CHAPTER

Structural Steel Design
LRFD Method

Third Edition

INTRODUCTION TO AXIALLY LOADED COMPRESSION MEMBERS

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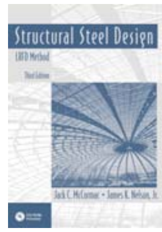
Part II – Structural Steel Design and Analysis

5b

FALL 2002



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Prentice Hall

ENCE 355 - Introduction to Structural Design

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Development of Column Formulas

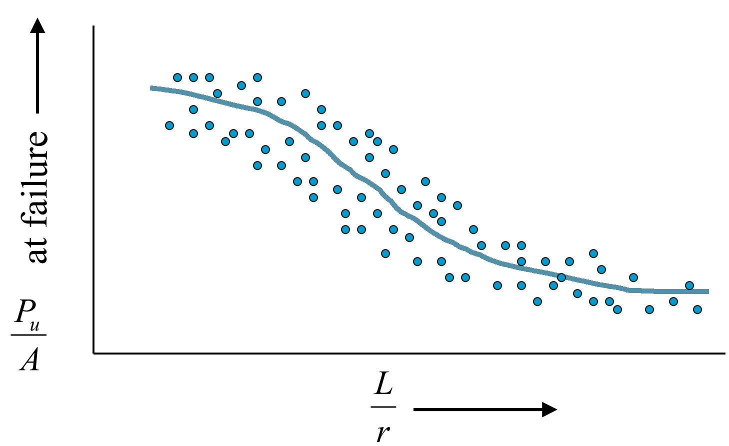
- In 1757, Leonhard Euler, A Swiss mathematician wrote a paper of great value concerning the buckling of columns
- He was probably the first person to realize the significance of buckling.
- The Euler formula, the most famous of all column equations will be derived in the following viewgraphs.

Development of Column Formulas

- This formula marked the real beginning of theoretical and experimental investigation of columns.
- Practical column design is based primarily on formulas that have been developed to fit with reasonable accuracy test-result curves.
- The testing of columns with various slenderness ratios results in a scattered range of values as shown in Fig. 1.

Development of Column Formulas

Figure 1. Test Result Curve



Development of Column Formulas

- The dots in Fig. 1 will not fall on a smooth curve even if all of the testing is performed in the same laboratory because of the difficulty of
  - Exactly centering the loads
  - Lack of perfect uniformity of the materials
  - Varying dimensions of the sections
  - Residual stresses
  - End restraint variations
  - Etc.

Development of Column Formulas

- The practical approach is to attempt to develop formulas which give results represented by an approximate average of the test results.
- It is to be noted also that the laboratory conditions are not field conditions and column tests probably give the limiting values of column strengths.

Development of Column Formulas

- Yield Strength and Length of Column
  - Short Columns
    - The yield stresses of the section tested are quite important for short columns as their failure stresses are close to those yield stresses.
  - Columns with Intermediate L/r
    - The yield stresses are of lesser importance on their effect on failure stresses. Also residual stresses have more effect on the results.
  - Long Slender Columns
    - The yield stresses are of no significance, but the column strength is very sensitive to end conditions.

The Euler Formula

- Buckling
  - Buckling is a mode of failure generally resulting from structural instability due to *compressive* action on the structural member or element involved.
  - Examples
    - Overloaded metal building columns.
    - Compressive members in bridges.
    - Roof trusses.
    - Hull of submarine.

The Euler Formula

- Buckling
  - Examples (cont'd)
    - Metal skin on aircraft fuselages or wings with excessive torsional and/or compressive loading.
    - Any thin-walled torque tube.
    - The thin web of an I-beam with excessive shear load
    - A thin flange of an I-beam subjected to excessive compressive bending effects.

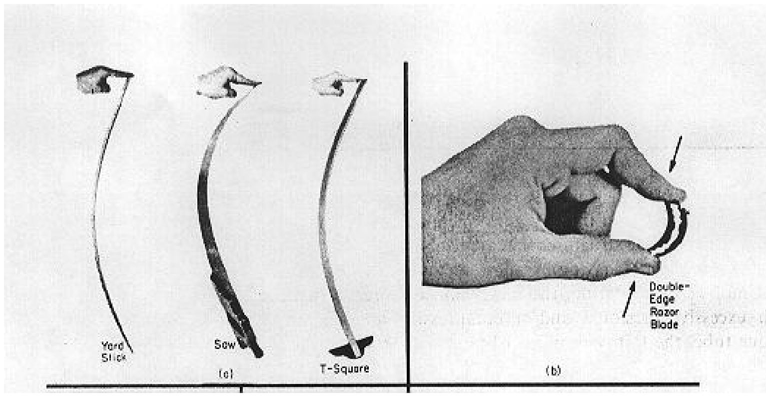
The Euler Formula

- Buckling
  - In view of the above-mentioned examples, it is clear that buckling is a result of compressive action.
  - Overall torsion or shear may cause a localized compressive action that could lead to buckling.
  - Examples of buckling for commonly seen and used tools (components) are provided in the next few viewgraphs.

The Euler Formula

- Buckling

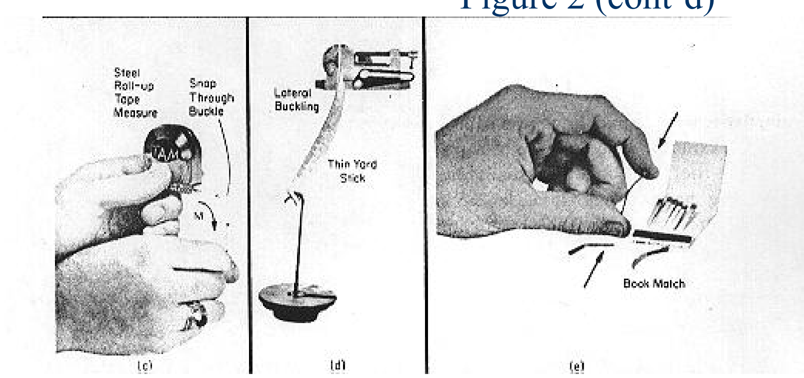
Figure 2



The Euler Formula

- Buckling

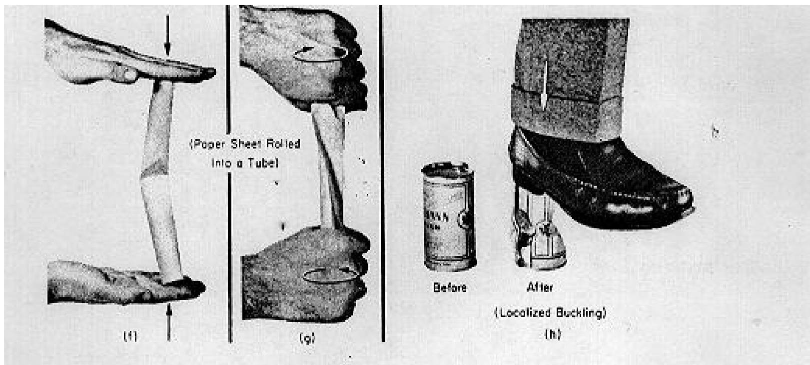
Figure 2 (cont'd)



The Euler Formula

- Buckling

Figure 2 (cont'd)



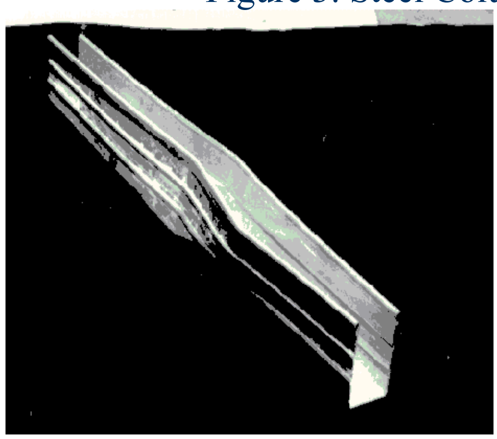
The Euler Formula

- Buckling
  - In Fig. 2, (a) to (d) are examples of temporary or elastic buckling.
  - While (e) to (h) of the same figure are examples of plastic buckling
  - The distinctive feature of buckling is the *catastrophic* and often spectacular nature of failure.

The Euler Formula

- Buckling

Figure 3. Steel Column Buckling



The Euler Formula

- Buckling
  - The collapse of a column supporting stands in a stadium or the roof of a building usually draws large headlines and cries of engineering negligence.
  - On a lesser scale, the reader can witness and get a better understanding of buckling by trying to understand a few of the tests shown in Fig. 2.

The Euler Formula

- The Nature of Buckling
  - For non-buckling cases of axial, torsional, bending, and combined loading, the stress or deformation was the significant quantity in failure.
  - Buckling of a member is uniquely different in that the quantity significant in failure is the buckling load itself.
  - The failure (buckling) load bears no unique relationship to the stress and deformation at failure.

The Euler Formula

- The Nature of Buckling
  - Buckling is unique from our other structural-element considerations in that it results from a state of unstable equilibrium.
  - For example, buckling of a long column is not caused by failure of the material of which the column is composed, but by determination of what was a stable state of equilibrium to an unstable one.

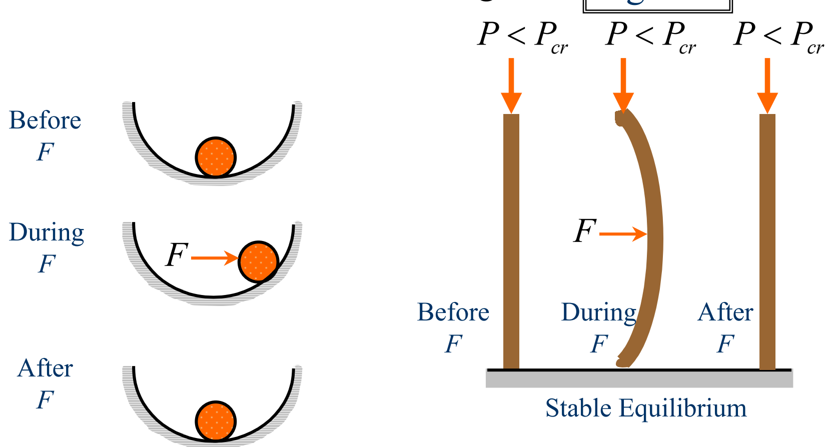
The Euler Formula

- Mechanism of Buckling
  - Let's consider Fig. 4, 5, and 6, and study them very carefully.
  - In Fig. 4, some axial load P is applied to the column.
  - The column is then given a small deflection by applying the small lateral force F.
  - If the load P is sufficiently small, when the force F is removed, the column will go back to its original straight condition.

The Euler Formula

- Mechanism of Buckling

Figure 4



The Euler Formula

- Mechanism of Buckling
  - The column will go back to its original straight condition just as the ball returns to the bottom of the curved container.
  - In Fig. 4 of the ball and the curved container, gravity tends to restore the ball to its original position, while for the column the elasticity of the column itself acts as restoring force.
  - This action constitutes stable equilibrium.

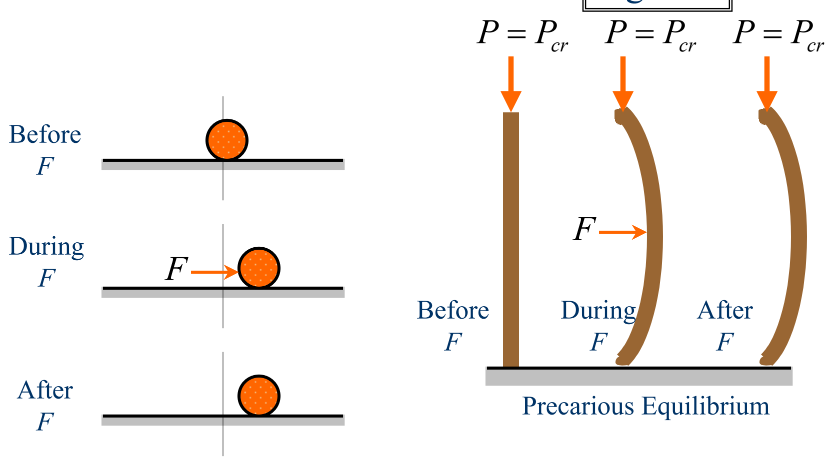
The Euler Formula

- Mechanism of Buckling
  - The same procedure can be repeated for increased value of the load P until some critical value P_{cr} is reached, as shown in Fig. 5.
  - When the column carries this load, and a lateral force F is applied and removed, the column will remain in the slightly deflected position. The elastic restoring force of the column is not sufficient to return the column to its original straight position but is sufficient to prevent excessive deflection of the column.

The Euler Formula

■ Mechanism of Buckling

Figure 5



The Euler Formula

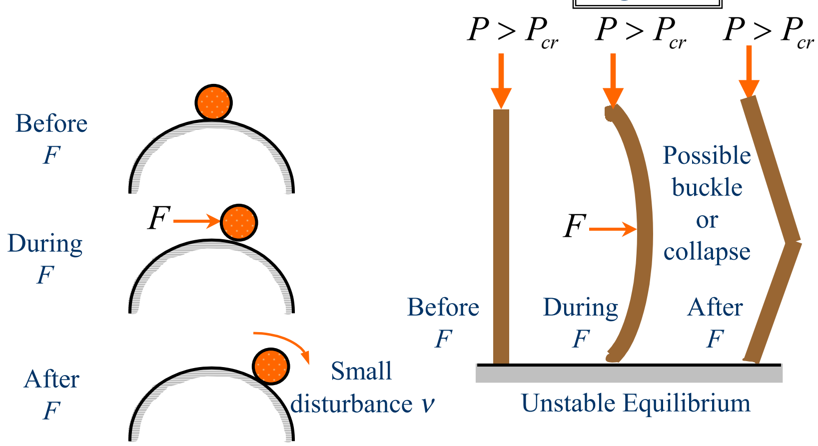
■ Mechanism of Buckling

- In Fig. 5 of the ball and the flat surface, the amount of deflection will depend on the magnitude of the lateral force F.
- Hence, the column can be in equilibrium in an infinite number of slightly bent positions.
- This action constitutes neutral or precarious equilibrium.
- If the column is subjected to an axial compressive load P that exceeds P_{cr}, as shown in Fig. 6, and a lateral force F is applied and removed, the column will bend considerably.

The Euler Formula

■ Mechanism of Buckling

Figure 6



The Euler Formula

■ Mechanism of Buckling

- That is, the elastic restoring force of the column is not sufficient to prevent a small disturbance from growing into an excessively large deflection.
- Depending on the magnitude of P, the column either will remain in the bent position or will completely collapse and fracture, just as the ball will roll off the curved surface in Fig. 6.

The Euler Formula

- Mechanism of Buckling
  - This type of behavior indicates that for axial loads greater than P_{cr}, the straight position of a column is one of *unstable equilibrium* in that a small disturbance will tend to grow into an excessive deformation.

The Euler Formula

- Definition

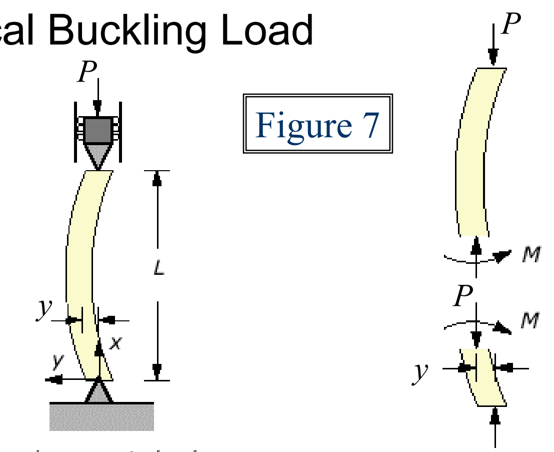
"Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased."

The Euler Formula

- Critical Buckling Load
  - The purpose of this analysis is to determine the minimum axial compressive load for which a column will experience lateral deflection.
  - Governing Differential Equation:
    - Consider a buckled simply-supported column of length L under an external axial compression force P, as shown in the left schematic of Fig. 7. The transverse displacement of the buckled column is represented by δ.

The Euler Formula

- Critical Buckling Load



(a) Simply supported column subjected to axial load F

(b) Free body diagram

The Euler Formula

- Critical Buckling Load
  - Governing Differential Equation:
    - The right schematic of Fig. 7 shows the forces and moments acting on a cross-section in the buckled column. Moment equilibrium on the lower free body yields a solution for the internal bending moment *M*,

$$Py + M = 0 \qquad (1)$$

The Euler Formula

- Critical Buckling Load
  - Governing Differential Equation (cont'd):
    - Recall the relationship between the moment *M* and the transverse displacement *y* for the elastic curve,

$$EI\frac{dy^2}{dx^2} = M \qquad (2)$$

    - Eliminating *M* from Eqs. 1 and 2 results in the governing equation for the buckled slender column,

The Euler Formula

- Critical Buckling Load
  - Governing Differential Equation (cont'd):

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0 \qquad (3)$$

  - Buckling Solution:
    - The governing equation is a second order homogeneous ordinary differential equation with constant coefficients and can be solved by the method of characteristic equations. The solution is found to be,

The Euler Formula

- Critical Buckling Load
  - Buckling Solution (cont'd):

$$y(x) = A\sin px + B\cos px \qquad (4)$$

- Where $p^2 = P/EI$. The coefficients *A* and *B* can be determined by the two boundary conditions, $y(0) = 0$ and $y(L) = 0$, which yields,

$$B = 0$$

$$A\sin pL = 0 \qquad (5)$$

The Euler Formula

- Critical Buckling Load
  - Buckling Solution (cont'd):
    - The coefficient *B* is always zero, and for most values of $m \times L$ the coefficient *A* is required to be zero. However, for special cases of $m \times L$, *A* can be nonzero and the column can be buckled. The restriction on $m \times L$ is also a restriction on the values for the loading *F*; these special values are mathematically called eigenvalues. All other values of *F* lead to trivial solutions (i.e. zero deformation).

The Euler Formula

- Critical Buckling Load
  - Buckling Solution (cont'd):

$$\sin pL = 0$$

$$\Rightarrow pL = 0, \pi, 2\pi, 3\pi, \cdots, n\pi$$

or

$$p = 0, \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \cdots, \frac{n\pi}{L} \qquad (6)$$

- Since $p^2 = P/EI$, therefore,

$$P = 0, \frac{\pi^2 EI}{L^2}, \frac{(2)^2\pi^2 EI}{L^2}, \frac{(3)^2\pi^2 EI}{L^2}, \cdots, \frac{n^2\pi^2 EI}{L^2} \qquad (7)$$

The Euler Formula

- Critical Buckling Load
  - Buckling Solution (cont'd):
    - Or

$$P = EI\left(\frac{n\pi}{L}\right)^2 \quad \text{for } n = 0,1,2,3\cdots \qquad (8)$$

    - The lowest load that causes buckling is called critical load (n = 1).

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (9)$$

The Euler Formula

- Critical Buckling Load, P_{cr}

The critical buckling load (Euler Buckling) for a long column is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (9)$$

where

E = modulus of elasticity of the material

I = moment of inertia of the cross section

L = length of column

The Euler Formula

- Critical Buckling Stress
  - The critical buckling normal stress F_e is found as follows:

When the moment of inertia I in Eq. 9 is replaced by Ar^2, the result is

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} = F_e \qquad (10)$$

where

A = cross-sectional area of column

r = radius of gyration = $\sqrt{\frac{I}{A}}$

The Euler Formula

- Critical Buckling Stress

The critical buckling normal stress is given by

$$F_e = \frac{\pi^2 E}{(L/r)^2} \qquad (11)$$

Where

r = radius of gyration = $\sqrt{\frac{I}{A}}$

(L/r) = slenderness ratio of column

The Euler Formula

- Critical Buckling Load and Stress
  - The Euler buckling load and stress as given by Eq. 9 or Eq. 11 agrees well with experiment if the slenderness ratio is large (L/r > 140 for steel columns).
  - Short compression members (L/r < 140 for steel columns) can be treated as compression blocks where yielding occurs before buckling.

The Euler Formula

- Critical Buckling Load and Stress
  - Many columns lie between these extremes in which neither solution is applicable.
  - These intermediate-length columns are analyzed by using empirical formulas to be described later.
  - *When calculating the critical buckling for columns, I (or r) should be obtained about the weak axis.*

The Euler Formula

- Example 1

  A W10 × 22 is used as a 15-long pin-connected column. Using Euler expression (formula),

  a. Determine the column's critical or buckling load, assuming the steel has a proportional limit of 36 ksi.

  b. Repeat part (a) if the length of the column is changed to 8 ft.

The Euler Formula

- Example 1 (cont'd)

  Using a W10 × 22, the following properties can be obtained from the LRFD Manual:

  $A = 6.49 \text{ in}^2$, $r_x = 4.27$ in, and $r_x = 1.33$ in

  Therefore, minimum $r = r_y = 1.33$ in.

a.

$$\frac{L}{r} = \frac{15 \times 12}{1.33} = 135.34$$

$$F_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (29 \times 10^3)}{(135.34)^2} = 15.63 \text{ ksi} < 36 \text{ ksi}$$

OK column is in elastic range

The Euler Formula

- Example 1 (cont'd)
  b. Using an 8-ft W10 × 22:

$$\frac{L}{r} = \frac{8\times 12}{1.33} = 72.18$$

$$F_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2\left(29\times 10^3\right)}{(72.18)^2} = 54.94\text{ ksi} > 36\text{ ksi}$$

$\therefore$ column is in inelastic range and Euler equation is not applicable

The Euler Formula

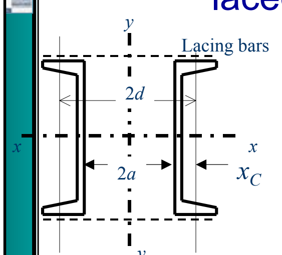
- Review of Parallel-Axis Theorem for Radius of Gyration
  - In dealing with columns that consist of several rolled standard sections, it is sometimes necessary to compute the radius of gyration for the entire section for the purpose of analyzing the buckling load.
  - It was shown that the parallel-axis theorem is a useful tool to calculate the second

The Euler Formula

- Review of Parallel-Axis Theorem for Radius of Gyration
  - Moment of area (moment of inertia) about other axes not passing through the centroid of the overall section.
  - In a similar fashion, the parallel-axis theorem can be used to find radii of gyration of a section about different axis not passing through the centroid.

The Euler Formula

- Review of Parallel-Axis Theorem for Radius of Gyration
  - Consider the two channels, which are laced a distance of 2*a* back to back.



$$I_x = 2I_{x_C} \Rightarrow r_x = \sqrt{\frac{I_x}{A_{\text{overall}}}} = \sqrt{\frac{2I_{x_C}}{2A_{\text{sec}}}} = \sqrt{\frac{I_{x_C}}{A_{\text{sec}}}} = r_{x_C} \quad (12)$$

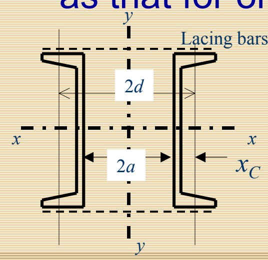
$$I_y = 2\left(I_{y_C} + A_{\text{sec}}d^2\right) = 2\left(A_{\text{sec}}r_{y_C}^2 + A_{\text{sec}}d^2\right) = 2A_{\text{sec}}\left(r_{y_C}^2 + d^2\right)$$

$$\Rightarrow r_y = \sqrt{\frac{I_y}{A_{\text{overall}}}} = \sqrt{\frac{2A_{\text{sec}}\left(r_{y_C}^2 + d^2\right)}{2A_{\text{sec}}}} = \sqrt{r_{y_C}^2 + d^2} \quad (13)$$

The Euler Formula

- Parallel-Axis Theorem for Radius of Gyration

  Eqs. 12 and 13 indicate that the radius of gyration for the two channels is the same as that for one channel, and



$$r_y = \sqrt{r_{y_C}^2 + (a + x_C)^2} \quad (14)$$

where $a + x_C = d$

The Euler Formula

- Example 2

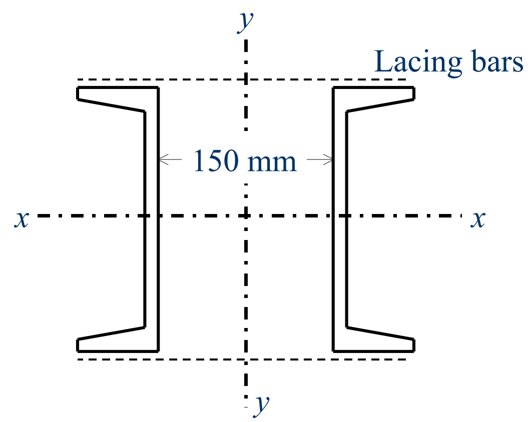
  Two C229 × 30 structural steel channels are used for a column that is 12 m long. Determine the total compressive load required to buckle the two members if

  (a) They act independently of each other. Use E = 200 GPa.

  (b) They are laced 150 mm back to back as shown in Fig. 10.

The Euler Formula

■ Example 2 (cont'd)



The Euler Formula

■ Example 2 (cont'd)

(a) Two channels act independently:

- If the two channels are not connected and each acts independently, the slenderness ratio is determined by using the minimum radius of gyration r_{min} of the individual section
- For a C229 × 30 section (see Fig 8):

$$r_{min} = r_y = 16.3 \text{ mm} \qquad A = 3795 \text{ mm}^2$$

Figure 8

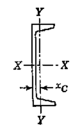


TABLE B-6 Standard Channels (SI Units)

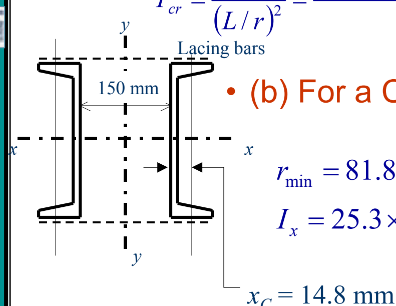
Designation*	Area (mm^2)	Depth (mm)	FLANGE Width (mm)	FLANGE Thickness (mm)	Web Thickness (mm)	AXIS X–X I (10^6 mm^4)	AXIS X–X S (10^3 mm^3)	AXIS X–X r (mm)	AXIS Y–Y I (10^6 mm^4)	AXIS Y–Y S (10^3 mm^3)	AXIS Y–Y r (mm)	x_C (mm)
C457 × 86	11030	457.2	106.7	15.9	17.8	281	1230	160	7.41	87.2	25.9	21.9
× 77	9870	457.2	104.1	15.9	15.2	261	1140	163	6.83	83.1	26.4	21.8
× 68	8710	457.2	101.6	15.9	12.7	241	1055	167	6.29	79.0	26.9	22.0
× 64	8130	457.2	100.3	15.9	11.4	231	1010	169	5.99	76.9	27.2	22.3
C381 × 74	9485	381.0	94.4	16.5	18.2	168	882	133	4.58	61.9	22.0	20.3
× 60	7615	381.0	89.4	16.5	13.2	145	762	138	3.84	55.2	22.5	19.7
× 50	6425	381.0	86.4	16.5	10.2	131	688	143	3.38	51.0	23.0	20.0
C305 × 45	5690	304.8	80.5	12.7	13.0	67.4	442	109	2.14	33.8	19.4	17.1
× 37	4740	304.8	77.4	12.7	9.8	59.9	395	113	1.86	30.8	19.8	17.1
× 31	3930	304.8	74.7	12.7	7.2	53.7	352	117	1.61	28.3	20.3	17.7
C254 × 45	5690	254.0	77.0	11.1	17.1	42.9	339	86.9	1.64	27.0	17.0	16.5
× 37	4740	254.0	73.3	11.1	13.4	38.0	298	89.4	1.40	24.3	17.2	15.7
× 30	3795	254.0	69.6	11.1	9.6	32.8	259	93.0	1.17	21.6	17.6	15.4
× 23	2895	254.0	66.0	11.1	6.1	28.1	221	98.3	0.949	19.0	18.1	16.1
C229 × 30	3795	228.6	67.3	10.5	11.4	25.3	221	81.8	1.01	19.2	16.3	14.8
× 22	2845	228.6	63.1	10.5	7.2	21.2	185	86.4	0.803	16.6	16.8	14.9
× 20	2540	228.6	61.8	10.5	5.9	19.9	174	88.4	0.733	15.7	17.0	15.3
C203 × 28	3555	203.2	64.2	9.9	12.4	18.3	180	71.6	0.824	16.6	15.2	14.4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	148	75.9	0.637	14.0	15.6	14.0

The Euler Formula

- Example 2 (cont'd)

$$\frac{L}{r}=\frac{12\times 10^3}{16.3}=736.2\ \text{(slender)}$$

$$P_{cr}=\frac{\pi^2 EA}{(L/r)^2}=\frac{\pi^2\left(200\times 10^9\right)\left[(2)\left(3795\times 10^{-6}\right)\right]}{(736.2)^2}=27.64\times 10^3\ \text{N}=27.6\ \text{kN}$$



- (b) For a C229 × 30 section (see Fig 8):

$r_{min}=81.8$ mm $\qquad x_C=14.8$ mm

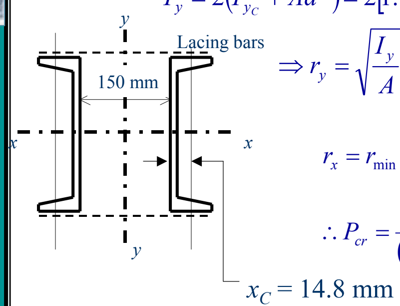
$I_x=25.3\times 10^6\ \text{mm}^4$ $\qquad I_y=1.01\times 10^6$ mm

The Euler Formula

■ Example 2 (cont'd)

$$I_x = 2I_{x_C} = 2\left(25.3\times10^6\right) = 50.6\times10^6\ \text{mm}^2 \Rightarrow r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{50.6\times10^6}{2(3795)}} = 81.7$$

$$I_y = 2\left(I_{y_C} + Ad^2\right) = 2\left[1.01\times10^6 + 3795(75+14.8)^2\right] = 63.23\times10^6\ \text{mm}^2$$



$$\Rightarrow r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{63.23\times10^6}{2(3795)}} = 91.3\ \text{mm}$$

$$r_x = r_{\min} = 81.7,\ \text{therefore,}\ \frac{L}{r_{\min}} = \frac{12\times10^3}{81.7} = 146.9$$

$$\therefore P_{cr} = \frac{\pi^2 EA}{(L/r_{\min})^2} = \frac{\pi^2\left(200\times10^9\right)\left[2\left(3795\times10^{-6}\right)\right]}{(146.9)^2} = 694.3\ \text{kN}$$

The Euler Formula

■ Example 2 (cont'd)

– An alternate solution for finding r_x and r_y:

• Using Eqs. 12 and 14,

$$r_x = r_{x_C} = 81.8\ \text{mm}$$

$$r_y = \sqrt{r_{y_C}^2 + (a + x_C)^2} = \sqrt{(16.3)^2 + (75+14.8)^2}$$

$$= 91.3\ \text{mm}$$

• Therefore, $r_{\min} = r_x = 81.8\ \text{mm}$

The slight difference in the result is due to round-off errors.