Introduction

- The members that can be designed for compression include:
  - Single shapes
  - W sections with cover plates
  - Built-up sections constructed with channels
  - Sections whose unbraced lengths in the \( x \) and \( y \) directions.
  - Lacing and tie plates for built-up sections with open sides.
Introduction

The Design Process for Columns

- It is to be noted that the design of columns using formulas involves a trial-and-error process.
- The design stress $\phi_c F_{cr}$ is not known until a column size is selected and vice versa.
- Once a trial section is assumed, the $r$ value for that section can be obtained and substituted into the appropriate column equation to determine its design stress.

In the design of columns, the factored load $P_u$ is computed for a particular column and then divided by an assumed design stress to give an estimated column area $A$, that is

$$A_{\text{estimated}} = \frac{P_u}{\text{assumed stress}} = \frac{P_u}{\phi_c F_{cr}}$$

(1)
Introduction

The Design Process for Columns

- After an estimated column area is determined, a trial section can be selected with approximately that area.
- The design stress for the selected section can be computed and multiplied by the cross-sectional area of the section to obtain the member’s design strength.
- This design strength is compared with the factored load $P_u$. It must be equal or greater than the load $P_u$.

Introduction

General Notes on Column Design

- The effective slenderness ratio ($KL/r$) for the average column of 10 to 15 ft in length will generally fall between 40 and 60.
- A value for $KL/r$ in this range can be assumed and substituted into the appropriate column equation.
- Or instead of the column equation, tables in LRFD manual can be consulted to give the design strength for that particular $KL/r$ value. ($KL/r$ ranges from 1 to 200 in LRFD)
Introduction

Example 1

Using $F_y = 50$ ksi, select the lightest W14 section available for the service column loads $P_D = 130$ k and $P_L = 210$ k. Assume $KL = 10$ ft.

$$P_u = 1.2P_D = 1.2(130) =$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(130) + 1.6(210) = 492 \text{k}$$

Assume $\frac{KL}{r} = 50$

Example 1 (cont’d)

$\phi_c F_{cr}$ form Table 3.50 (Part 16 of Manual) = 35.4 ksi

$A_{\text{required}} = \frac{P_u}{\phi_c F_{cr}} = \frac{492}{35.4} = 13.90 \text{ in}^2$

Try W14×48 ($A = 14.1 \text{ in}^2$, $r_z = 5.85 \text{ in}$, $r_y = 1.91 \text{ in}$)

$K_L = \frac{12 \times 10}{1.91} = 62.83$

$\phi_c F_{cr}$ form Table 3.50 (Part 16 of Manual)

and by interpolation = 31.85 ksi
Introduction


### TABLE 3-50

Design Stress for Compression Members of 50 ksi Specified Yield Stress Steel, $\alpha_c = 0.85^{[g]}$

<table>
<thead>
<tr>
<th>$\alpha F_{cy}$ ksf</th>
<th>$K_I$</th>
<th>$\theta F_{cy}$ ksf</th>
<th>$K_I$</th>
<th>$\phi F_{cy}$ ksf</th>
<th>$K_I$</th>
<th>$\phi F_{cy}$ ksf</th>
<th>$K_I$</th>
<th>$\phi F_{cy}$ ksf</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.6</td>
<td>41</td>
<td>27.6</td>
<td>81</td>
<td>26.3</td>
<td>121</td>
<td>14.9</td>
<td>161</td>
<td>8.23</td>
</tr>
<tr>
<td>42.5</td>
<td>42</td>
<td>27.4</td>
<td>82</td>
<td>26.2</td>
<td>122</td>
<td>14.9</td>
<td>163</td>
<td>8.13</td>
</tr>
<tr>
<td>42.3</td>
<td>43</td>
<td>27.1</td>
<td>83</td>
<td>25.7</td>
<td>123</td>
<td>14.1</td>
<td>163</td>
<td>8.03</td>
</tr>
<tr>
<td>42.8</td>
<td>44</td>
<td>26.9</td>
<td>84</td>
<td>25.4</td>
<td>124</td>
<td>14.0</td>
<td>164</td>
<td>7.93</td>
</tr>
<tr>
<td>42.0</td>
<td>45</td>
<td>26.7</td>
<td>85</td>
<td>25.1</td>
<td>125</td>
<td>13.7</td>
<td>165</td>
<td>7.84</td>
</tr>
<tr>
<td>42.4</td>
<td>46</td>
<td>26.4</td>
<td>86</td>
<td>24.8</td>
<td>126</td>
<td>13.4</td>
<td>166</td>
<td>7.74</td>
</tr>
<tr>
<td>42.4</td>
<td>47</td>
<td>26.2</td>
<td>87</td>
<td>24.4</td>
<td>127</td>
<td>13.0</td>
<td>167</td>
<td>7.66</td>
</tr>
<tr>
<td>42.3</td>
<td>48</td>
<td>26.0</td>
<td>88</td>
<td>24.1</td>
<td>128</td>
<td>13.0</td>
<td>168</td>
<td>7.56</td>
</tr>
<tr>
<td>42.3</td>
<td>49</td>
<td>25.7</td>
<td>89</td>
<td>23.8</td>
<td>129</td>
<td>12.0</td>
<td>169</td>
<td>7.47</td>
</tr>
<tr>
<td>42.2</td>
<td>50</td>
<td>25.4</td>
<td>90</td>
<td>23.5</td>
<td>130</td>
<td>12.0</td>
<td>170</td>
<td>7.38</td>
</tr>
</tbody>
</table>
Introduction

Example 1 (cont’d)

\[ \phi_c P_n = (\phi_c F_{cr}) A_g = 31.85(14.1) = 449 \text{k} < 492 \text{k} \]

\[ \therefore \text{try next larger W14} \]

Try W14×53 \( A = 15.6 \text{in}^2, r_x = 5.89 \text{in}, r_y = 1.92 \text{in} \)

\[ r_y = 1.92 \text{ controls} \]

\[ \frac{KL}{r_y} = \frac{12 \times 10}{1.92} = 62.5 \]

\[ \phi_c F_{cr} \text{ form Table 3.50 (Part 16 of Manual)} \]

and by interpolation = 31.95 ksi

Example 1 (cont’d)

\[ \therefore \phi_c P_n = (\phi_c F_{cr}) A_g = 31.95(15.6) = 498 \text{k} < 492 \text{k} \]

OK

Checking width-thickness ratio for W14 × 53:

W14 × 53 \( b_f = 8.060 \text{in}, t_f = 0.660 \text{in}, \)

\( k = 1.25 \text{in}, d = 13.9 \text{in}, t_w = 0.370 \text{in} \)

\[ \frac{b_f}{2t_f} = \frac{8.060}{2(0.660)} = 6.11 < 0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{29 \times 10^3 / 50} = 13.49 \]

OK

\[ \frac{h}{t_w} = \frac{13.9 - 2(1.25)}{0.370} = 30.81 < 1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{29 \times 10^3 / 50} = 35.88 \]

OK
**LRFD Design Tables**

- The LRFD Manual can be used to select various column sections from tables without the need of using a trial-and-error procedures.
- These tables provide axial design strengths $\phi_cP_n$ for various practical effective lengths of the steel sections commonly used as columns.

---

**LRFD Design Tables**

- LRFD Manual Design Tables (P. 4-25)
The values are given with respect to the least radii of gyration for W’s and WT’s with 50 ksi steel. Other grade steels are commonly used for other types of sections as shown in the Manual and listed there. These include 35 ksi for steel pipe, 36 ksi for L’s, 42 ksi for round HSS sections, and 46 ksi for square and rectangular HSS sections.

For most columns consisting of single steel shapes, the effective slenderness ratio with respect to the y axis \((KL/r)_y\) is larger than the effective slenderness ratio with respect to the x axis \((KL/r)_x\). As a result, the controlling or smaller design stress is for the y axis. Because of this, the LRFD tables provide design strengths of columns with respect to their y axis.
**Example 2**

Using the LRFD column tables with their given yield strengths:

a. Select the lightest W section available for the loads, steel, and $KL$ of Example 1. Use $F_y = 50$ ksi.

b. Select the lightest satisfactory standard (S), extra strong (XS), and double extra strong (XXS) pipe columns described in part (a) of this example. Use $F_y = 35$ ksi.

c. Select the lightest satisfactory rectangular and square HSS sections for the situation in part (a). Use $F_y = 46$ ksi.

d. Select the lightest round HSS section for part (a). Use $F_y = 42$ ksi.

--

**Example 2 (cont’d)**

a. Enter LRFD tables with $K_y L_y = 10$ ft., $P_u = 492$ k, and $F_y = 50$ ksi.
Example 2 (cont’d)

Lightest suitable section in each W series:

Lightest

\[ W_{14} \times 53(\phi_c P_n) = 498 \text{ k} \]

\[ W_{12} \times 53(\phi_c P_n) = 559 \text{ k} \]

\[ W_{10} \times 49(\phi_c P_n) = 520 \text{ k} \]

Page 4-26 of Manual

Therefore, USE W10×49

b. Pipe Columns:

Page 4-76 of Manual

\[ S : \text{not available} \]

\[ X_{S12} \times 0.500(65.5 \text{ lb/ft}) = 549 \text{ k} \]

\[ X_{S8} \times 0.875(72.5 \text{ lb/ft}) = 575 \text{ k} \]

c. Rectangular and square HSS sections:

Page 4-49 of Manual

\[ HSS_{14\times14} \times \frac{5}{16} \times (57.3 \text{ lb/ft}) = 530 \text{ k} \]

Page 4-51 of Manual

\[ HSS_{12\times10} \times \frac{3}{8} \times (52.9 \text{ lb/ft}) = 537 \text{ k} \]

d. Round HSS section:

Page 4-66 of Manual

\[ HSS_{16} \times 0.312(52.3 \text{ lb/ft}) = 500 \text{ k} \]
LRFD Design Tables

How to handle the situation when \((KL/r)_x\) is larger than \((KL/r)_y\)?

- Two methods can be used:
  - Trial-and-error method
  - Use of LRFD Tables

- An axially loaded column is laterally restrained in its weak direction as shown in Figs. 1 and 2

---

Figure 1

This brace must be a section which prevents lateral movement and twisting of the column. A rod or bar is not satisfactory.
How to handle the situation when $(KL/r)_x$ is larger than $(KL/r)_y$?

**Trial-and-error Procedure:**

- A trial section can be selected as described previously.
- Then the slenderness values $(KL/r)_x$ and $(KL/r)_x$ are computed.
- Finally, $\phi_c F_{cr}$ is determined for the larger value of $(KL/r)_x$ and $(KL/r)_x$ and multiplied by $A_g$ to obtain $\phi_c P_n$.
- Then if necessary, another size can be tried, and so on.
LRFD Design Tables

■ How to handle the situation when \((KL/r)_x\) is larger than \((KL/r)_y\)?

– It is assumed that \(K\) is the same in both directions. Then, if equal strengths about the \(x\) and \(y\) axis to be obtained, the following relation must hold:

\[
\frac{L_x}{r_x} = \frac{L_y}{r_y}
\]

(2)

LRFD Design Tables

■ How to handle the situation when \((KL/r)_x\) is larger than \((KL/r)_y\)?

– For \(L_y\) to be equivalent to \(L_x\), the following relation would hold true:

\[
L_x = L_y \cdot \frac{r_x}{r_y}
\]

(3)

– If \(L_y \cdot (r_x/r_y)\) is less than \(L_x\), then \(L_x\) controls.
– If \(L_y \cdot (r_x/r_y)\) is greater than \(L_x\), then \(L_y\) controls.
LRFD Design Tables

How to handle the situation when \((KL/r)_x\) is larger than \((KL/r)_y\) ?

Use of LRFD Tables:

- Based on the preceding information, the LRFD Manual provides a method with which a section can be selected from tables with little trial and error when the unbraced lengths are different.
- The designer enters the appropriate table with \(K_yL_y\), selects a shape, takes \(r_x/r_y\) value in the table for that shape, and multiplies it by \(L_y\).
- If the result is larger than \(K_xL_x\), then \(K_yL_y\) controls and the shape initially selected is the correct one.

Use of LRFD Tables (cont’d):

- If the result of the multiplication is less than \(K_xL_x\), then \(K_xL_x\) controls and the designer will reenter the tables with a larger \(K_yL_y\) equal to \(K_xL_x/(r_x/r_y)\) and select the final section.
LRFD Design Tables

Example 3

Select the lightest satisfactory W12 for the following conditions: $F_y = 50$ ksi, $P_u = 900$ k, $K_xL_x = 26$ ft, and $K_yL_y = 13$ ft.

a. By trial and error
b. Using LRFD tables

---

Using trial and error:

Assume $\frac{KL}{r} = 50$

---

Example 3 (cont’d)

$K = 1$

$KL = L = 26$ ft

Bracing

---
LRFD Design Tables

Example 3 (cont’d)

\[ \phi_e F_{cr} = 35.40 \text{ ksi (from Table 3.50 of Manual)} \]

\[ A_{\text{required}} = \frac{P}{\phi_e F_{cr}} = \frac{900}{35.40} = 25.42 \text{ in}^2 \]

Try W12 \times 87 (\( A = 25.6 \text{ in}^2, r_x = 5.38 \text{ in}, r_y = 3.07 \text{ in} \))

\[ \frac{KL}{r_x} = \frac{12 \times 26}{5.38} = 57.99 \approx 58 \quad \text{controls} \]

\[ \frac{KL}{r_y} = \frac{12 \times 13}{3.07} = 50.81 \]

\[ \phi_e F_{cr} = 33.2 \text{ ksi} \quad \therefore \phi_e P_n = 33.2(25.6) = 850 \text{ k} < 900 \text{ k} \]

NG

A subsequent check of the next larger W section (W12 \times 96) shows it will work.

Therefore, USE W12 \times 96

---

LRFD Design Tables

LRFD Design Tables

- LRFD Manual Design Tables (P. 16.1-145)

### TABLE 3-50
Design Stress for Compression Members of 50 ksi Specified Yield Stress Steel, $\phi = 0.85$[8]

<table>
<thead>
<tr>
<th>$\phi F_y$ ksl</th>
<th>$K_l$</th>
<th>$\phi F_y$ ksl</th>
<th>$K_l$</th>
<th>$\phi F_y$ ksl</th>
<th>$K_l$</th>
<th>$\phi F_y$ ksl</th>
<th>$K_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.6</td>
<td>55</td>
<td>34.1</td>
<td>95</td>
<td>22.0</td>
<td>135</td>
<td>11.7</td>
<td>175</td>
</tr>
<tr>
<td>41.7</td>
<td>55</td>
<td>33.8</td>
<td>95</td>
<td>21.7</td>
<td>136</td>
<td>11.5</td>
<td>176</td>
</tr>
<tr>
<td>41.6</td>
<td>57</td>
<td>33.5</td>
<td>97</td>
<td>21.4</td>
<td>137</td>
<td>11.4</td>
<td>177</td>
</tr>
<tr>
<td>41.5</td>
<td>55</td>
<td>33.2</td>
<td>98</td>
<td>21.1</td>
<td>138</td>
<td>11.2</td>
<td>178</td>
</tr>
<tr>
<td>41.4</td>
<td>59</td>
<td>33.0</td>
<td>99</td>
<td>20.8</td>
<td>139</td>
<td>11.0</td>
<td>179</td>
</tr>
<tr>
<td>41.3</td>
<td>61</td>
<td>32.7</td>
<td>100</td>
<td>20.5</td>
<td>140</td>
<td>10.9</td>
<td>180</td>
</tr>
<tr>
<td>41.2</td>
<td>61</td>
<td>32.4</td>
<td>101</td>
<td>20.2</td>
<td>141</td>
<td>10.7</td>
<td>181</td>
</tr>
<tr>
<td>41.0</td>
<td>62</td>
<td>32.1</td>
<td>102</td>
<td>19.9</td>
<td>142</td>
<td>10.6</td>
<td>182</td>
</tr>
<tr>
<td>40.9</td>
<td>63</td>
<td>31.8</td>
<td>103</td>
<td>19.6</td>
<td>143</td>
<td>10.4</td>
<td>183</td>
</tr>
<tr>
<td>40.8</td>
<td>64</td>
<td>31.5</td>
<td>104</td>
<td>19.3</td>
<td>144</td>
<td>10.3</td>
<td>184</td>
</tr>
</tbody>
</table>

**Example 3 (cont’d)**

b. **Using LRFD tables:**

Enter tables with $K_y L_y = 13$ ft, $F_y = 50$ ksi, and $P_u = 900$ k.

Try $W12 \times 87 \left( \frac{r_x}{r_y} = 1.75 \right)$ with $\phi P_n$ based on $K_y L_y$

Equivalent $K_y L_y = \frac{K_y L_y}{r_x/r_y} = 13(1.75) = 22.75$ ft $< K_y L_y$

Therefore, $K_y L_y$ controls.

Reenter tables with $K_y L_y = \frac{K_y L_y}{r_x/r_y} = \frac{26}{1.75} = 14.86$

**USE W12 × 96** $\phi P_n = P_u = 935$ k

**OK**
Built-up Columns

- Compression members may be constructed with more shapes built-up into a single member.
- They may consist of parts in contact with each other, such as cover-plated sections:

![Cover-Plated Section Diagram](image)

Built-up Columns

- Or they may consist of parts in near contact with each other, such as pair of angles:

![Pair of Angles Diagram](image)

- These pairs of angles may be separated by a small distance from each other equal the thickness of the end connection or gusset plates between them.
Built-up Columns

- They may consist of parts that are spread well apart, such as pairs of channels:

- Or four angles, and so on.

Built-up Columns

- Two-angle sections probably are the most common type of built-up members. They are frequently used as the members of light trusses.

- When a pair angles are used as a compression member, they need to be fastened together so they will act as a unit.

- Welds may be used at intervals or they may be connected with bolts.
Built-up Columns

- For long columns, it may be suitable to use built-up sections where the parts of the columns are spread out or widely separated from each other.

- These types of built-up columns are commonly used for crane booms and for compression members of various kinds of towers.

- The widely spaced parts of these types must be carefully laced or tied together.