Lateral Support of Beams

- Most beams are designed in such a way that their flanges are restrained against lateral buckling.
- The upper flanges of beams used to support concrete building and bridge floors are often incorporated in these concrete floors.
- Therefore, these type of beams fall into Zone 1.
Lateral Support of Beams

- If the compression flange of a beam is without lateral support for some distance, it will have a stress distribution similar to that of columns.
- When the compression flange of a beam is long enough and slender enough, it may buckle unless lateral support is provided.

Twisting or Torsion of Beams

- When the compression flange begin to buckle, twisting or torsion will occur, and the smaller the torsional strength of the beam the more rapid will be the failure.
- Standard shapes such as W, S, and channels used for beam sections do not have a great deal of resistance to lateral buckling and the resulting torsion.
- Some other shapes, notably the built-up box shapes are tremendously stronger.
Lateral Support of Beams

- Lateral Support not Provided by Slab
  - Should lateral support of the compression flange not be provided by a floor slab, it is possible that such support may be provided with connecting beams or with special members inserted for that purpose.
  - Depending on the spacing of the support, the beam will fall into Zones 1, 2, or 3.

Introduction to Inelastic Buckling, Zone 2

- Inelastic buckling can occur when the bracing is insufficient to permit the member to develop and reach a full plastic strain distribution before buckling occurs.
- Because of the presence of residual stresses, yielding will begin in a section at applied stresses equal to

\[ F_y - F_r \]  

(1)
Introduction to Inelastic Buckling, Zone 2

- In Eq. 1, $F_y =$ yield stress of the web, and $F_r =$ compressive residual stress, and assumed equal to 10 ksi for rolled shapes and 16.5 ksi for welded shapes.
- When a constant moment occurs along the unbraced length $L_b$ of a compact I- or C-shaped section and $L_b$ is larger than $L_p$, the beam will fail inelastically unless $L_b$ is greater than a distance $L_r$.

Introduction to Inelastic Buckling, Zone 2

- Lateral Buckling of Beams
  - Fig. 1 shows that beams have three distinct ranges or zones of behavior depending on their lateral bracing situation:
    - Zone 1: closely spaced lateral bracing, beams fail plastically.
    - Zone 2: moderate unbraced lengths, beams fail inelastically.
    - Zone 3: Larger unbraced lengths, beams fail elastically
Introduction to Inelastic Buckling, Zone 2

Figure 1. $M_n$ as a function of $L_b$

- Plastic behavior-full plastic moment ($Zone 1$)
- Inelastic buckling ($Zone 2$)
- Elastic buckling ($Zone 3$)

$M_n$ $\rightarrow L_b$ (laterally unbraced length of compression flange)

Introduction to Inelastic Buckling, Zone 2

- **Bending Coefficients**
  - A moment coefficient, designated by $C_b$, is included in design formulas to account for the effect of different moment gradients on lateral-torsional buckling.
  - The use of this coefficient is to take into account the effect of the end restraint and loading condition of the member on lateral buckling.
Introduction to Inelastic Buckling, Zone 2

**Bending Coefficients (cont’d)**

- In Fig 2a, the moment in the unbraced beam causes a worse compression flange situation than does the moment in the unbraced beam of Fig. 2b.

- For one reason, the upper flange in Fig. 2a is in compression for its entire length, while in Fig. 2b the length of the “column”, that is the length of the upper flange that is in compression is much less (shorter column).

![Figure 2](a) Single curvature  (b) Double curvature
Introduction to Inelastic Buckling, Zone 2

■ Bending Coefficients (cont’d)

− Values of $C_b$:
  • For the simply supported beam of Fig. 2a:
    \[ C_b = 1.14 \]
  • For the fixed-end beam of Fig. 2b:
    \[ C_b = 2.38 \]

− The basic moment capacity equations for Zones 2 and 3 were developed for laterally unbraced beams subjected to single curvature with
  \[ C_b = 1.0 \]

Introduction to Inelastic Buckling, Zone 2

■ LRFD Specification

− LRFD Specification provides moment or $C_b$ coefficients larger than 1.0 which are to be multiplied by the computed $M_n$ values.
− The results are higher moment capacities.
− The value of $C_b = 1.0$ is a conservative value.
− It should be noted that that value obtained by multiplying $M_n$ by $C_b$ may not be larger than the plastic moment $M_p$ of Zone 1, which is equal to $F_y Z$. 
Introduction to Inelastic Buckling, Zone 2

**LRFD Specification**

The Manual provides an equation for calculating the coefficient $C_b$ as follows:

$$
C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}
$$

(2)

- $M_{\text{max}}$ = largest moment in unbraced segment of a beam
- $M_A$ = moment at the $\frac{1}{4}$ point
- $M_B$ = moment at the $\frac{1}{2}$ point
- $M_C$ = moment at the $\frac{3}{4}$ point

---

- $C_b = 1.0$ for cantilevers or overhangs where the free end is unbraced.
- Some special values of $C_b$ calculated with Eq. 2 are shown in Fig. 3 for various beam moment situations.
- Most of these values are also given in Table 5.1 of the LRFD Manual.
Introduction to Inelastic Buckling, Zone 2

Figure 3a

\[
C_a = \frac{32.1}{30.1} = \frac{b_C}{L^2} \quad \text{End section} \quad C_a = 1.30
\]

\[
C_a = \frac{67.1}{30.1} = \frac{b_C}{L^2} \quad \text{Midsection} \quad C_a = 1.32
\]

\[
C_a = \frac{67.1}{30.1} = \frac{3}{L^2} \quad \text{For two center sections} \quad C_a = 1.11
\]

\[
C_a = \frac{67.1}{30.1} = \frac{4}{L^2} \quad \text{For two end sections} \quad C_a = 1.67
\]

Figure 3b

\[
P_u \quad w_r (k/ft)
\]

\[
L/3 \quad L/3 \quad L/3
\]

\[
C_v \quad \text{varies}
\]

\[
P_u
\]

\[
L/4 \quad L/4 \quad L/4 \quad L/4
\]

\[
C_v = 1.0
\]
Introduction to Inelastic Buckling, Zone 2

Figure 3c

\[ C_h = 2.27 \]

\[ C_h = 2.38 \]

\[ w_c \text{ (k/ft)} \]

\[ L/2 \quad L/2 \]

\[ C_h = 2.38 \]

\[ C_h = 1.92 \]

Figure 3d

\[ C_h = 2.27 \]

\[ P_u \]

\[ L/2 \quad L/2 \]

\[ C_h = 1.32 \]
Moment Capacity, Zone 2

- If the distance between points of torsional bracing is increased beyond $L_p$ (see Fig. 1), the moment capacity of the section will become smaller and smaller.

- Finally, at an unbraced length $L_r$, the section will buckle elastically as soon as the yield stress is reached.

Effect of Residual Stresses

- Due to the rolling operation on steel shapes, there is residual stress in the section equal to $F_r$.

- Thus, the elastically computed stress caused by bending can only reach $F_y - F_r$ as given by Eq. 1. Assuming $C_b = 1$, the design moment for a compact I- or C-shaped section may be determined as follows if $L_b = L_r$:

$$
\phi_b M_r = \phi_b S_x (F_y - F_r)
$$

(3)
Moment Capacity, Zone 2

- Provisions for Zone 2 Design by the LRFD
  - If decrease the unbraced length $L_b$ from $L_r$ to $L_p$, buckling does not not occur when the yield stress is first reached.
  - This range between $L_r$ and $L_p$ is called Zone 2 and is illustrated in Fig. 1.
  - For these cases, when the unbraced length falls between $L_r$ and $L_p$, the design moment strength will fall approximately on a straight line between

\[
\phi M_n = \phi_y F_y Z \text{ at } L_p \quad \text{and} \quad \phi_y S_x (F_y - F_r) \text{ at } L_r
\]

- For intermediate values of the unbraced length, the moment capacity may be determined by proportions or by substituting into expressions.
  - If $C_b$ is larger than 1.0, the section will resist additional moment but not more than

\[
\phi_y F_y Z = \phi_y M_p \quad (4)
\]
Moment Capacity, Zone 2

Provisions for Zone 2 Design by the LRFD

The moment capacity can be determined by the following two expressions:

\[ \phi_b M_{nx} = C_b \left[ \phi_b M_{px} - BF \left( L_b - L_p \right) \right] \leq \phi_b M_{px} \quad (5) \]

or

\[ M_n = C_b \left[ M_p - \left( M_p - M_r \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) \right] \leq M_p \quad (6) \]

Provisions for Zone 2 Design by the LRFD

- In Eq. 5, \( BF \) is a factor given in LRFD Table 5-3 for each section, which enables us to do the proportioning with simple formula.
- Note that in Eq. 6, after the moment \( M_n \) has been computed, it should be multiplied by \( \phi_b \) to obtain \( \phi_b M_n \).
Example 1

Determine the moment capacity of a W24 \times 62 with \( F_y = 50 \text{ ksi} \) if \( L_b = 8.0 \text{ ft} \) and \( C_b = 1.0 \).

For \( F_y = 50 \text{ ksi} \), the LRFD Table 5-3 (P. 5-46) gives the following for W24 \times 62:

\[
\begin{align*}
L_p &= 4.84 \text{ ft}, \quad L_r = 13.3 \text{ ft}, \\
\phi_b M_r &= 396 \text{ kip-ft} \\
\phi_b M_p &= 578 \text{ kip-ft}, \quad \text{and} \\
BF &= 21.6 \text{ kips}
\end{align*}
\]

Example 1 (cont’d)

Since \((L_p = 4.84') < (L_b = 8.0') < (L_r = 13.3')\), the moment capacity falls in Zone 2, and

\[
\begin{align*}
\phi_b M_{nx} &= C_b \left[ \phi_b M_{px} - BF(L_b - L_p) \right] \leq \phi_b M_{px} \quad \text{Eq. 5} \\
\phi_b M_n &= C_b \left[ \phi_b M_p - BF(L_b - L_p) \right] \leq \phi_b M_p \\
\phi_b M_n &= 1.0 \left[ 578 - 21.6 (8.0 - 4.84) \right] = 509.7 \leq \phi_b M_p = 578
\end{align*}
\]

Therefore,

The moment capacity = 509.7 ft-kip
Moment Capacity, Zone 2

Example 1 (cont’d)

Figure 1. $M_n$ as a function of $L_b$

- Plastic Behavior-full
- Inelastic buckling (Zone 2)
- Elastic buckling (Zone 3)

$M_n$ = Plastic moment

- $L_{pd}$ = 4.84 ft
- $L_p = 8.0$ ft
- $L_r = 13.3$ ft
- $L_b$ (laterally unbraced length of compression flange)

Moment Capacity, Zone 2

Example 2

Select the lightest available section for a factored moment of 290 ft-kips if $L_b = 10.0$ ft. Use 50 ksi steel and assume $C_b = 1.0$.

Enter LRFD Table 5-3 (P. 5-47) and notice that $\phi_b M_p$ for W18 x 40 is 294 ft-kip.

For this section:

- $L_p = 4.49$ ft, $L_r = 12.0$ ft,
- $\phi_b M_p = 205$ kip-ft
- $\phi_b M_p = 294$ kip-ft, and
- $BF = 11.7$ kips
Example 2 (cont’d)

Since \( L_p = 4.49' \) < \( L_b = 10.0' \) < \( L_r = 12.0' \)

The moment capacity falls in Zone 2, and

\[
\phi_b M_{nx} = C_b [ \phi_b M_{px} - BF (L_b - L_p) ] \leq \phi_b M_{px}
\]

\[
\phi_b M_n = C_b [ \phi_b M_p - BF (L_b - L_p) ] \leq \phi_b M_p
\]

\[
\phi_b M_n = 1.0 [ 294 - 11.7 (10.0 - 4.49) ] = 229.5 \leq \phi_b M_p = 294
\]

Therefore,

The moment capacity = 229 ft - kip < \( M_u = 290 \) ft - kip

NG

Example 2 (cont’d)

Moving up in the table and after several trials, try a W21 × 48 that has the following properties:

\[ L_p = 6.09 \text{ ft}, \quad L_r = 15.4 \text{ ft}, \]

\[ \phi_b M_p = 279 \text{ kip - ft} \]

\[ \phi_b M_p = 401 \text{ kip - ft}, \quad \text{and} \]

\[ BF = 13.2 \text{ kips} \]

Since \( L_p = 6.09' \) < \( L_b = 10.0' \) < \( L_r = 15.4' \)

The moment capacity falls in Zone 2, and
Moment Capacity, Zone 2

- Example 2 (cont’d)

\[
\phi_b M_{nx} = C_b \left[ \phi_b M_{px} - BF(L_b - L_f) \right] \leq \phi_b M_{px}
\]

\[
\phi_b M_{nx} = C_b \left[ \phi_b M_p - BF(L_b - L_f) \right] \leq \phi_b M_p
\]

\[
\phi_b M_n = 1.0 \left[ 401 - 13.2(10.0 - 6.09) \right] = 249 \leq \phi_b M_p = 401
\]

Therefore,

The moment capacity = 349 ft - kip > \( M_u = 290 \) ft - kip  

\[ \text{OK} \]

Hence, USE W21 \( \times 48 \)

---

Elastic Buckling, Zone 3

- When a beam is not fully braced, it may fail due to buckling of the compression portion of the cross section laterally about the weak axis.

- This will be accompanied also with twisting of the entire cross section about the beam’s longitudinal axis between points of lateral bracing.

- For the moment capacity to fall into Zone 3, \( L_b \geq L_f \)
Elastic Buckling, Zone 3

Figure 1. $M_n$ as a function of $L_b$

- Plastic Behavior—full Plastic moment (Zone 1)
- Inelastic buckling (Zone 2)
- Elastic buckling (Zone 3)

$M_n$ as a function of $L_b$

$L_b$ (laterally unbraced length of compression flange)

CHAPTER 9b. DESIGN OF BEAMS FOR MOMENTS

Slide No. 35

Elastic Buckling, Zone 3

- LRFD Specifications for Zone 3
  - The classic equation for determining the flexural-torsional buckling moment is given by

  $$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

  (7)

  - $G$ = shear modulus of steel = 11,200 ksi
  - $J$ = torsional constant (in$^4$)
  - $C_w$ = warping constant (in$^6$)

  NOTE: These properties are provided in Tables 1.25 to 1.35 Of the LRFD Manual for rolled sections
Elastic Buckling, Zone 3

Example 3

Compute $\phi M_{cr}$ for a W18 × 97 consisting of 50 ksi steel if the unbraced length $L_b$ is 38 ft. Assume $C_b = 1.0$.

For W18 × 97, Table 5-3 (P. 5-46) of the Manual gives

- $L_p = 9.36$ ft, $L_r = 27.5$ ft,
- $\phi_b M_r = 564$ kip-ft
- $\phi_b M_p = 791$ kip-ft, and
- $BF = 12.8$ kips

Example 3 (cont'd)

Since $(L_p = 9.36') < (L_r = 27.5') < (L_b = 38')$

The moment capacity falls in Zone 3, and

- $J = 5.86$ in$^4$, and $C_w = 15,800$ in$^6$

From Part 1 of the Manual, tables for torsion properties (P. 1-91):

From P.1-17

- $L_r = 201$ in$^4$, $J = 5.86$ in$^4$, and $C_w = 15,800$ in$^6$

Therefore, using Eq. 7

$$M_{cr} = \left[\frac{\pi}{12 \times 38}\right] \frac{29 \times 10^3 (201)(12,200)(5.86) + \left(\frac{\pi}{38 \times 12}\right)^2 (201)(15,800)}{29 \times 10^3 (201)(12,200)(5.86) + \left(\frac{\pi}{38 \times 12}\right)^2 (201)(15,800)} = 4,916.9$\text{ in $\cdot$kip} = 410$ ft $\cdot$kip

Therefore, $\phi_b M_{cr} = 0.9 (410) = 369$ ft-kip