UNIVERSITY OF MARYLAND

Department of Civil and Environmental Engineering

College Park Campus

Fall Semester 2002

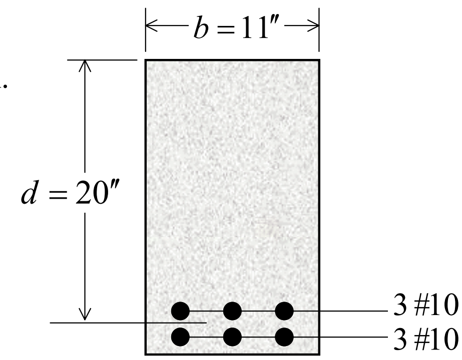
Solution to QUIZ #1

ENCE 355 – Introduction to Structural Design **NAME:**______________________

For the reinforced concrete beam cross section shown in the figure, determine

1- The amount of steel required to create the balanced condition.
2- Whether this beam is balanced, underreinforced, or overreinforced? Why?
3- The practical moment capacity ϕM_n of the beam according to the ACI Code.

Use f_y = 40,000 psi for steel and $f_c' = 3{,}000$ psi for concrete.



****** SOLUTION ******

1- Amount of Steel:

From Table 2, ρ_{max} = 0.0278, therefore, $\rho_b = \dfrac{\rho_{max}}{0.75} = \dfrac{0.0278}{0.75} = 0.03707$

Hence, Steel required for balanced condition $= A_{sb} = \rho_b bd = 0.03707(11)(20) = \underline{8.16\text{ in}^2}$

(Note: ρ_b can also be obtained using $\rho_b = \dfrac{0.85 f_c' \beta_1}{f_y}\left(\dfrac{87{,}000}{f_y + 87{,}000}\right)$

2- The Condition of the Beam:

From Table 1, the area for one #10 bar = 1.27 in^2

Therefore, $A_s = 6 \times 1.27 = 7.62\text{ in}^2$

The beam is underreinforced because

$$\left(A_s = 7.62\text{ in}^2\right) < \left(A_{sb} = 8.16\text{ in}^2\right)$$

3- Practical moment capacity of the beam:

According to the ACI Code, the practical moment capacity ϕM_n of the beam should be determined based on $A_{s,max}$, and not $A_s = 7.62\text{ in}^2$. Therefore,

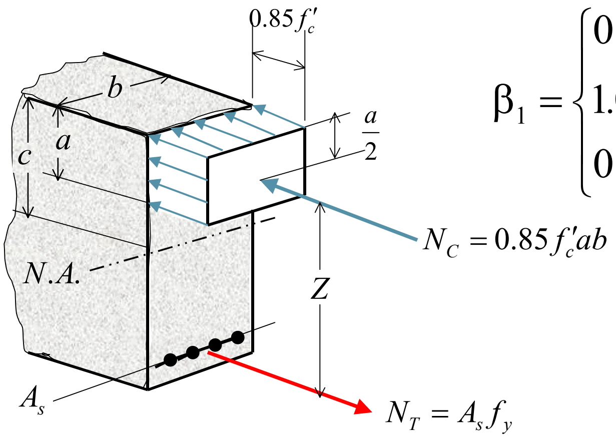
$$A_{s,max} = \rho_{max} bd = 0.0278(11)(20) = 6.12\text{ in}^2$$

$$a = \frac{A_{s,max} f_y}{0.85 f_c' b} = \frac{6.12(40)}{0.85(3)(11)} = 8.73\text{ in.}$$

$$Z = 20 - \frac{a}{2} = 20 - \frac{8.73}{2} = 15.64\text{ in.}$$

$$\varphi M_n = (0.90)(A_{s,max} f_y Z) = 0.9(6.12)(40)(15.64) = 3{,}445.8\text{ in - kips} = \underline{287\text{ ft - kips}}$$

Formulas, Tables, and Figures



$$\beta_1 = \begin{cases} 0.85 & \text{for } f'_c \le 4{,}000 \text{ psi} \\ 1.05 - 5\times 10^{-5} f'_c & \text{for } 4{,}000 \text{ psi} < f'_c \le 8{,}000 \text{ psi} \\ 0.65 & \text{for } f'_c > 8{,}000 \text{ psi} \end{cases}$$

$$a = \beta_1 c \qquad \rho_{max} = 0.75\rho_b$$

$$\rho = \frac{A_s}{bd} \qquad A_{s_{max}} = 0.75 A_{sb}$$

Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

$$\rho_b = \frac{0.85 f'_c \beta_1}{f_y}\left(\frac{87{,}000}{f_y + 87{,}000}\right)$$

Table 2. Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \ge \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\ \rho_b$	Recommended Design Values	
			ρ	$\bar{k}$ (ksi)
		F_y = 40,000 psi		
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
		F_y = 50,000 psi		
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
		F_y = 60,000 psi		
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
		F_y = 75,000 psi		
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Table 3. Strength Reduction Factors

Type of Loading	ϕ
Bending	0.90
Shear and Torsion	0.85
Compression members (spirally reinforced)	0.75
Compression Members (tied)	0.70
Bearing on Concrete	0.70