Structural Steel Design
LRFD Method

Third Edition

CHAPTER

8d

INTRODUCTION TO BEAMS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part II – Structural Steel Design and Analysis

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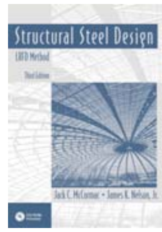


By
Dr. Ibrahim. Assakkaf

ENCE 355 - Introduction to Structural Design

Department of Civil and Environmental Engineering
University of Maryland, College Park

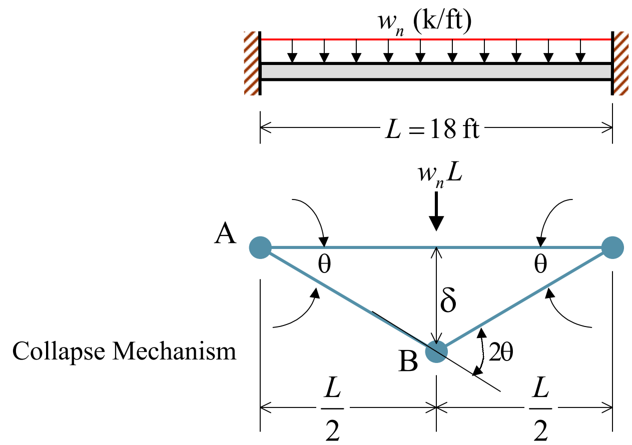
Prentice Hall

Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge
  - For a uniformly loaded fixed-end beam shown in Fig. 1, the location of the plastic hinge along the length of the beam is at the midspan of the beam.
  - This was concluded due to the fact that beam is symmetrical in terms of both the uniform loading and the end supports.

Location of Plastic Hinge for Uniform Loadings

Figure 1. Uniformly Loaded Fixed-end Beam



Location of Plastic Hinge for Uniform Loadings

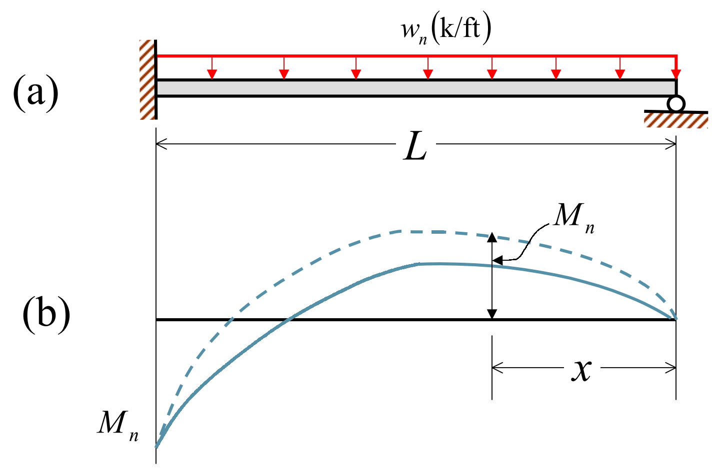
- Location of Plastic Hinge (cont'd)
  - For other beams with uniform loads, such as propped or continuous beams, the determination of the location of plastic hinge may be rather difficult.
  - For this reason, a value, expressed as fraction of the length L, that determines the location of the plastic hinge is needed for the analysis of both the propped and continuous beams.

Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge (cont'd)
  - Consider the propped beam of Fig. 2.
  - The elastic moment diagram for this beam is shown as the solid line in part (b) of the figure.
  - As the uniform load is increased in magnitude, a plastic hinge will first form at the fixed end.

Location of Plastic Hinge for Uniform Loadings

Figure 2. Propped Beam



Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge (cont'd)
  - At this time the beam will, in effect, be a "simple" beam with a plastic hinge on one end and a real hinge on the other.
  - Subsequent increases in the load will cause the moment to change as represented by the dashed line in part (b) of Fig. 2.

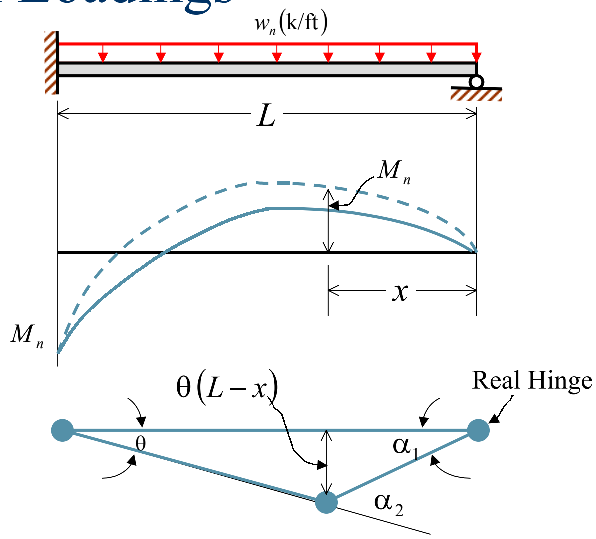
Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge (cont'd)
  - The process will continue until the moment at some other point (a distance x from the right support in the figure) reaches M_n and create another plastic hinge.
  - The virtual-work expression for the collapse mechanism for the beam shown in Fig. 3 is written as follows:

$$[w_n L]\left[\frac{1}{2}(\theta)(L-x)\right] = M_n\left(\theta + \theta + \frac{L-x}{x}\theta\right) \qquad (1)$$

Location of Plastic Hinge for Uniform Loadings

Figure 3. Collapse Mechanism



$$\alpha_1 = \frac{L-x}{x}\theta$$

$$\alpha_2 = \theta + \alpha_1$$

$$= \theta + \frac{L-x}{x}\theta$$

Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge (cont'd)

Or

$$M_n\left(\theta + \theta + \frac{L-x}{x}\theta\right) = [w_n L]\left[\frac{1}{2}(\theta)(L-x)\right] \qquad (2)$$

$$M_n = \frac{[w_n L]\left[\frac{1}{2}(\theta)(L-x)\right]}{\left(\theta + \theta + \frac{L-x}{x}\theta\right)} \qquad (3)$$

Solving for M_n by taking the derivative of M_n with respect to x and equate it to zero, that is

$$\frac{dM_n}{dx} = 0 \qquad (4)$$

Location of Plastic Hinge for Uniform Loadings

- Location of Plastic Hinge (cont'd)
  - Solving for x, it can be shown that Eq. 4 yields a value of x as given by

$$x = 0.414L \qquad (5)$$

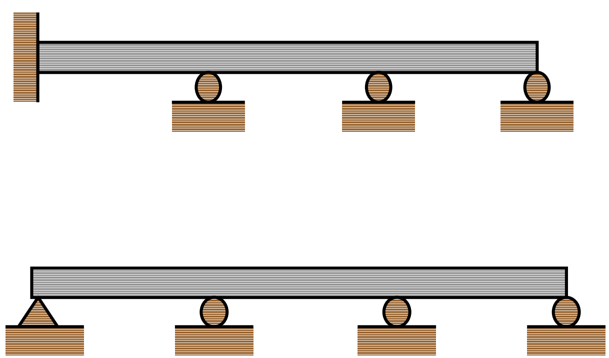
  - This value is applicable to uniformly loaded end spans of both propped and continuous beams with simple supports.

Continuous Beams

- Continuous beams are very common in engineering structures (Fig. 4).
- They can be analyzed by both the elastic and the plastic theories.
- However, the plastic analysis can be more complicated.
- Plastic analysis can be applied to continuous beams as it is to one-span beams.

Continuous Beams

Figure 4. Continuous Beams



Continuous Beams

- The resulting values definitely give a more realistic picture of the limiting strength of a structure than can be obtained by elastic analysis.
- Continuous statically indeterminate beams can be handled by virtual-work method as they were for single-span statically indeterminate beams.

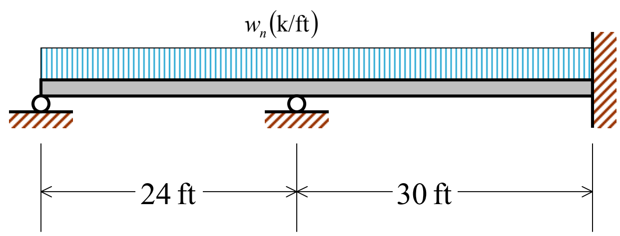
Continuous Beams

- Virtual-Work Method for Continuous Beams
  - For each span of the continuous beams, virtual-work expressions are written separately.
  - From the resulting expressions, it is possible to determine the limiting or maximum loads that the beams can support.

Continuous Beams

- Example 1

  A W18 × 55 (Z_x = 112 in^3) has been selected for the beam shown in the figure. Using 50 ksi steel and assuming full lateral support, determine the value of w_n.



Continuous Beams

- Example 1 (cont'd)
  - The nominal (plastic) moment of the beam is calculated first:

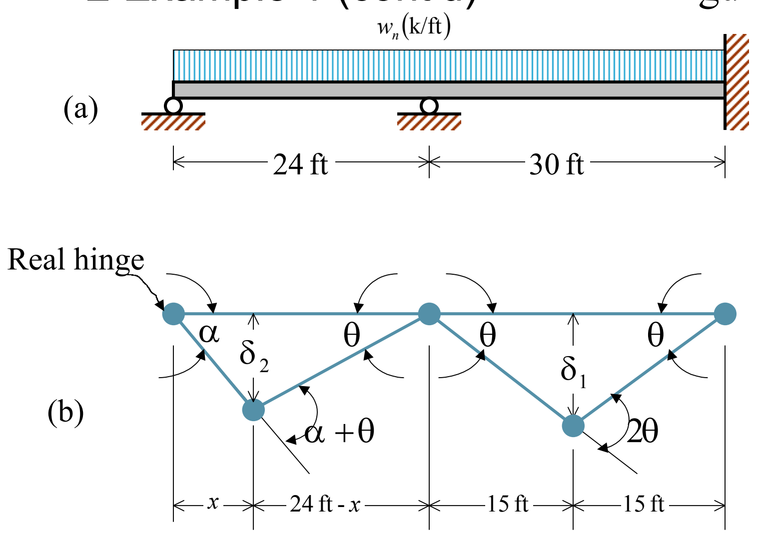
$$M_n = F_y Z = \frac{50(112)}{12} = 466.7 \text{ ft - k} \qquad (6)$$

  - The virtual-work expressions are written separately for each span of the continuous beams.
  - The collapse mechanisms for the two spans are drawn as shown in Fig. 5b.

Continuous Beams

- Example 1 (cont'd)

Figure 5



Continuous Beams

- Example 1 (cont'd)

  Calculation of rotation angles:

  Using Eq. 4, the location of the plastic hinge for the left span is

$$x = 0.414L = 0.414(24) = 9.94 \text{ ft}$$

- From the triangle of the right span:

$$\tan\theta \approx \theta = \frac{\delta_1}{15} \Rightarrow \delta_1 = 15\theta$$

- From the triangle of the left span:

$$24 - x = 24 - 9.94 = 14.06 \text{ ft}$$

Continuous Beams

- Example 1 (cont'd)
  - Also

$$\tan\theta \approx \theta = \frac{\delta_2}{14.06} \Rightarrow \delta_2 = 14.06\theta$$

  - Therefore,

$$\tan\alpha \approx \alpha = \frac{\delta_2}{9.94} = \frac{14.06\theta}{9.94} = 1.414\theta$$

– Virtual-work applied to right span:

$$\text{External work} = \text{Internal work}$$
$$W_{\text{ext.}} = W_{\text{int.}}$$

Continuous Beams

- Example 1 (cont'd)

$$(w_n L_1)\times(\delta_1)_{\text{avg}} = M_n(\theta+\theta+2\theta)$$

$$(w_n \times 30)\left(\frac{1}{2}\times 15\theta\right) = M_n(4\theta)$$

Controls

$$\therefore w_n = \frac{2(4M_n)}{30(15)} = 0.01778M_n = 0.01778(466.7) = 8.30\,\frac{\text{k}}{\text{ft}}$$

– Virtual-work applied to left span:

$$(w_n L_2)\times(\delta_2)_{\text{avg}} = M_n(\alpha+\theta+\theta)$$

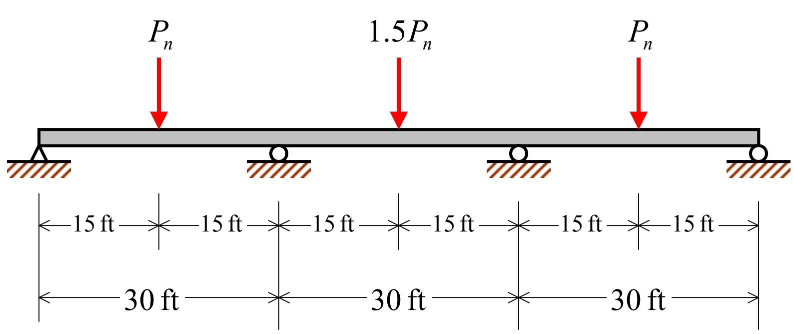
$$(w_n \times 24)\left(\frac{1}{2}\times 14.06\theta\right) = M_n(1.414\theta+\theta+\theta)$$

$$\therefore w_n = \frac{2(3.414M_n)}{24(14.06)} = 0.0.02023M_n = 0.02023(466.7) = 9.44\,\frac{\text{k}}{\text{ft}}$$

Continuous Beams

- Example 2

Using a W21 × 44 (Z_x = 95.4 in^3) consisting of A992 steel, determine the value of P_n for the beam shown.

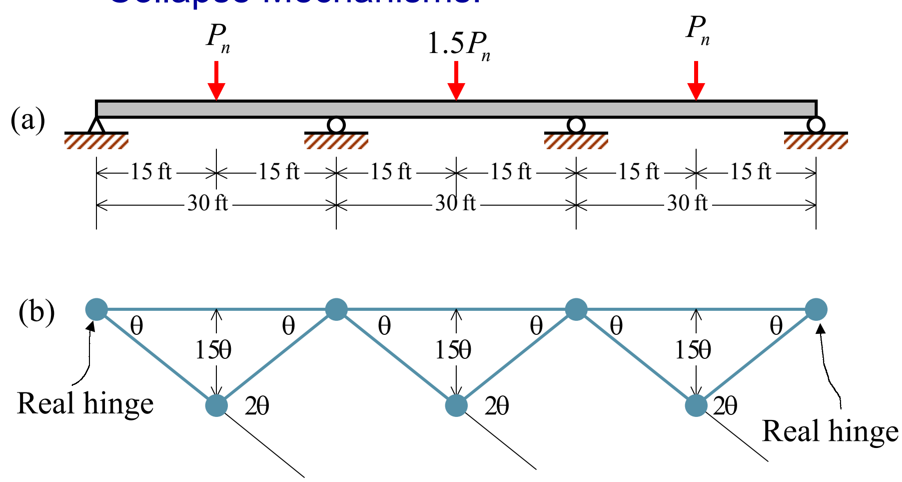


Continuous Beams

- Example 2 (cont'd)

Figure 6

Collapse Mechanisms:



Continuous Beams

- Example 2 (cont'd)
  - The nominal (plastic) moment of the beam is calculated first:

$$M_n = F_y Z = \frac{50(95.4)}{12} = 397.5 \text{ ft-k}$$

  - The virtual-work expressions are written separately for each span of the continuous beams.
  - The collapse mechanisms for the three spans are drawn as shown in Fig. 6b.

Continuous Beams

- Example 2 (cont'd)

  For the first and third spans:

$$W_{\text{ext.}} = W_{\text{int.}}$$
$$P_n(15\theta) = M_n(\theta + 2\theta)$$
$$P_n = \frac{M_n(3)}{15} = 0.2M_n = 0.2(397.5) = 79.5 \text{ kips}$$

  For the center span:

$$W_{\text{ext.}} = W_{\text{int.}}$$
$$1.5P_n(15\theta) = M_n(\theta + \theta + 2\theta)$$
$$P_n = \frac{M_n(4)}{1.5(15)} = 0.1778M_n = 0.1778(397.5) = 70.7 \text{ kips}$$

Controls → 70.7 kips