Structural Steel Design
LRFD Method

Third Edition

CHAPTER

INTRODUCTION TO BEAMS

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Part II – Structural Steel Design and Analysis

8b

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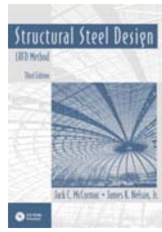

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ENCE 355 - Introduction to Structural Design

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Elastic Design

- For many years the elastic theory has been the basis for steel structural design and analyses. This theory is based on the yield stress of a steel structural element.
- However, nowadays, it has been replaced with a more rational and realistic theory, the ultimate stress design that is based on the plastic capacity of a steel structure.

Elastic Design

- In the elastic theory, the maximum load that a structure could support is assumed to equal the load that caused a stress somewhere in the structure equal the yield stress F_y of the material.
- The members were designed so that computed bending stresses for service loads did not exceed the yield stress divided a factor of safety (e.g., 1.5 to 2)

Elastic Design

- Elastic Versus Ultimate-based Design of Steel Structures

$$\frac{R_n}{\text{FS}} \geq \sum_{i=1}^{m} L_i$$

ASD

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_i$$

LRFD

  - According to ASD, one factor of safety (FS) is used that accounts for the entire uncertainty in loads and strength.
  - According to LRFD (probability-based), different partial safety factors for the different load and strength types are used.

Elastic Design

- Engineering structures have been designed for many years by the allowable stress design (ASD), or elastic design with satisfactory results.
- However, engineers have long been aware that ductile members (e.g., steel) do not fail until a great deal of yielding occurs after yield stress is first reached.
- This mean that such members have greater margin of safety against collapse than the elastic theory would seem to suggest.

The Elastic Modulus

- The yield moment M_y equals the yield stress F_y times the elastic modulus S:

$$M_y = F_y S \qquad (1)$$

where

$$S = \frac{I}{c}$$

I = moment of inertia

c = distance from N.A. to outer fiber of cross section

The Elastic Modulus

- The elastic modulus for a rectangular section $b \times d$ as shown in Fig. 1 can be computed by using:
  - The flexural formula, or
  - The internal couple method

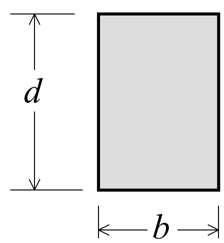


Figure 1

The Elastic Modulus

- Using the Flexural Formula
  - Rectangular Cross Section:

$$F_y = \frac{M_y c}{I} = \frac{M_y}{I/c} = \frac{M_y}{S}$$

$$M_y = F_y S$$

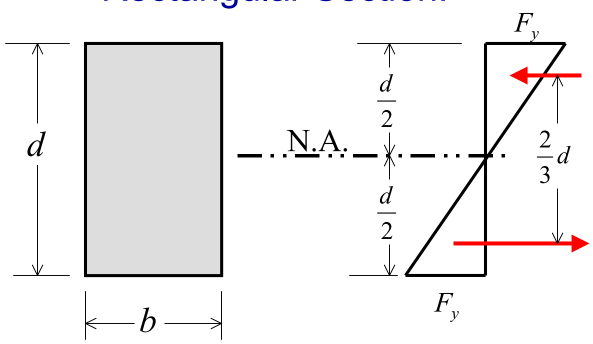
$$I = \frac{bd^3}{12},\ c = \frac{d}{2} \Rightarrow S = \frac{I}{c} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$$

$$\therefore M_y = F_y S = \frac{F_y bd^2}{6}$$

The Elastic Modulus

- Using the Internal Couple Method:
  - Rectangular Section:

Figure 2



$$C = \frac{1}{2}F_y\left(\frac{d}{2}\times b\right) = \frac{F_y db}{4}$$

$$T = \frac{1}{2}F_y\left(\frac{d}{2}\times b\right) = \frac{F_y db}{4}$$

$$M_y = \text{Force}\times\text{moment arm} = \left(\frac{F_y db}{4}\right)\times\left(\frac{2}{3}d\right) = \frac{F_y bd^2}{6}$$

The Plastic Modulus

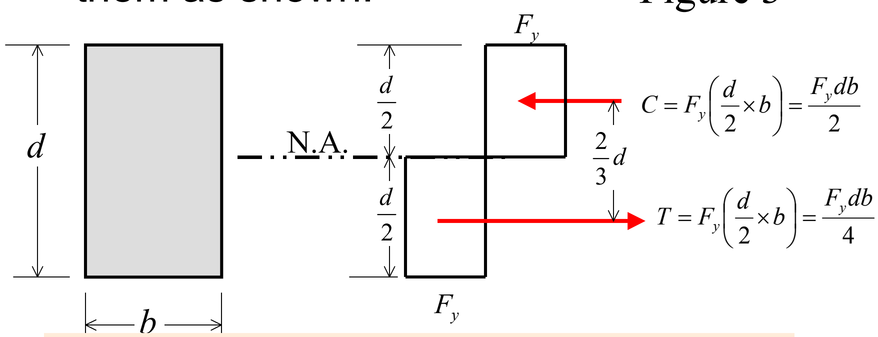
- The resisting moment at full plasticity can be determined in a similar manner.
- The result is the so-called plastic moment M_p.
- It is also the nominal moment of the section, M_n

$$M_p = M_n \qquad (2)$$

The Plastic Modulus

- The plastic (or nominal) moment equals T or C times the lever arm between them as shown.

Figure 3



$$M_p = \text{Force} \times \text{lever arm} = T\frac{d}{2} = C\frac{d}{2} = \left(\frac{F_y db}{2}\right)\times\frac{d}{2} = F_y\frac{bd^2}{4}$$

The Plastic Modulus

- The plastic moment is equal to the yield stress F_y times the plastic modulus Z.
- From the foregoing expression for a rectangular section, the plastic modulus Z can be seen to equal $bd^2/4$.

$$M_p = F_y Z = F_y\left(\frac{bd^2}{4}\right)$$

$$Z = \frac{bd^2}{4}$$

The Plastic Modulus

- The shape factor, which is equal

$$\frac{M_p}{M_y} = \frac{F_y Z}{F_y S} = \frac{Z}{S}$$

- Is also equal to

$$\text{Shape Factor} = \frac{Z}{S} = \frac{\frac{bd^2}{4}}{\frac{bd^2}{6}} = 1.5$$

So, for rectangular section, the shape factor equal 1.5.

The Plastic Modulus

- Shape Factor
  - Definition

    "The shape factor of a member cross section can be defined as the ratio of the plastic moment M_p to yield moment M_y".
  - The shape factor equals 1.50 for rectangular cross sections and varies from about 1.10 to 1.20 for standard rolled-beam sections

The Plastic Modulus

■ Shape Factor

The shape factor Z can be computed from the following expressions:

$$\text{Shape Factor} = \frac{M_P}{M_y} \qquad (3)$$

Or from

$$\text{Shape Factor} = \frac{Z}{S} \qquad (4)$$

The Plastic Modulus

■ Neutral Axis for Plastic Condition

- The neutral axis for plastic condition is different than its counterpart for elastic condition.
- Unless the section is symmetrical, the neutral axis for the plastic condition will not be in the same location as for the elastic condition.
- The total internal compression must equal the total internal tension.

The Plastic Modulus

- Neutral Axis for Plastic Condition
  - As all fibers are considered to have the same stress F_y in the plastic condition, the areas above and below the plastic neutral axis must be equal.
  - This situation does not hold for unsymmetrical sections in the elastic condition.

The Plastic Modulus

- Plastic Modulus
  - Definitions

"The plastic modulus *Z* is defined as the ratio of the plastic moment M_p to the yield stress F_Y."

"It can also be defined as the first moment of area about the neutral axis when the areas above and below the neutral axis are equal."

The Plastic Modulus

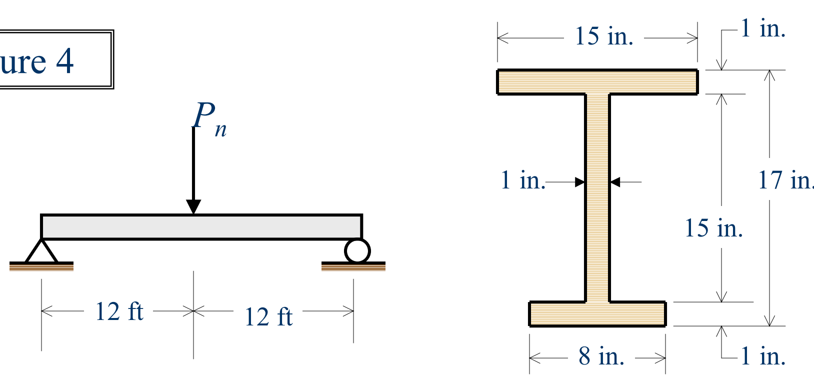
- Example 1

  Determine the yield moment M_y, the plastic or nominal moment M_p (M_n), and the plastic modulus Z for the simply supported beam having the cross section shown in Fig. 4b. Also calculate the shape factor and nominal load P_n acting transversely through the midspan of the beam. Assume that F_Y = 50 ksi.

The Plastic Modulus

- Example 1 (cont'd)

Figure 4

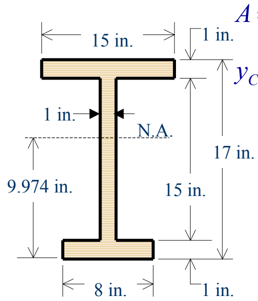


(a) (b)

The Plastic Modulus

- Example 1 (cont'd)

Elastic Calculations:



$$A = 15(1) + 15(1) + 8(1) = 38 \text{ in}^2$$

$$y_C = \frac{15(1)(16.5) + 15(1)(8.5) + 8(1)(0.5)}{38} = 9.974 \text{ in from lower base}$$

$$I_x = \frac{8(9.974)^3}{3} - \frac{7(8.974)^3}{3} + \frac{15(7.026)^3}{3} - \frac{14(6.026)^3}{3}$$

$$= 1{,}672.64 \text{ in}^4$$

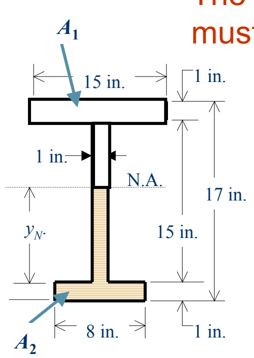
$$S = \frac{I}{c} = \frac{1{,}672.64}{9.974} = 167.7 \text{ in}^3 \quad M_y = F_Y S = \frac{50(167.7)}{12} = \boxed{698.75 \text{ ft-kip}}$$

The Plastic Modulus

- Example 1 (cont'd)

Plastic Calculations:

- The areas above and below the neutral axis must be equal for plastic analysis



$$A_1 = A_2$$

$$15(1) + (15 - y_N)(1) = 8(1) + y_N(1)$$

$$15 + 15 - y_N = 8 + y_N$$

$$2y_N = 15 + 15 - 8 = 22$$

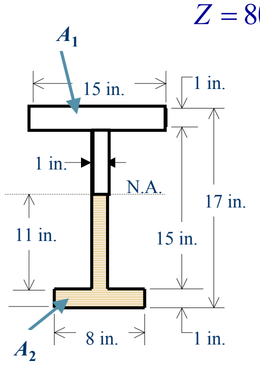
$$y_N = 11 \text{ in}$$

The Plastic Modulus

- Example 1 (cont'd)

Plastic Calculations (cont'd):

$$Z = 8(1)(11.5) + 11(1)(5.5) + 15(1)(4.5) + 4(1)(2) = 228 \text{ in}^3$$



$$M_p = M_n = F_y Z = \frac{50(228)}{12} = 950 \text{ ft-kip}$$

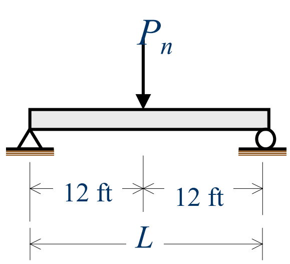
$$\text{Shape Factor} = \frac{M_n}{M_y} = \frac{950}{698.75} = 1.36$$

Note, the shape factor can also be calculated from

$$\text{Shape Factor} = \frac{Z}{S} = \frac{228}{167.7} = 1.36$$

The Plastic Modulus

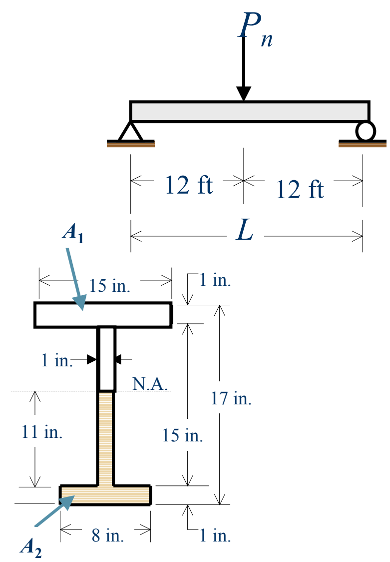
- Example 1 (cont'd)
  - In order to find the nominal load P_n, we need to find an expression that gives the maximum moment on the beam. This maximum moment occurs at midspan of the simply supported beam, and is given by



$$M_P = M_{L/2} = \frac{P_n L}{4}$$

The Plastic Modulus

- Example 1 (cont'd)



$$M_P = M_{L/2} = \frac{P_n L}{4}$$

$$950 = \frac{P_n(24)}{4}$$

Therefore,

$$P_n = \frac{4(950)}{24} = \boxed{158.3 \text{ kips}}$$

Theory of Plastic Analysis

- The basic theory of plastic analysis is considered a major change in the distribution of stresses after the stresses at certain points in a structure reach the yield stress F_y.
- The plastic theory implies that those parts of the structure that have been stressed to the yield stress F_y cannot resist additional stresses.

Theory of Plastic Analysis

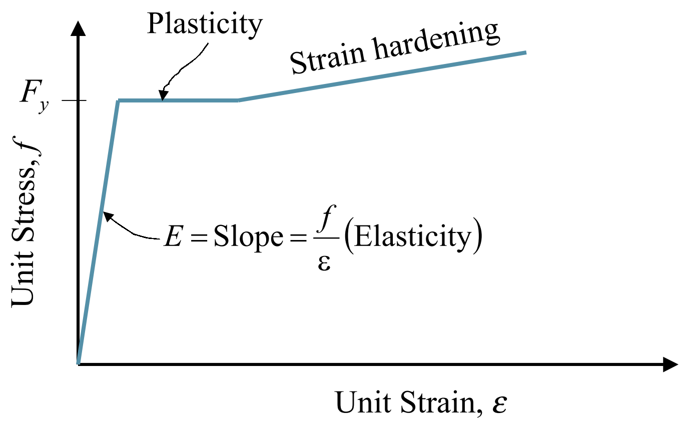
- They instead will yield the amount required to permit the extra load or stresses to be transferred to other parts of the structure where the stresses are below the yield stress F_y, and thus in the elastic range and able to resist increased stress.
- Plasticity can be said to serve the purpose of equalizing stresses in cases of overload.

Theory of Plastic Analysis

- Idealized Stress-Strain Diagram for Steel
  - The stress-strain diagram is assumed to have the idealized shape shown in Fig. 5.
  - The yield stress and the proportional limit are assumed to occur at the same point for this steel.
  - Also, the stress-strain diagram is assumed to be a perfectly straight line in the plastic range.
  - Beyond the plastic range there is a range of strain hardening.

Theory of Plastic Analysis

Figure 5. Stress-Strain Diagram for Steel



Theory of Plastic Analysis

- Idealized Stress-Strain Diagram for Steel
  - The strain hardening range could theoretically permit steel members to withstand additional stress.
  - However, from a practical standpoint, the stains occurring are so large that they cannot be considered.
  - Furthermore, inelastic buckling will limit the ability of a section to develop a moment greater than M_p, even if strain hardening is significant.