CHAPTER

Structural Steel Design
LRFD Method

Third Edition

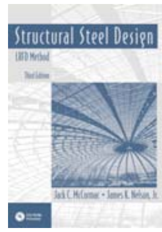
INTRODUCTION TO BEAMS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part II – Structural Steel Design and Analysis

8c

FALL 2002



By
Dr. Ibrahim. Assakkaf

ENCE 355 - Introduction to Structural Design

Department of Civil and Environmental Engineering
University of Maryland, College Park

Prentice Hall

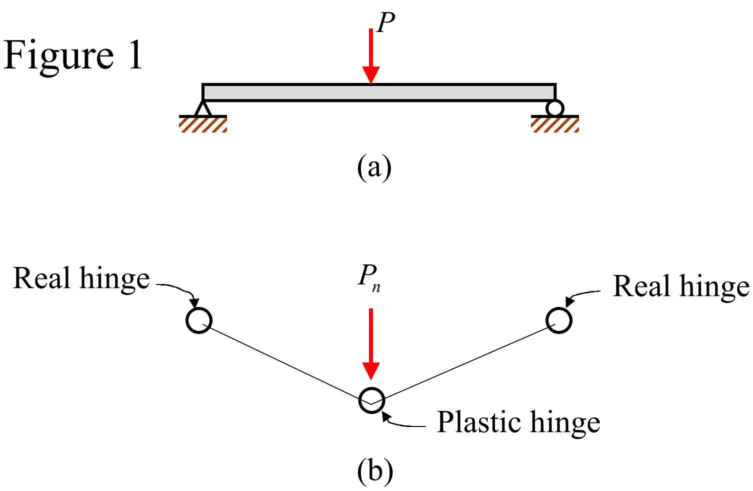
The Collapse Mechanism

- Statically Determinate Beam
  - A statically determinate beam will fail if one plastic hinge developed.
  - Consider the simply-supported beam of Fig. 1. That has a constant cross section and loaded with a concentrated load P at midspan.
  - If P is increased until a plastic hinge is developed at the point of maximum moment (just underneath P), an unstable structure will be created as shown in Fig 1b.

The Collapse Mechanism

- Statically Determinate Beam (cont'd)

Figure 1



The Collapse Mechanism

- Statically Determinate Beam (cont'd)
  - Any further increase in the load will cause collapse.
  - P_n represents the nominal or theoretical maximum load that the beam can support.

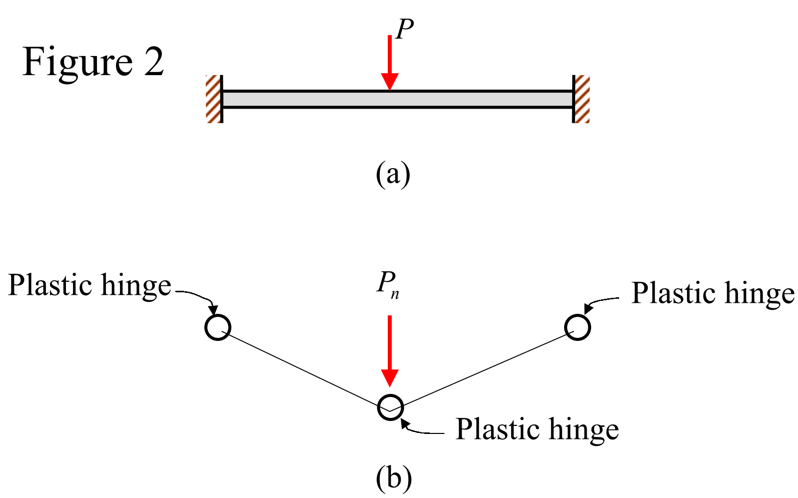
The Collapse Mechanism

- Statically Indeterminate Beam
  - For statically indeterminate beam to fail, it is necessary for more than one plastic hinge to form.
  - The number of plastic hinges required for failure of statically indeterminate structure will be shown to vary from structure to structure, but never be less than two.
  - The fixed-end beam of Fig. 2 cannot fail unless the three hinges shown in the figure are developed.

The Collapse Mechanism

- Statically Indeterminate Beam (cont'd)

Figure 2



The Collapse Mechanism

- Statically Indeterminate Beam (cont'd)
  - Although a plastic hinge may have formed in a statically indeterminate structure, the load can still be increased without causing failure if the geometry of he structure permits.
  - The plastic hinge will act like a real hinge as far as the increased loading is concerned.
  - As the load is increased, there is a redistribution of moment because the plastic hinge can resist no more moment.

The Collapse Mechanism

- Statically Indeterminate Beam (cont'd)
  - As more plastic hinges are formed in the structure, there will eventually be a sufficient number of them to cause collapse.
  - Actually, some additional load can be carried after this time before collapse occurs as the stresses go into the strain hardening range.
  - However, deflections that would occur are too large to be permissible in the design.

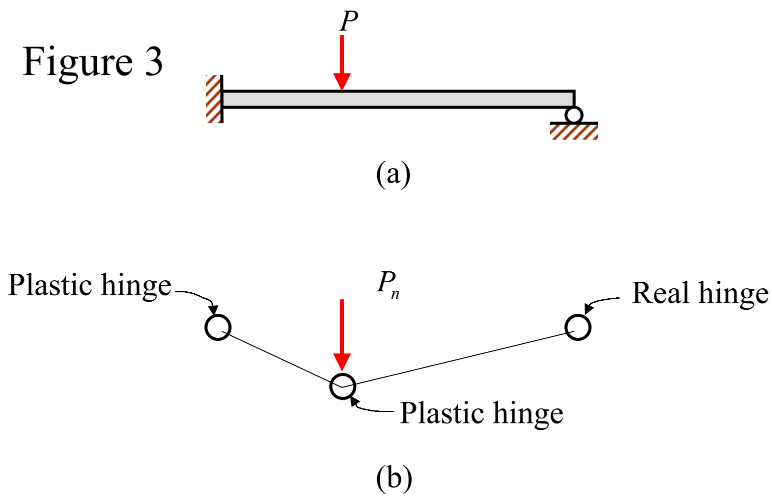
The Collapse Mechanism

- Statically Indeterminate Beam (cont'd)
  - The propped beam of Fig. 3 is an example of a structure that will fail after two plastic hinges develop.
  - Three hinges are required for collapse, but there is a real hinge on the right end.
  - In this beam the largest elastic moment caused by the design concentrated load is at the fixed end.
  - As the magnitude of the load is increased a plastic hinge will form at that point.

The Collapse Mechanism

- Statically Indeterminate Beam (cont'd)

Figure 3



The Collapse Mechanism

- The Mechanism
  - The load may be further increased until the moment at some point (here it will be at the concentrated load) reaches the plastic moment.
  - Additional load will cause the beam to collapse.
  - Therefore, the Mechanism is defined as the arrangement of plastic hinges and perhaps real hinges which permit the collapse in a structure as shown in part (b) of Figs. 1, 2, and 3.

Plastic Analysis of Structure

- There are various methods that can be used to perform plastic analysis for a given structure.
- Two satisfactory method for this type of analysis are
  - The virtual-work Method (Energy Method)
  - Equilibrium Method
- In this course, we will focus on the virtual-work method.

Plastic Analysis of Structure

- The Virtual-Work Method
  - The structure under consideration is assumed to be loaded to its nominal capacity, M_n.
  - Then, it is assumed to deflect through a small additional displacement after the ultimate load is reached.
  - The work performed by the external loads during this displacement is equated to internal work absorbed by the hinges.

Plastic Analysis of Structure

- The Virtual-Work Method

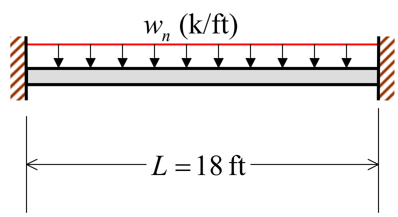
$$\text{External work} = \text{Internal work} \qquad W_{\text{ext.}} = W_{\text{int.}} \tag{1}$$

  - For this case, the small-angle theory is used.
  - For this theory, the sine of a small angle equals the tangent of that angle and also equals the same angle expressed in radians.

Plastic Analysis of Structure

- Example 1

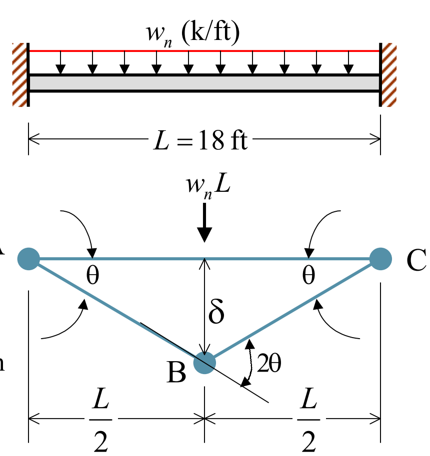
  Determine the plastic limit (or nominal) distributed load w_n in terms of the plastic (or nominal) moment M_n developed at the hinges.



Plastic Analysis of Structure

- Example 1 (cont'd)

  The collapse mechanism for the beam is sketched.



Collapse Mechanism

Plastic Analysis of Structure

- Example 1 (cont'd)
  - Because of the symmetry, the rotations θ at the end plastic hinges are equal.
  - The work done by the external load ($w_n L$) is equal $w_n L$ times the average deflection δ_{avg} of the mechanism at the center of the beam.
  - The deflection δ is calculated as follows:

$$\tan\theta \approx \theta = \frac{\delta}{L/2} \quad \text{(small angle theory)}$$

$$\therefore \delta = \frac{\theta L}{2}$$

Plastic Analysis of Structure

- Example 1 (cont'd)
  - The internal work absorbed by the hinges is equal the sum of plastic moments M_n at each plastic hinge times the angle through which it works.
  - The average deflection δ_{avg} throughout the length of the beam is equals one-half the deflection δ at the center of the beam, that is

$$\delta_{avg} = \frac{1}{2}\delta = \frac{1}{2}\left(\frac{\theta L}{2}\right) = \frac{\theta L}{4}$$

Plastic Analysis of Structure

- Example 1 (cont'd)
  - Applying Eq. 1 (conservation of energy), yield a relationship between w_n and M_n as follows:

$$\text{External work} = \text{Internal work}$$

$$W_{\text{ext.}} = W_{\text{int.}}$$

$$(w_n L)\delta_{\text{avg}} = \underbrace{(M_n)\theta}_{\substack{\text{Left}\\ \text{A}}} + \underbrace{(M_n)2\theta}_{\substack{\text{Middle}\\ \text{B}}} + \underbrace{(M_n)\theta}_{\substack{\text{Right}\\ \text{C}}}$$

$$w_n L\left(\frac{\theta L}{4}\right) = 4\theta M_n$$

Plastic Analysis of Structure

- Example 1 (cont'd)
  - Therefore,

$$w_n L\left(\frac{L}{4}\right) = 4M_n$$

$$w_n = \frac{16M_n}{L^2}$$

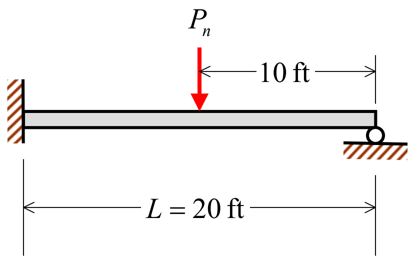
  - For 18-ft span, the plastic limit distributed load is computed as

$$w_n = \frac{16M_n}{L^2} = \frac{16M_n}{(18)^2} = \frac{M_n}{20.25}$$

Plastic Analysis of Structure

- Example 2

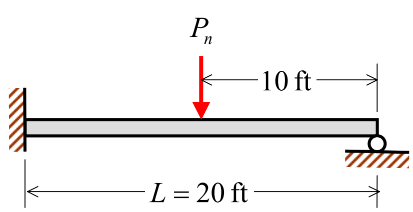
  For the propped beam shown, determine the plastic limit (or nominal) load P_n in terms of the plastic (or nominal) moment M_n developed at the hinges.



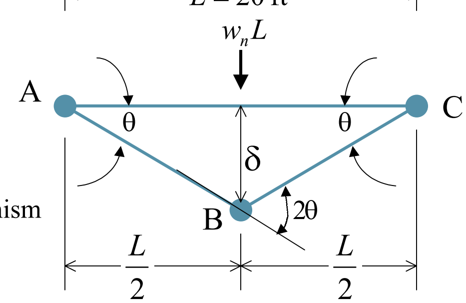
Plastic Analysis of Structure

- Example 2 (cont'd)

  The collapse mechanism for the beam is sketched.



Collapse Mechanism



Plastic Analysis of Structure

- Example 2 (cont'd)
  - Because of the symmetry, the rotations θ at the end plastic hinges are equal.
  - The work done by the external load (P_n) is equal P_n times the deflection δ of the mechanism at the center of the beam.
  - The deflection δ is calculated as follows:

$$\tan\theta \approx \theta = \frac{\delta}{L/2} \quad \text{(small angle theory)}$$

$$\therefore \delta = \frac{\theta L}{2}$$

Plastic Analysis of Structure

- Example 2 (cont'd)
  - The internal work absorbed by the hinges is equal the sum of plastic moments M_n at each plastic hinge times the angle through which it works.
  - Note that in example, we have only two plastic hinges at points A and B of the mechanism. Point C is a real hinge, and no moment occurs at that point.
  - Also note that the external work is calculated using δ and not δ_{avg}. because of the concentrated load P_n in that location.

Plastic Analysis of Structure

- Example 2 (cont'd)
  - Applying Eq. 1 (conservation of energy), yield a relationship between P_n and M_n as follows:

$$\text{External work} = \text{Internal work}$$

$$W_{\text{ext.}} = W_{\text{int.}}$$

$$(P_n)\delta = \underbrace{(M_n)\theta}_{\substack{\text{Left} \\ \text{A}}} + \underbrace{(M_n)2\theta}_{\substack{\text{Middle} \\ \text{B}}}$$

$$P_n\left(\frac{\theta L}{2}\right) = 3\theta M_n$$

Plastic Analysis of Structure

- Example 2 (cont'd)
  - Therefore,

$$P_n\left(\frac{L}{2}\right) = 3M_n$$

$$P_n = \frac{6M_n}{L}$$

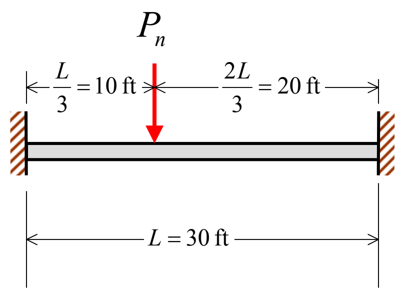
  - For 20-ft span, the plastic limit load P_n is computed as

$$P_n = \frac{6M_n}{L} = \frac{6M_n}{20} = \frac{3M_n}{10} = 0.3M_n$$

Plastic Analysis of Structure

■ Example 3

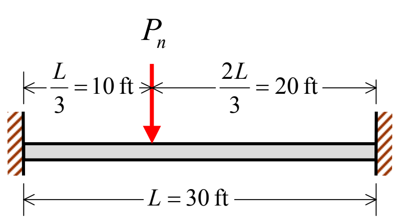
For the fixed-end beam shown, determine the plastic limit (or nominal) load P_n in terms of the plastic (or nominal) moment M_n developed at the hinges.

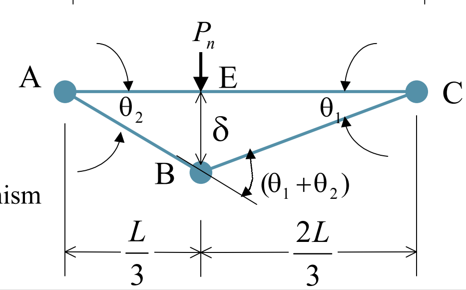


Plastic Analysis of Structure

■ Example 3 (cont'd)

The collapse mechanism for the beam is sketched.





Collapse Mechanism

Plastic Analysis of Structure

- Example 3 (cont'd)
  - Because of the unsymmetry, the rotations θ_1 and θ_2 at the end plastic hinges are not equal.
  - We need to find all rotations in terms, say θ_1
  - The work done by the external load (P_n) is equal P_n times the deflection δ of the mechanism at the center of the beam.

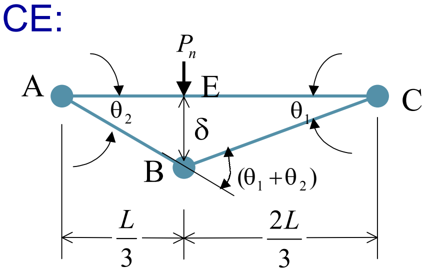
Plastic Analysis of Structure

- Example 3 (cont'd)

From triangles ABE and BCE:

$$\text{ABE}: \tan\theta_1 \approx \theta_1 = \frac{\delta}{2L/3} \Rightarrow \delta = \frac{2L}{3}\theta_1 \qquad (2)$$

$$\text{BCE}: \tan\theta_2 \approx \theta_2 = \frac{\delta}{L/3} \Rightarrow \delta = \frac{L}{3}\theta_2 \qquad (3)$$



Thus, from Eqs. 2 and 3:

$$\frac{2L}{3}\theta_1 = \frac{L}{3}\theta_2 \qquad \text{or} \qquad \theta_2 = 2\theta_1$$

Therefore,

At C: $\theta_C = \theta_1$

At A: $\theta_A = \theta_2 = 2\theta_1$

At B: $\theta_B = \theta_1 + \theta_2 = \theta_1 + 2\theta_1 = 3\theta_1$

Plastic Analysis of Structure

- Example 3 (cont'd)
  - The internal work absorbed by the hinges is equal the sum of plastic moments M_n at each plastic hinge times the angle through which it works.
  - Note that in example, we have three plastic hinges at points A, B, and C of the mechanism. Also there is no real hinge.
  - Also note that the external work is calculated using δ and not δ_{avg}. because of the concentrated load P_n in that location.

Plastic Analysis of Structure

- Example 3 (cont'd)
  - Applying Eq. 1 (conservation of energy), yield a relationship between P_n and M_n as follows:

$$\text{External work} = \text{Internal work}$$

$$W_{ext.} = W_{int.}$$

$$(P_n)\delta = \underbrace{(M_n)2\theta_1}_{\text{Left A}} + \underbrace{(M_n)3\theta_1}_{\text{Middle B}} + \underbrace{(M_n)\theta_1}_{\text{Right C}}$$

$\delta = \frac{2L}{3}\theta_1$ from Eq. 2

$$P_n\left(\frac{2L}{3}\theta_1\right) = 6\theta_1 M_n$$

Plastic Analysis of Structure

- Example 3 (cont'd)
  - Therefore,

$$P_n\left(\frac{2L}{3}\right) = 6M_n$$

$$P_n = \frac{9M_n}{L}$$

  - For 30-ft span, the plastic limit load P_n is computed as

$$P_n = \frac{9M_n}{L} = \frac{9M_n}{30} = 0.3M_n$$

Plastic Analysis of Structure

- Complex Structures
  - If a structure (beam) has more than one distributed or concentrated loads, there would be different ways in which this structure will collapse.
  - To illustrate this, consider the propped beam of Fig. 4.
  - The virtual-work method can be applied to this beam with various collapse mechanisms.

Plastic Analysis of Structure

- Complex Structures (cont'd)

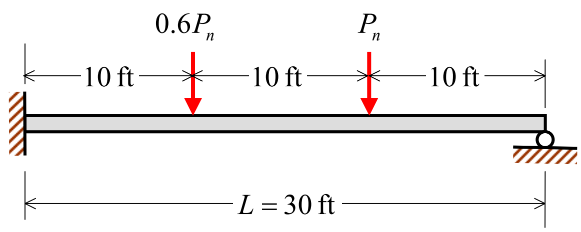


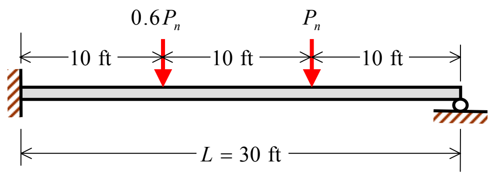
Figure 4

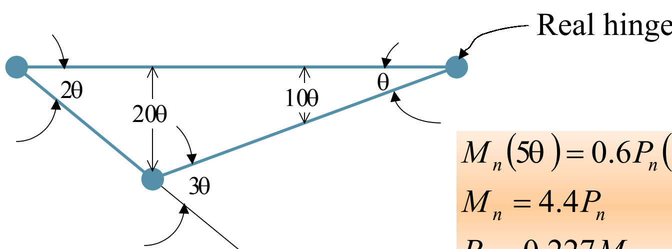
Plastic Analysis of Structure

- Complex Structures (cont'd)
  - The beam with its two concentrated loads is shown in Fig. 5 together with four possible collapse mechanisms and the necessary calculations.
  - It is true that the mechanisms of parts (a), (c), and (d) of Fig. 5 do not control, but such a fact is not obvious for those taking an introductory course in plastic analysis.
  - Therefore, it is necessary to consider all cases.

Plastic Analysis of Structure

Figure 5a. Various Cases of Collapse Mechanism

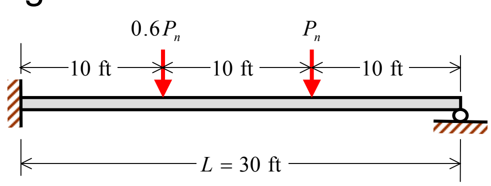


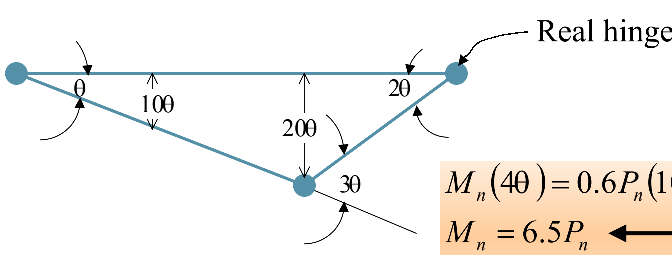


$$M_n(5\theta) = 0.6P_n(20\theta) + P_n(10\theta)$$
$$M_n = 4.4P_n$$
$$P_n = 0.227M_n$$

Plastic Analysis of Structure

Figure 5b. Various Cases of Collapse Mechanism



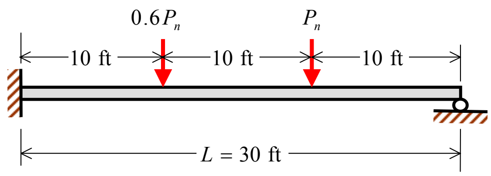


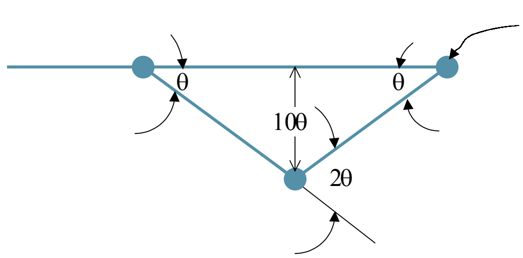
$$M_n(4\theta) = 0.6P_n(10\theta) + P_n(20\theta)$$
$M_n = 6.5P_n$ ⟵ **Controls**

$P_n = 0.154M_n$ ⟵ **Controls**

Plastic Analysis of Structure

Figure 5c. Various Cases of Collapse Mechanism

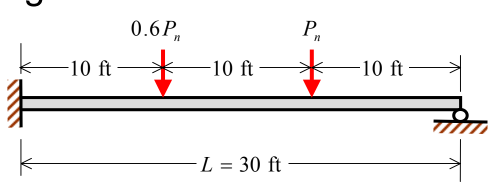


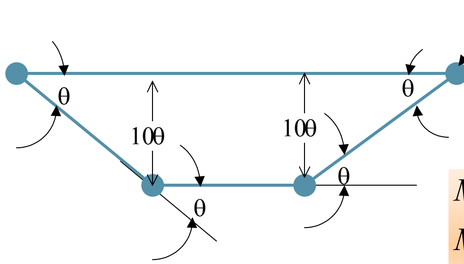


$$M_n(3\theta) = P_n(10\theta)$$
$$M_n = 3.33P_n$$
$$P_n = 0.3M_n$$

Plastic Analysis of Structure

Figure 5d. Various Cases of Collapse Mechanism





$$M_n(3\theta) = 0.6P_n(10\theta) + P_n(10\theta)$$
$$M_n = 5.33P_n$$
$$P_n = 0.1875M_n$$

Plastic Analysis of Structure

- Complex Structures (cont'd)
  - The value for which the collapse load P_n is the *smallest* in terms of M_n is the correct value.
  - or the value where M_n is the *greatest* in terms of P_n.
  - For this beam, the second plastic hinge forms at the concentrated load P_n, and P_n equals 0.154 M_n.