Reinforced Concrete Design — Fifth Edition

CHAPTER 9c

COLUMNS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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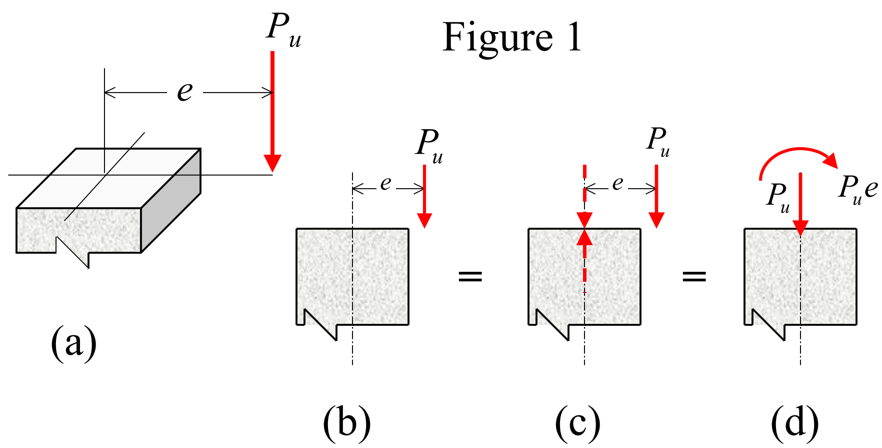
Prentice Hall

The Load-Moment Relationship

- Axial Load-Moment Combination
  - Assume that P_u is applied to a cross section at an eccentricity e from the centroid, as shown in Fig. 1a and 1b.
  - Add equal and opposite forces P_u at the centroid of the cross section, as shown in Fig. 1c.
  - The original eccentric force P_u may noe be combined with the upward force P_u to form a couple P_ue, that is a pure moment.

The Load-Moment Relationship

- Axial Load-Moment Combination



The Load-Moment Relationship

- Axial Load-Moment Combination
  - This will leave remaining one force, P_u acting downward at the centroid of the cross section.
  - It can be therefore be seen that if a force P_u is applied with an eccentricity e, the situation that results is identical to the case where an axial load of P_u at the centroid and a moment of $P_u e$ are simultaneously applied as shown in Fig. 1d.

The Load-Moment Relationship

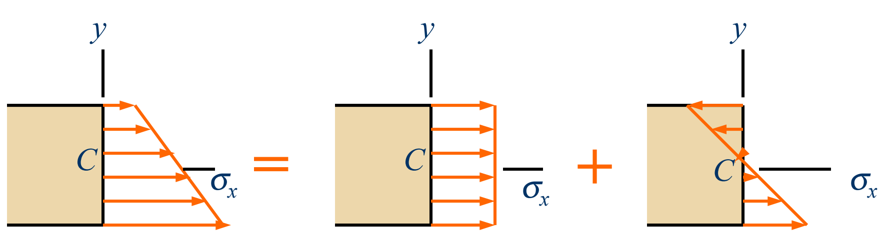
- Axial Load-Moment Combination
  - If M_u is defined as the factored moment to be applied on a compression member along with a factored axial load of P_u at the centroid, the relationship between the two can expressed as

$$e = \frac{M_u}{P_u} \qquad (1)$$

The Load-Moment Relationship

- Eccentric Axial Loading in A Plane of Symmetry

Figure 2



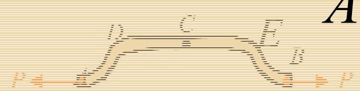
$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$

The Load-Moment Relationship

- Eccentric Axial Loading in A Plane of Symmetry

  The stress due to eccentric loading on a beam cross section is given by

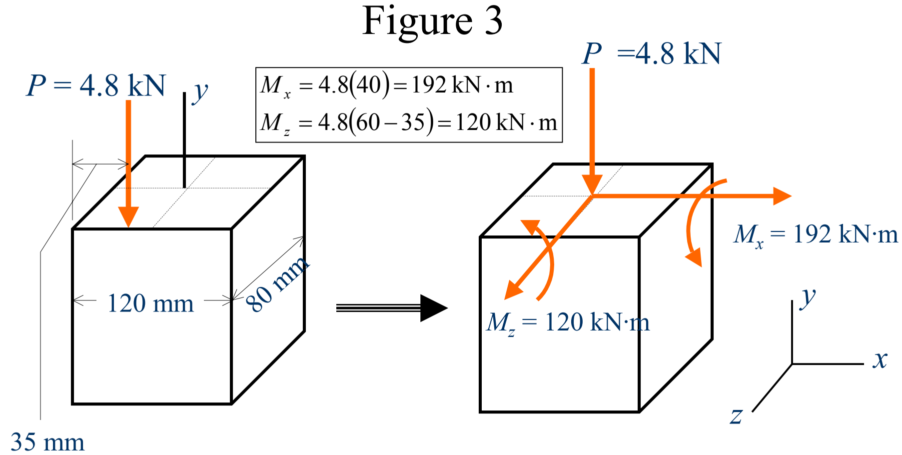
$$\sigma_x = \frac{P}{A} \pm \frac{My}{I} \qquad (2)$$



The Load-Moment Relationship

- Equivalent Force System for Eccentric Loading

Figure 3



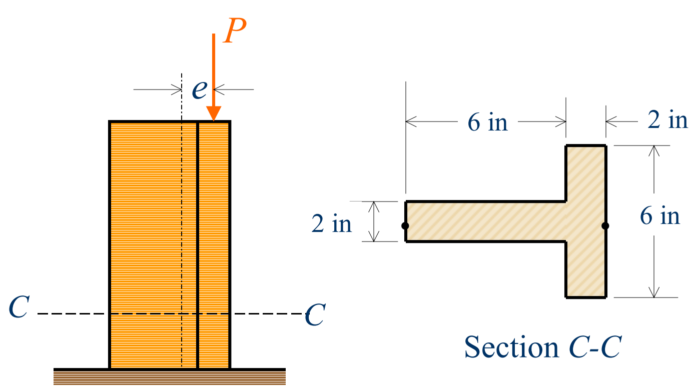
The Load-Moment Relationship

- Example 1

  The T-section shown in Fig. 50 is used as a short post to support a compressive load *P* of 150 kips. The load is applied on centerline of the stem at a distance *e* = 2 in. from the centroid of the cross section. Determine the normal stresses at points *A* and *B* on a transverse plane *C-C* near the base of the post.

The Load-Moment Relationship

- Example 1 (cont'd)



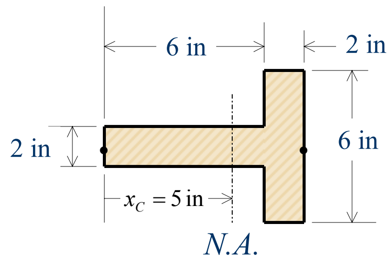
The Load-Moment Relationship

- Example 1 (cont'd)

Computing the cross-sectional properties:

$$\text{Area} = A = 2[6 \times 2] = 24 \text{ in}^2$$

$$x_C = \frac{3(6 \times 2) + (6+1)(6 \times 2)}{24} = 5 \text{ in. from point } A$$



$$I_y = \frac{2(5)^3}{3} + \frac{6(3)^3}{3} - \frac{4(1)^3}{3} = 136 \text{ in}^4$$

The Load-Moment Relationship

- Example 1 (cont'd)

Equivalent force system:

$$P = 150 \text{ kip acts through centroid}$$

$$M = Pe = (150)(2) \times 12 = 3{,}600 \text{ kip} \cdot \text{in}$$

Computations of normal stresses:

$$\sigma_A = -\frac{P}{A} + \frac{My}{I_x} = -\frac{150}{24} + \frac{300(5)}{136} = \boxed{4.78 \text{ ksi (T)}}$$

$$\sigma_B = -\frac{P}{A} - \frac{My}{I_x} = -\frac{150}{24} - \frac{300(3)}{136} = \boxed{-12.87 \text{ ksi (C)}}$$

Analysis of Short Columns: Large Eccentricity

- The first step in the investigation of short columns carrying loads at eccentricity is to determine the strength of given column cross section that carries load at various eccentricities.
- For this, the design axial load strength ϕP_n is found, where P_n is defined as the nominal axial load strength at a given eccentricity.

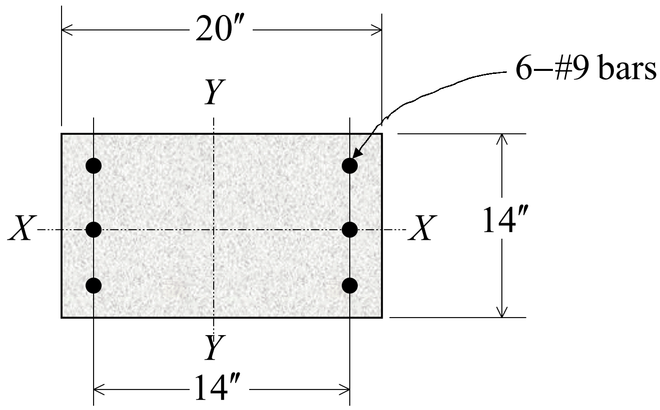
Analysis of Short Columns: Large Eccentricity

- Example 2

  Find the design axial load strength ϕP_n for the tied column for the following conditions: (a) small eccentricity, (b) pure moment, (c) e = 5 in., and (d) the balanced condition. The column cross section is shown. Assume a short column. Bending about the Y-Y axis. Use f_c' = 4000 psi and f_y = 60,000 psi.

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

(a) Small Eccentricity:

$$A_g = 14(20) = 280\ \text{in}^2$$

$$A_{st} = 6\ \text{in}^2\ (\text{area of } 6\text{–}\#9 \text{ bars})$$

$$\begin{aligned}\phi P_n &= \phi P_{n(\max)} \\ &= 0.80\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \\ &= 0.80(0.70)[0.85(4)(280-6) + (60)(6)] \\ &= 723\ \text{kips}\end{aligned}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

(b) Pure Moment:

The analysis of the pure moment condition is similar to the analysis of the case where the eccentricity e is infinite as shown in Fig. 4.

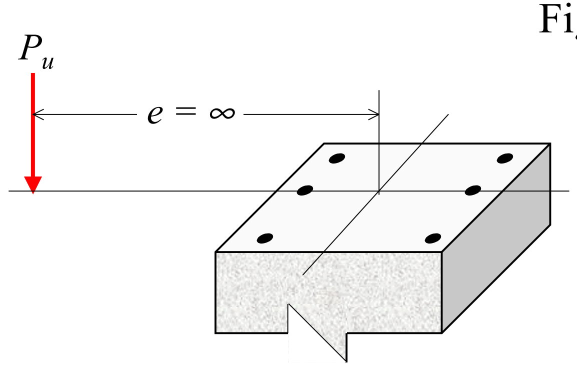
The design moment ϕM_n will be found since P_u and ϕP_n will both be zero.

Assume that A_s is at yield, and then with reference to Fig. 5, then

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Figure 4



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

  C_1 = concrete compressive force
  C_2 = steel compressive force
  T = steel tensile force

$$\frac{\varepsilon_s'}{c-3} = \frac{0.003}{c} \Rightarrow \varepsilon_s' = 0.003\frac{c-3}{c} \qquad (3)$$

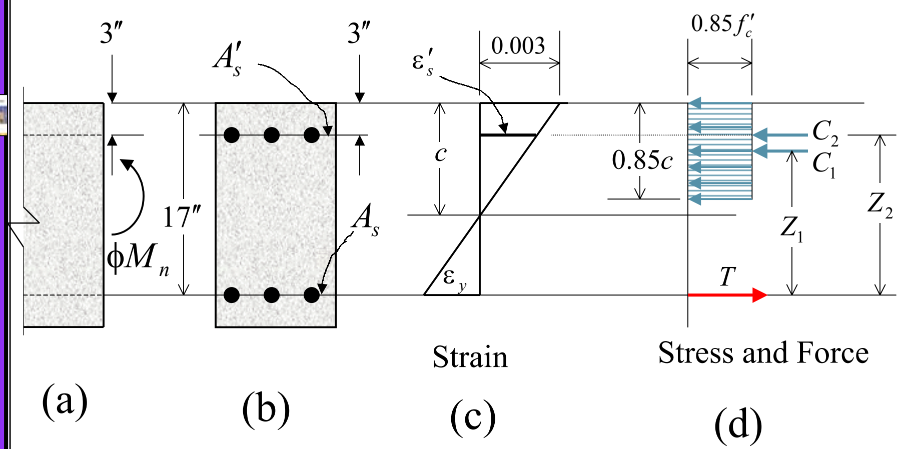
Since

$$f_s' = E_s \varepsilon_s' \qquad (4)$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Figure 5



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Substituting E_s = 29 × 10[6] psi and ε_s' given by Eq. 3 into Eq. 4, gives

$$f_s' = 29\times10^6(0.003)\frac{c-3}{c} = 87\frac{c-3}{c} \qquad (4)$$

For equilibrium in Fig 4d,

$$C_1 + C_2 = T \qquad (5)$$

Substituting into above equation, yields

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

$$(0.85f_c)(0.85c)b + f_s'A_s' - 0.85f_c'A_s' = f_yA_s \qquad (6a)$$

$$(0.85)(4)(0.85c) + 87\frac{c-3}{c}(3) - 0.85(4)(3) = 3(60) \qquad (6b)$$

– The above equation can be solved for c to give

$$c = 3.62 \text{ in.}$$

and thus,

$$f_s' = 87\frac{3.62-3}{3.62} = 14.90 \text{ ksi (compression)} \qquad (7)$$

Analysis of Short Columns: Large Eccentricity

Table 1. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Table A-2 Textbook

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Therefore, the forces will be

$$C_1 = 0.85 f_c'(0.85c)b = 0.85(4)(0.85)(3.62)(14) = 146.5 \text{ kips}$$

$$C_2 = f_s'A_s' - 0.85 f_c'A_s' = 14.9(3) - 0.85(4)(3) = 34.5 \text{ kips}$$

– The internal Moments are

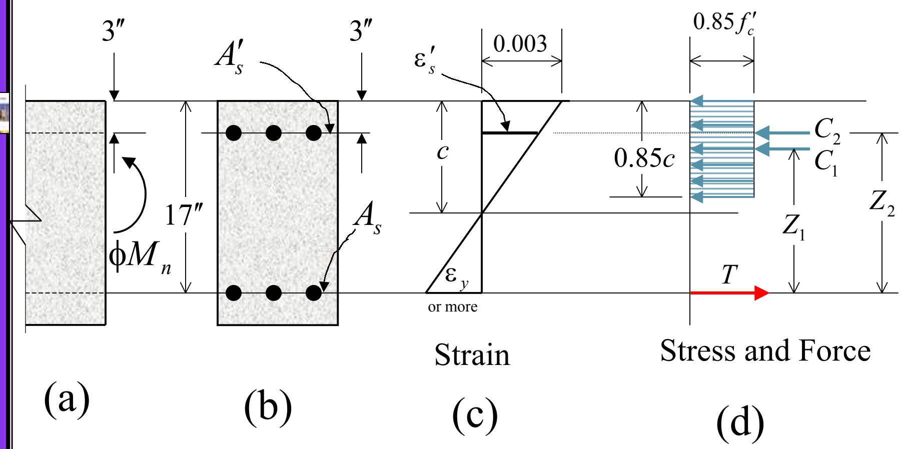
$$M_{n1} = C_1 Z_1 = \frac{146.5}{12}\left[17 - \frac{0.85(3.62)}{2}\right] = 188.8 \text{ ft-kips}$$

$$M_{n2} = C_2 Z_2 = \frac{34.5(14)}{12} = 40.3 \text{ ft-kips}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Figure 5



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Therefore,

$$M_n = M_{n1} + M_{n2} = 188.8 + 40.3 = 229 \text{ ft - kips}$$

and

$$\phi M_n = 0.7(229) = 160 \text{ ft - kips}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

  (c) The eccentricity e = 5 in:

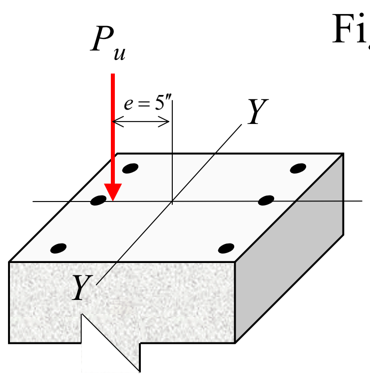
  The situation of e = 5 in. is shown in Fig. 6

  Note that in Part (a), all steel was in *compression* and in Part (b), the steel on the side of the column away from the load was in *tension*. Therefore, there is some value of the eccentricity at which steel will change from tension to compression. Since this is not known, the strain in Fig. 7 is assumed.

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

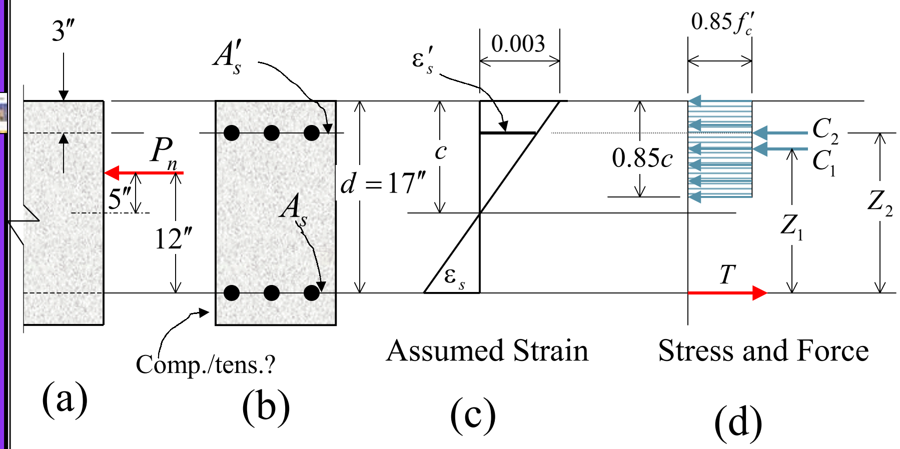
Figure 6



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Figure 7



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

The assumptions at ultimate load are

1. Maximum concrete strain = 0.003
2. $\varepsilon'_s > \varepsilon_y$, therefore, $f'_s = f_y$
3. ε_s is tensile
4. $\varepsilon_s < \varepsilon_y$ and thus $f_s < f_y$

These assumptions will be verified later.

The unknown quantities are P_u and c.

The forces will be evaluated as follows:

Analysis of Short Columns: Large Eccentricity

■ Example 2 (cont'd)

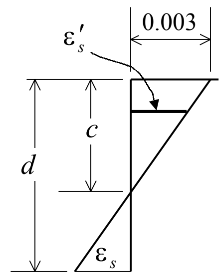
$$C_1 = 0.85 f_c' ab = 0.85(4)(0.85c)(14) - 40.46c$$

$$C_2 = f_y A_s' - 0.85 f_c' A_s'$$
$$= 60(3) - 0.85(4)(3) = 169.8 \text{ kips}$$

$$T = f_s A_s = \varepsilon_s E_s A_s = 87\left(\frac{d-c}{c}\right) A_s$$
$$= 87\left(\frac{17-c}{c}\right)3 = 261\frac{17-c}{c}$$

From $\sum$ moments = 0 in Fig. 7c:

$$P_n = C_1 + C_2 - T$$
$$= 40.46c + 169.8 - 261\frac{17-c}{c} \qquad (8)$$



$$\frac{\varepsilon_s}{0.003} = \frac{d-c}{c}$$

$$\varepsilon_s = 0.003\frac{d-c}{c}, \text{ and}$$

$$f_s = \varepsilon_s E_s = \left(0.003\frac{d-c}{c}\right)29\times10^3 = 87\frac{d-c}{c}$$

Analysis of Short Columns: Large Eccentricity

■ Example 2 (cont'd)

From $\sum$ moments = 0, taking moments about T in Fig. 7d

$$P_n(12) = C_1\left(d - \frac{a}{2}\right) + C_2(14) \qquad (9)$$
$$= \frac{1}{12}\left[40.46c\left(17 - \frac{0.85c}{2}\right) + 169.8(14)\right]$$

Eqs. 8 and 9 can be solved simultaneously for c to give

$$c = 14.86 \text{ in.}$$

$$P_n = 733 \text{ kips}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Now, the assumptions can be checked:

$$\varepsilon_s' = \left(\frac{14.86 - 3}{14.86}\right)(0.003) = 0.0024$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29 \times 10^6} = 0.00207 < (\varepsilon_s' = 0.0024) \quad \text{OK}$$

Therefore, $f_s' = f_y$, and based on the location of the neutral axis:

$$f_s = 87\left(\frac{17 - 14.86}{14.86}\right) = 12.53 \text{ ksi} < 60 \text{ ksi} \quad \text{OK}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)
  - The design moment for an eccentricity of 5 in. can be computed as follows:

$$P_u = \phi P_n = 0.7(733) = 513 \text{ kips}$$

$$\phi M_n = \phi P_n e = \frac{513(5)}{12} = 214 \text{ ft-kips}$$

  - Therefore, the given column has a design load-moment combination strength of 513 kips axial load and 214 ft-kips moment.

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

  (d) The Balanced Condition Case:

  The balanced condition is defined when the concrete reaches a strain of 0.003 at the same time that the tension steel reaches its yield strian, as shown in Fig. 8c.

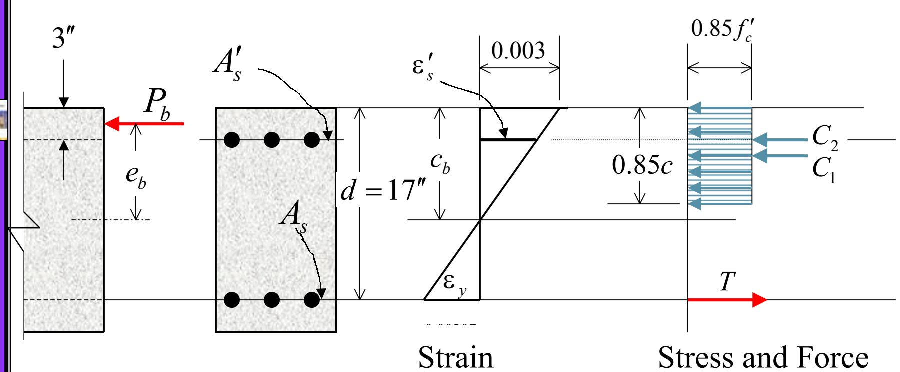
  The value of c_b can be calculated from

$$c_b = \frac{87}{87 + f_y} d = \frac{87}{87 + 60}(17) = 10.06 \text{ in.}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

Figure 8



(a) (b) (c) (d)

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)

$$\varepsilon_s' = \frac{10.06-3}{10.06}(0.003) = 0.0021 > \varepsilon_y = 0.00207$$

Therefore, $f_s' = f_y$ = 60 ksi

The forces can computed as follows:

$$C_1 = 0.85(4)(0.85)(10.06)(14) = 407 \text{ kips}$$
$$C_2 = 60(3) - 0.85(4)(3) = 170 \text{ kips}$$
$$T = 60(3) = 180 \text{ kips}$$
$$P_b = C_1 + C_2 - T = 407 + 170 - 180 = 397 \text{ kips}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)
  - The value of e_b may be calculated by summing moments about T as follows:

$$P_e(e_b + 7) = C_1\left(d - \frac{0.85c_b}{2}\right) + C_2(14)$$

$$397(e_b + 7) = 407\left[17 - \frac{0.85(10.06)2}{2}\right] + 170(14)$$

  - From which, e_b = 12.0 in. Therefore, at the balanced condition:

$$\phi P_b = 0.70(397) = 278 \text{ kips}$$

$$\phi M_n = \phi P_b e_b = \frac{278(12)}{12} = 278 \text{ ft - kips}$$

Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)
  - The results of the four parts can be tabulated (see Table 2) and plotted as shown in Fig. 9.
  - This plot is called an "***interaction diagram***".
  - In the plot, any point on the solid line represents an allowable combination of load and moment.
  - Any point within the solid line represents a load-moment combination that is also allowable, but for which this column is *overdesigned*.

Analysis of Short Columns: Large Eccentricity

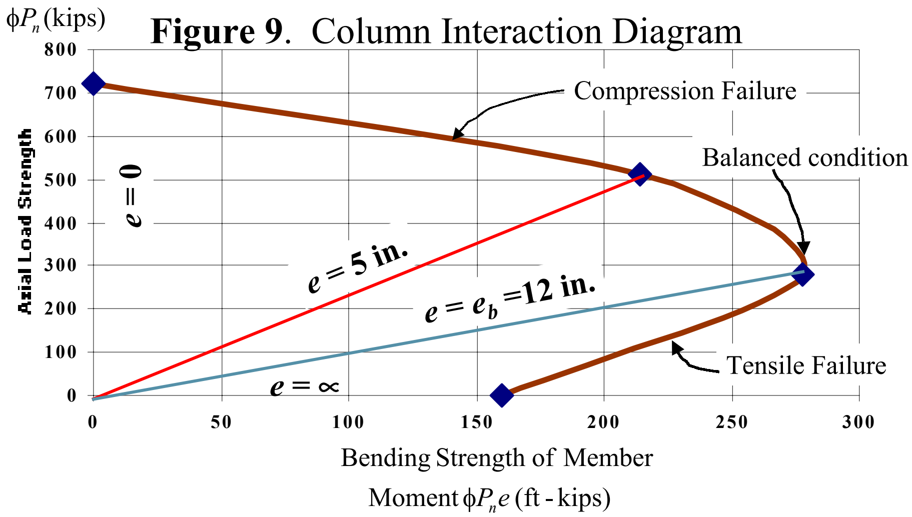
- Example 2 (cont'd)

Table 2

e	Axial load strength (ϕP_n, kips)	Moment strength ($\phi P_n e$, ft- kips)
Small	723	0 (small)
Infinite	0	160
5 in.	513	214
12 in	278	278

Analysis of Short Columns: Large Eccentricity

Figure 9. Column Interaction Diagram



Analysis of Short Columns: Large Eccentricity

- Example 2 (cont'd)
  - Any point outside the solid line represents an unaccepted load-moment combination or a load-moment combination for which this column is *underdesigned*.
  - Radial lines from the origin represent various eccentricities (slope = $\phi P_n / \phi P_n$ or $1/e$).
  - Any eccentricity less than e_b will result in compression controlling the column, and any eccentricity greater than e_b will result in tension controlling the column.

Design of Short Columns: Large Eccentricity

- The design of a column cross section using the previous calculation approach would be a trial-and-error method and would become exceedingly tedious.
- Therefore, design and analysis aids have been developed that shorten the process to a great extent.
- A chart approach has been developed in ACI Publication SP-17 (97), *ACI Design Handbook.*

Design of Short Columns: Large Eccentricity

- The charts take on the general form of Figure 9 but are set up to be more general so that they will remain applicable if various code criteria undergo changes.
- These charts can be used for both analysis and design of columns.
- There are also computer programs available to aid in the design process.