	131	9.23	34.5	9.80	1.3
W21 × 101	29.8	21.36	12.290	0.800	0.500	2420	227	9.02	248	40.3	2.8
× 83	24.3	21.43	8.355	0.835	0.515	1830	171	8.67	81.4	19.5	1.8
× 62	18.3	20.99	8.240	0.615	0.400	1330	127	8.54	57.5	13.9	1.7
W18 × 97	28.5	18.59	11.145	0.870	0.535	1750	188	7.82	201	36.1	2.6
× 76	22.3	18.21	11.035	0.680	0.425	1330	146	7.73	152	27.6	2.6
× 60	17.6	18.24	7.555	0.695	0.415	984	108	7.47	50.1	13.3	1.6
W16 × 100	29.4	16.97	10.425	0.985	0.585	1490	175	7.10	186	35.7	2.5
× 67	19.7	16.33	10.235	0.665	0.395	954	117	6.96	119	23.2	2.4
× 40	11.8	16.01	6.995	0.505	0.305	518	64.7	6.63	28.9	8.25	1.5
× 26	7.68	15.69	5.500	0.345	0.250	301	38.4	6.26	9.59	3.49	1.1
W14 × 120	35.3	14.48	14.670	0.940	0.590	1380	190	6.24	495	67.5	3.7
× 82	24.1	14.31	10.130	0.855	0.510	882	123	6.05	148	29.3	2.4
× 43	12.6	13.66	7.995	0.530	0.305	428	62.7	5.82	45.2	11.3	1.8
× 30	8.85	13.84	6.730	0.385	0.270	291	42.0	5.73	19.6	5.82	1.4
W12 × 96	28.2	12.71	12.160	0.900	0.550	833	131	5.44	270	44.4	3.1
× 65	19.1	12.12	12.000	0.605	0.390	533	87.9	5.28	174	29.1	3.0
× 50	14.7	12.19	8.080	0.640	0.370	394	64.7	5.18	56.3	13.9	1.9
× 30	8.79	12.34	6.520	0.440	0.260	238	38.6	5.21	20.3	6.24	1.5
W10 × 60	17.6	10.22	10.080	0.680	0.420	341	66.7	4.39	116	23.0	2.5
× 45	13.3	10.10	8.020	0.620	0.350	248	49.1	4.33	53.4	13.3	2.0
× 30	8.84	10.47	5.810	0.510	0.300	170	32.4	4.38	16.7	5.75	1.3
× 22	6.49	10.17	5.750	0.360	0.240	118	23.2	4.27	11.4	3.97	1.
W8 × 40	11.7	8.25	8.070	0.560	0.360	146	35.5	3.53	49.1	12.2	2.
× 31	9.13	8.00	7.995	0.435	0.285	110	27.5	3.47	37.1	9.27	2.
× 24	7.08	7.93	6.495	0.400	0.245	82.8	20.9	3.42	18.3	5.63	1.
× 15	4.44	8.11	4.015	0.315	0.245	48.0	11.8	3.29	3.41	1.70	0.
W6 × 25	7.34	6.38	6.080	0.455	0.320	53.4	16.7	2.70	17.1	5.61	1.
× 16	4.74	6.28	4.030	0.405	0.260	32.1	10.2	2.60	4.43	2.20	0.
W5 × 16	4.68	5.01	5.000	0.360	0.240	21.3	8.51	2.13	7.51	3.00	1.
W4 × 13	3.83	4.16	4.060	0.345	0.280	11.3	5.46	1.72	3.86	1.90	1.

Problem 3:
The unit weight of normal weight reinforced concrete is commonly assumed to be 150 lb/ft^3. Find the weight per lineal foot (lb/ft) for normal weight reinforced concrete beam that (a) has a rectangular cross section 16 in. wide and 28 in. deep, (b) has a cross section as shown in the figure.

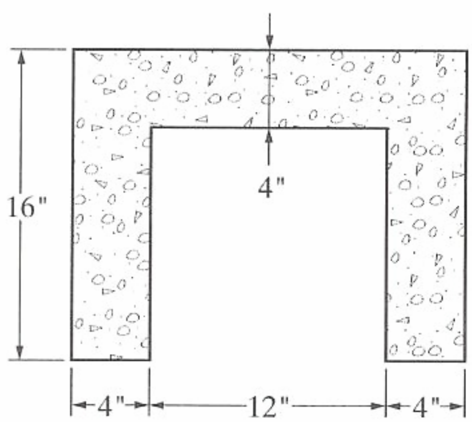


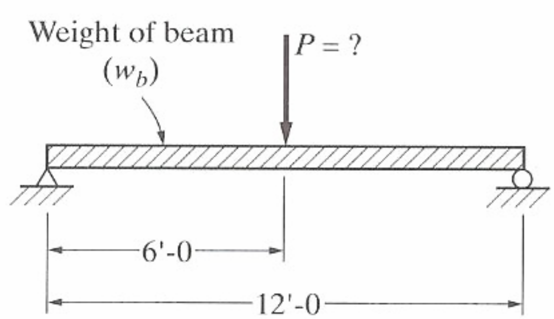
***** SOLUTION *****

(a) $\frac{16(28)}{144}(150) = 467$ #/FT

(b) $\frac{12(26-6)}{144}(150) + \frac{6(38)}{144}(150) = 488$ #/FT

Problem 4:
The plain concrete beam shown is used on a 12-ft simple span. The concrete is normal weight (145 pcf) with f_c' = 3000 psi. (a) Calculate the cracking moment, (b) Calculate the value of the concentrated load *P* at midspan that would cause the concrete beam to crack. (include the weight of the beam).





*** SOLUTION ***

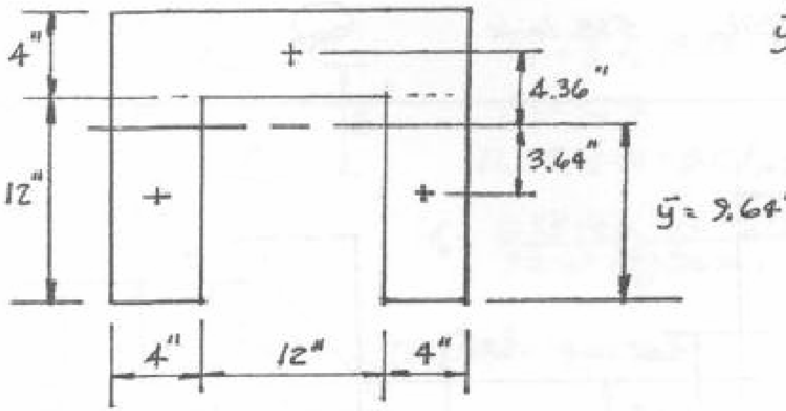
$f'_c = 3000$ psi

$f_r = \dfrac{7.5\sqrt{3000}}{1000} = 0.411$ ksi

$\bar{y} = \dfrac{\Sigma Ay}{\Sigma A}$

$= \dfrac{2(4)(12)(6) + 4(20)(14)}{2(4)(12) + 4(20)}$

$= 9.64''$



$$I = 2\left(\frac{4(12)^3}{12}\right) + 2(4)(12)(3.64)^2 + \frac{20(4)^3}{12} + 4(20)(4.36)^2$$

$$= 4051 \text{ in.}^4$$

(a) $M_{cr} = \dfrac{f_r I}{c} = \dfrac{0.411(4051)}{9.64} = 172.7$ in.-k

(b) BEAM WT $= \dfrac{4(20) + 2(4)(12)}{144}(0.150) = 0.1833$ k/ft

BEAM WT MOMENT $= \dfrac{0.1833(12)^2}{8} = 3.30$ ft-k $= 39.6$ in.-k

$\dfrac{PL}{4} = M_{cr} - 39.6 = 172.7 - 39.6 = 133.1$ in.-k

$P = \dfrac{4(133.1 \text{ in.-k})}{12 \text{ ft } (12 \text{ in./ft})} = 3.70$ k