**Introduction**

- A fairly long, simply supported beam can be subjected to gravity transverse loading.
- Due to the application of this loading, the beam will bend downward, and its upper part will be placed in compression and will act as a compression member.
- The cross section of this “column” will consist of the portion of the cross section above the neutral axis.
Introduction

Figure 1. Deformation of Beam due to Lateral Loading

Figure 2. Distribution of Normal Stress in a Beam Cross Section

Figure 9

This part acts as a compression member
**Introduction**

- For the usual beam the “column” will have a much smaller moment of inertia about its $y$ or vertical axis than its $x$ or horizontal axis.
- If its $y$ axis is not braced perpendicularly, it will buckle laterally at a much smaller load than would otherwise have been required to produce a vertical failure.

**Lateral Buckling of Beams**

- Lateral buckling will not occur if the compression flange is braced laterally or if twisting of the beam is prevented at frequent intervals.
- Types of beams with respect to lateral buckling:
  1. The beams can be assumed to have continuous lateral bracing for their compression flanges.
  2. Next, the beams can be assumed to be braced laterally at short intervals.
  3. Finally, the beams can be braced laterally at larger intervals.
Introduction

■ Lateral Buckling of Beams (cont’d)
  – Fig. 3 shows that beams have three distinct ranges or zones of behavior depending on their lateral bracing situation:
    • Zone 1: closely spaced lateral bracing, beams fail plastically.
    • Zone 2: moderate unbraced lengths, beams fail inelastically.
    • Zone 3: larger unbraced lengths, beams fail elastically

Introduction

Figure 3. $M_n$ as a function of $L_b$

Plastic Behavior-full
Plastic moment (Zone 1)
Inelastic buckling (Zone 2)
Elastic buckling (Zone 3)

$L_p$ (laterally unbraced length of compression flange)
Yielding Behavior - Full Plastic Moment, Zone 1

- LRFD Specification
  - The full plastic moment $M_p$ (or $M_n$) is limited to a value of $1.5 M_y$.
  - If $L_b$ of the compression flange of a compact I- or C-shaped section does not exceed $L_p$ (for elastic analysis) or $L_{pd}$ (for plastic analysis), then the member bending strength about its major axis (e.g., $x$) may be determined as follows:

\[
M_n = M_p = F_y Z \leq 1.5 M_y \quad (1)
\]

\[
M_u = \phi_b M_n \text{ with } \phi_b = 0.90
\]

- When elastic analysis is used, $L_b$ may not exceed the value $L_p$ to follow if $M_n$ is to equal $F_y Z$:

\[
L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} \quad (2)
\]
Yielding Behavior - Full Plastic Moment, Zone 1

- LRFD Specification (cont’d)
  - For solid rectangular bars and box beams with \( A \) = cross-sectional area (in²) and \( J \) = torsional constant (in⁴), \( L_b \) may not exceed the value \( L_p \) to follow if \( M_n \) is to equal \( F_yZ \):

\[
L_p = \frac{0.13r_yE}{M_p} \sqrt{JA}
\]  
(3)

Yielding Behavior - Full Plastic Moment, Zone 1

- LRFD Specification (cont’d)
  - When plastic analysis is used to establish member forces for symmetric I-shaped members with compression flanges larger than their tension flanges loaded in the plane of the web, \( L_b \) may not exceed the value \( L_{pd} \) to follow if \( M_n \) is to equal \( F_yZ \):

\[
L_{pd} = \left[ 0.12 + 0.076 \left( \frac{M_1}{M_2} \right) \right] \left( \frac{E}{F_y} \right) r_y
\]  
(4)
Yielding Behavior - Full Plastic Moment, Zone 1

- In Eq. 4, $M_1$ is the smaller moment at the end of the unbraced length of the beam and $M_2$ is the larger moment at the end of the unbraced length.
- The ratio $M_1 / M_2$ is positive when the moments cause the member to be bent in double curvature and negative if they bend it in single curvature.
- According to LRFD, only steels with $F_y$ value of 65 ksi or less may be considered.

Design of Beams, Zone 1

- Beams are generally designed so that they will provide sufficient design moment capacities $\phi M_n$ and checked to see if any of the following items are critical:
  1. Shear
  2. Deflections
  3. Crippling
  4. Lateral bracing for compression flanges
  5. Fatigue
Design of Beams, Zone 1

- The factored moment will be computed, and a section having that much design moment capacity will be initially selected from the LRFD Manual.
- Table 5-3 of the Manual, entitled “W-Shaped Selection by $Z_x$” can be used.
- From this table, steel shapes having sufficient plastic moduli to resist certain moments can quickly be selected.

Design of Beams, Zone 1

- Two items to be considered when using the LRFD table in selecting shapes:
  1. Steel sections cost so many cents per pound and it is therefore desirable to select the lightest possible shape having the required plastic modulus. The table has sections arranged in various groups having certain ranges of plastic moduli. The heavily typed section at the top of each group is the lightest in that group.
2. The plastic moduli values in the table are given about the horizontal axes for beams in their upright positions. If a beam is to be turned on its side, the proper plastic modulus can be found in Table 5-3 of the Manual or LRFD tables giving dimensions and properties of shapes in Part 1 of the LRFD Manual. A W shape turned on its side may only be 10 to 30 percent as strong as one in the upright position when subjected to gravity loads.

Beam Weight Estimates

- Beam design should include the weight of the beam.
- However, because this information is not possibly available before the design, a simple procedure or method for estimating the beam weight should be used.
  - This method involves:
    1. Calculating the maximum factored bending moment $M_{u'}$. 
2. Select a section from LRFD Table 5-3.
3. Then, the weight of that section or a little bit more (since the beam’s weight will increase the moment somewhat) can be used as the estimated beam weight.
4. Finally, this estimated load can be added to the external dead load acting on the beam.

Example 1

Select a beam section for the span and loading shown in the figure, assuming full lateral support is provided for the compression flange by the floor slab above (that is $L_b = 0$) and $F_y = 50$ ksi.

- $D = 1$ k/f (not including beam weight)
- $L = 3$ k/f

![Beam diagram with span 21 ft]
Example 1 (cont’d)

**Beam weight estimate:**

\[ w_u (\text{beam weight excluded}) = 1.2(1.0) + 1.6(3.0) = 6.0 \frac{\text{kips}}{\text{ft}} \]

\[ M_u = \frac{w_u L^2}{8} = \frac{6(21)^2}{8} = 330.75 \text{ ft-kips} \]

\[ Z_{\text{required}} = \frac{M_u}{\phi F_y} = \left( \frac{330.75}{0.90(50)} \right) \times 12 = 88.2 \text{ in}^3 \]

Referring to Table 5-3 in Part 5 of the LRFD Manual, a **W21 x 44** \((Z_x = 95.8 \text{ in}^3)\) is the lightest section available.

Example 1 (cont’d)

Assume beam weight = 44 lb/ft, therefore the design distributed load \(w_u\) will be revised as follows:

\[ w_u = 1.2(1.044) + 1.6(3) = 6.05 \frac{\text{kips}}{\text{ft}} \]

\[ M_u = \frac{w_u L^2}{8} = \frac{6.05(21)^2}{8} = 333.5 \text{ ft-kips} \]

\[ Z_{\text{required}} = \left( \frac{333.5}{0.90(50)} \right) \times 12 = 88.9 \text{ in}^3 < Z = 95.8 \text{ in}^3 \quad \text{OK} \]

Therefore,

**USE W21 x 44 with } F_y = 50 \text{ ksi**
Example 2

The 5-in reinforced-concrete slab shown in the figure is to be supported with steel W sections 8 ft 0 in on centers. The beams, which will span 20 ft, are assumed to be simply supported. If the concrete slab is designed to support a live load of 100 psf, determine the lightest steel section required to support the slab. It is assumed that the compression flange of the beam will be fully supported laterally by the concrete slab. The concrete weighs 150 lb/ft³, and $F_y = 50$ ksi.

Example 2 (cont’d)

The concrete slab weighs 150 lb/ft³, and $F_y = 50$ ksi.
Design of Beams, Zone 1

Example 2 (cont’d)

\[ M_u = \frac{w_u L^2}{8} = \frac{1.906(20)^2}{8} = 95.3 \text{ ft-kips} \]

Span of Beam = 20 in.

Calculation of Dead and Live Loads:

Slab weight = \( \left( \frac{5}{12} \right)(8)(150) = 500 \text{ lb/ft} = 0.5 \text{ kips/ft} \)

Live load = \( 100(8) = 800 \text{ lb/ft} = 0.8 \text{ kips/ft} \)

Initial design excluding beam weight:

\[ w_i (\text{beam weight excluded}) = 1.2(0.5) + 1.6(0.8) = 1.88 \text{ kips/ft} \]

\[ M_u = \frac{w_u L^2}{8} = \frac{1.88(20)^2}{8} = 94.0 \text{ ft-kips} \]

\[ Z_{\text{required}} = \frac{M_u}{\phi F_y} = \left( \frac{94.0}{0.90(50)} \right) \times 12 = 25.07 \text{ in}^3 \]
Example 2 (cont’d)

Referring to Table 5-3 in Part 5 of the LRFD Manual, a **W10 × 22** ($Z_x = 26.0 \text{ in}^3$) is the lightest section available.

Assume beam weight = 44 lb/ft, therefore the design distributed load $w_u$ will be revised as follows:

$$w_u = 1.2[0.5 + 0.022] + 1.6(0.8) = 1.906 \text{ kips/ft}$$

Therefore,

$$M_u = \frac{w_u L^2}{8} = \frac{1.906(20)^2}{8} = 95.3 \text{ ft-kips}$$

$$Z_{\text{required}} = \left( \frac{95.3}{0.90(50)} \right) \times 12 = 25.4 \text{ in}^3 < Z = 26.0 \text{ in}^3 \quad \text{OK}$$

Therefore,

USE **W10 × 22** with $F_y = 50 \text{ ksi}$