CHAPTER

5c

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Structural Steel Design
LRFD Method

Third Edition

INTRODUCTION TO AXIALLY LOADED COMPRESSION MEMBERS

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Part II – Structural Steel Design and Analysis

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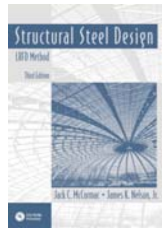

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ENCE 355 - Introduction to Structural Design

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End Restraint and Effective Lengths of Columns

- Limitations of Basic Euler Formula
  - The basic Euler formula is only useful if the end support conditions are carefully considered.
  - The results obtained by application of the formula to specific examples compare very well with test results for centrally loaded, long, slender columns with rounded ends.

End Restraint and Effective Lengths of Columns

- Limitations of Basic Euler Formula
  - In real life, these types of columns barely exists.
  - For example, the columns with which one works do not have rounded ends and are not free to rotate because their ends are bolted or welded to other members.
  - Furthermore, the axial load applied to these columns are not centric in most cases.

End Restraint and Effective Lengths of Columns

- Effect of End Restraint on Column Load Capacity
  - End restraint and its effect on the load-carrying capacity of columns is very important subject.
  - Columns with appreciable rotational and transnational end restraint can support considerably more load than those with little rotational end restraint as at hinged end.

End Restraint and Effective Lengths of Columns

- General Notes On Column Buckling

1. Boundary conditions other than simply-supported will result in different critical loads and mode shapes.
2. The buckling mode shape is valid only for small deflections, where the material is still within its elastic limit.
3. The critical load will cause buckling for slender, long columns. In contrast, failure will occur in short columns when the strength of material is exceeded. Between the long and short column

End Restraint and Effective Lengths of Columns

- General Notes On Column Buckling

limits, there is a region where buckling occurs after the stress exceeds the proportional limit but is still below the ultimate strength. These columns are classified as intermediate and their failure is called inelastic buckling.

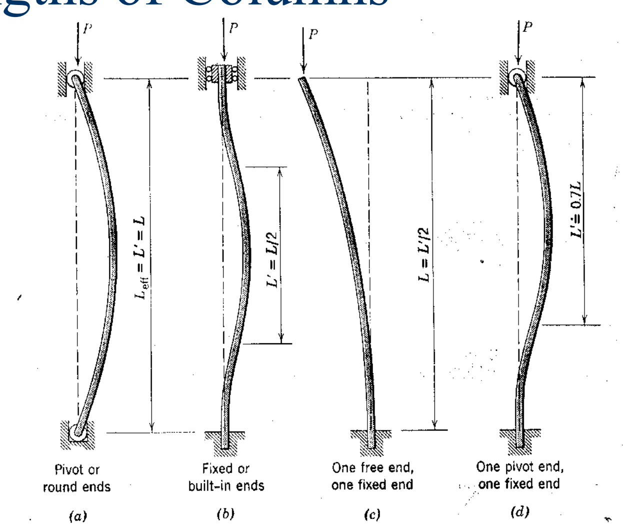
4. Whether a column is short, intermediate, or long depends on its geometry as well as the stiffness and strength of its material. This concept is addressed in the columns introduction page.

End Restraint and Effective Lengths of Columns

- The Concept of Effective Length
  - The Euler buckling formula, namely Eqs. 9 or 11 of Chapter 5b, were derived for a column with pivoted ends.
  - The Euler equation changes for columns with different end conditions, such as the four common ones found in Figs.1and 2.
  - While it is possible to set up the differential equation with appropriate boundary conditions to determine the Euler buckling

End Restraint and Effective Lengths of Columns

Figure 1



End Restraint and Effective Lengths of Columns

Figure 2

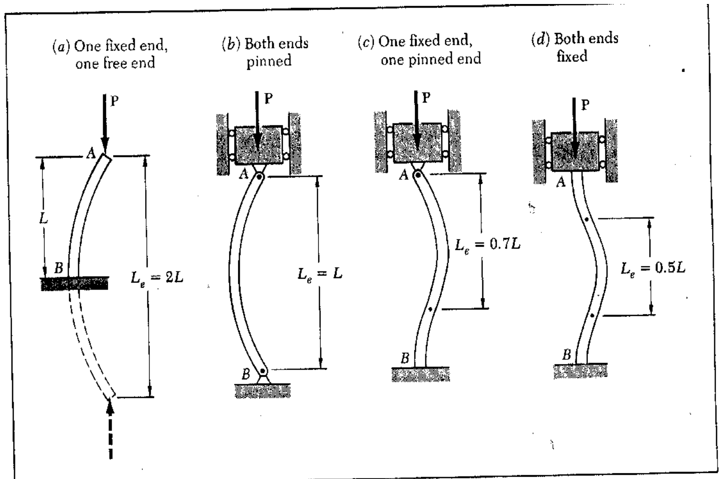


Fig. 11.17 Effective length of column for various end conditions. (Beer and Johnston 1992)

End Restraint and Effective Lengths of Columns

- The Concept of Effective Length

  formula for each case, a more common approach makes use of the concept of an "***effective length***".

  – The pivoted ended column, by definition, has zero bending moments at each end.

  – The length L in the Euler equation, therefore, is the distance between successive points of zero bending moment.

End Restraint and Effective Lengths of Columns

- The Concept of Effective Length
  - All that is needed to modify the Euler column formula for use with other end conditions is to replace L by L'.
  - L' is defined as the *effective length* of the column.

$$L' = L_e = KL \qquad (1)$$

where

K = effective length factor as defined by LRFD

End Restraint and Effective Lengths of Columns

- The Effective Length Concept

  Definition:

  The effective length L' (or L_e or KL) of a column is defined as the distance between successive inflection points or points of zero moment.

End Restraint and Effective Lengths of Columns

- The Effective Length Concept

  Based on the effective length concept, the Euler buckling load and stress formulas become, respectively

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad (2)$$

or

$$F_e = \frac{\pi^2 E}{(KL/r)^2} \qquad (3)$$

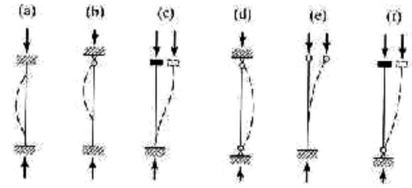
$KL = L_e = L' = \text{effective length}$

End Restraint and Effective Lengths of Columns

- AISC LRFD Specifications for End Restraint
  - Table C-C2.1 of the "Commentary on the LRFD Specification" gives recommended values of the effective length factors K when ideal conditions are approximated.
  - This table is reproduced here as Table 1 (Table 5.1, Textbook).
  - Two sets of K values are provided in the table.

End Restraint and Effective Lengths of Columns

Table 1



Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0

End condition code
- Rotation fixed and translation fixed
- Rotation free and translation fixed
- Rotation fixed and translation free
- Rotation free and translation free

Source: Load and Resistance Factor Design Specification for Structural Steel Buildings, December 27, 1999 (Chicago: AISC)

End Restraint and Effective Lengths of Columns

- AISC LRFD Specifications for End Restraint
  - One being the theoretical values and the other being the recommended design values.
  - The recommended values are based on the fact that perfectly pinned and fixed conditions are not always possible.
  - For continuous frames, the LRFD provides special *K*-value Charts for that purpose.

End Restraint and Effective Lengths of Columns

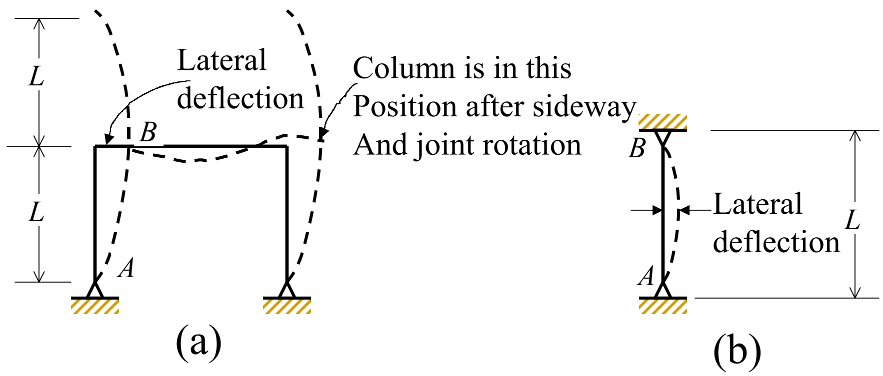
- Effect of Braced and Unbraced Structural Frames on Columns Strength
  - Structural steel columns can be parts of structural frames.
  - These frames are sometimes braced and sometimes unbraced.
  - A ***braced*** frame is one for which sideway (joint translation) by means of bracing, shear walls, or lateral support from adjoining structure (see Fig. 2a).

End Restraint and Effective Lengths of Columns

- Effect of Braced and Unbraced Structural Frames on Columns Strength
  - An ***unbraced*** frame does not have any of these types of bracing provided, and must depend on the stiffness of its own members and rotational rigidity of the joints between the frames members to prevent lateral buckling (see Fig. 2b)

End Restraint and Effective Lengths of Columns

■ Figure 2



End Restraint and Effective Lengths of Columns

■ Effect of Braced and Unbraced Structural Frames on Columns Strength

- Examination of Fig 2a will show that the effective length will exceed the actual length of the column as the elastic curve will theoretically take the shape of the curve of a pinned-end column of twice its length and K will theoretically equal 2.0.
- Notice in Fig 2b how much smaller the lateral deflection of column AB would be if it were pinned at both ends to prevent sideway.

End Restraint and Effective Lengths of Columns

- Effect of Braced and Unbraced Structural Frames on Columns Strength
  - For braced frames, K values can never be greater than 1.0, but for unbraced frames the K values will always be greater than 1.0 because of sideway.
  - The smaller the effective length (i.e., braced) of a particular column, the smaller its danger of lateral buckling and the greater its load-carrying capacity.

Stiffened and Unstiffened Elements

- Local Buckling
  - Up to this point, the overall stability of a particular column has been considered.
  - Yet, it is entirely possible for thin flanges or webs of a column or beam to ***buckle locally*** in compression well before the calculated buckling strength of the whole member is reached.
  - When thin plates are used to carry compressive stresses they are particularly susceptible to buckling about their weak axes due to small moment of inertia.

Stiffened and Unstiffened Elements

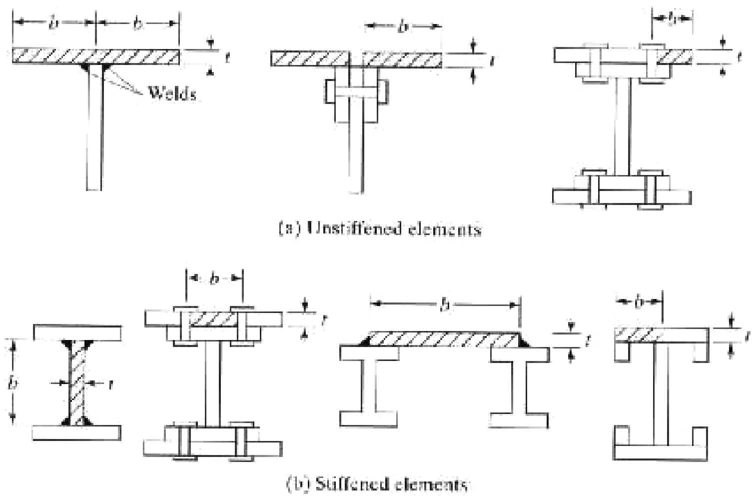
- LRFD Specification (Section B5)
  - The LRFD Specification provides limiting values for the width-thickness ratios of the individual parts of compression members and for the parts of beams in their compression regions.
  - Two categories are listed in the LRFD Manual:
    - Stiffened elements
    - Unstiffened elements

Stiffened and Unstiffened Elements

- LRFD Specification (Section B5)
  - An ***unstiffened element*** is a projecting piece with one free edge parallel to the direction of the compression force.
  - A ***stiffened element*** is supported along two edges in that direction.
  - These types of elements are shown in Figure 3. In each case, the width b and the thickness t of the elements in question are shown.

Stiffened and Unstiffened Elements

- Figure 3. Unstiffened and Stiffened Elements



Stiffened and Unstiffened Elements

- LRFD Specification (Section B5)
  – For establishing width-thickness ratio limits for elements of compression members, the LRFD Specification divides members into three distinct classifications as follows:
    1. Compact sections
    2. Noncompact sections
    3. Slender compression elements

Stiffened and Unstiffened Elements

- Compact Sections
  - A compact section is one that has a sufficiently stocky profile so that it is capable of developing a fully plastic stress distribution before buckling.
  - For a section to be compact, it has to have a width-thickness ratios equal to or less than the limiting values provided in Table 4 (Table 5.2, Text, or Table B5.1, LRFD Maual).

Stiffened and Unstiffened Elements

- Figure 4. Limiting Width-Thickness Ratios for Compression Elements

	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	Limiting Width-Thickness Ratios λ_r (noncompact)
Unstiffened Elements	Flanges of I-shaped rolled beams and channels in flexure	b/t	$0.38\sqrt{E/F_y}$ [c]	$0.83\sqrt{E/F_L}$ [e]
Unstiffened Elements	Flanges of I-shaped hybrid or welded beams in flexure	b/t	$0.38\sqrt{E/F_{yf}}$	$0.95\sqrt{E/(F_L/k_c)}$ [e], [f]
Unstiffened Elements	Flanges projecting from built-up compression members	b/t	NA	$0.64\sqrt{E/(F_y/k_c)}$ [f]
Unstiffened Elements	Flanges of I-shaped sections in pure compression, plates projecting from compression elements; outstanding legs of pairs of angles in continuous contact; flanges of channels in pure compression	b/t	NA	$0.56\sqrt{E/F_y}$
Unstiffened Elements	Legs of single angle struts; legs of double angle struts with separators; unstiffened elements, i.e., supported along one edge	b/t	NA	$0.45\sqrt{E/F_y}$
Unstiffened Elements	Stems of tees	d/t	NA	$0.75\sqrt{E/F_y}$

Stiffened and Unstiffened Elements

- Figure 4. (cont'd) Limiting Width-Thickness Ratios for Compression Elements

	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	Limiting Width-Thickness Ratios λ_r (noncompact)
Stiffened Elements	Flanges of rectangular box and hollow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t		
	for uniform compression		$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$
	for plastic analysis		$0.939\sqrt{E/F_y}$	•
	Unsupported width of cover plates perforated with a succession of access holes [b]	b/t	NA	$1.86\sqrt{E/F_y}$
	Webs in flexural compression [a]	h/t_w	$3.76\sqrt{E/F_y}$ [c], [g]	$5.70\sqrt{E/F_y}$ [h]

Stiffened and Unstiffened Elements

- Figure 4. (cont'd) Limiting Width-Thickness Ratios for Compression Elements

Stiffened Elements	Webs in combined flexural and axial compression	h/t_w	for $P_u/\phi_b P_y \leq 0.125$ [c], [g] $3.76\sqrt{\frac{E}{F_y}}\left(1 - \frac{2.75P_u}{\phi_b P_y}\right)$; for $P_u/\phi_b P_y > 0.125$ [c], [g] $1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$	[h] $5.70\sqrt{\frac{E}{F_y}}\left(1 - 0.74\frac{P_u}{\phi_b P_y}\right)$
	All other uniformly compressed stiffened elements, i.e., supported along two edges	b/t h/t_w	NA	$1.49\sqrt{E/F_y}$
	Circular hollow sections In axial compression In flexure	D/t	[d] NA $0.07E/F_y$	$0.11E/F_y$ $0.31E/F_y$

Stiffened and Unstiffened Elements

- Figure 4. (cont'd) Limiting Width-Thickness Ratios for Compression Elements

[a] For hybrid beams, use the yield strength of the flange F_{yf} instead of F_y.

[b] Assumes net area of plate at widest hole.

[c] Assumes an inelastic rotation capacity of 3 radians. For structures in zones of high seismicity, a greater rotation capacity may be required.

[d] For plastic design use $0.045E/F_y$.

[e] F_L = smaller of $(F_{yf} - F_r)$ or F_{yw}, ksi (MPa)
F_r = compressive residual stress in flange
= 10 ksi (69 MPa) for rolled shapes
= 16.5 ksi (114 MPa) for welded shapes

[f] $k_c = \dfrac{4}{\sqrt{h/t_w}}$ and $0.35 \le k_c \le 0.763$

[g] For members with unequal flanges use h_p instead of h when comparing to λ_p.

[h] For members with unequal flanges, see Appendix B5.1.

Stiffened and Unstiffened Elements

- Noncompact Sections
  - A noncompact section is one for which the yield stress can be reached in some but not all of its compression elements before buckling occurs.
  - It is not capable of reaching fully plastic stress distribution.
  - For a section to be noncompact, it has to have a width-thickness ratios greater than λ_p but less than λ_r as provided in Table 4 (Table 5.2, Text, or Table B5.1, LRFD Maual).

Stiffened and Unstiffened Elements

- Slender Compression Elements
  - A slender element with a cross section that does not satisfy the width-thickness ratio requirements of Table 4 (Table 5.2, Text, or Table B5.1, LRFD Maual).
  - For a section to be slender, it has to have a width-thickness ratios greater than λ_r as provided in Table 4 (Table 5.2, Text, or Table B5.1, LRFD Maual).