

CHAPTER

Prentice Hall Reinforced Concrete Design Fifth Edition

UNIVERSITY OF MARYLAND  
COLLEGE PARK

# COLUMNS

A. J. Clark School of Engineering • Department of Civil and Environmental Engineering  
Part I – Concrete Design and Analysis

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**ENCE 355 - Introduction to Structural Design**  
Department of Civil and Environmental Engineering  
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9a

Prentice Hall

CHAPTER 9a. COLUMNS

Slide No. 1

ENCE 355 ©Assakkaf

## Introduction

- Axial Compression
  - Columns are defined as members that carry loads in compression.
  - Usually they carry bending moments as well, about one or both axes of the cross section.
  - The bending action may produce tensile forces over a part of the cross section.
  - Despite of the tensile forces or stresses that may be produced, columns are



# Introduction

## ■ Axial Compression

- Generally referred to as “compression members” because the compression forces or stresses dominate their behavior.
- In addition to the most common type of compression members (vertical elements in structures), compression members include:
  - Arch ribs
  - Rigid frame members inclined or otherwise
  - Compression elements in trusses
  - shells



# Introduction





## Introduction



Reinforced Concrete Columns

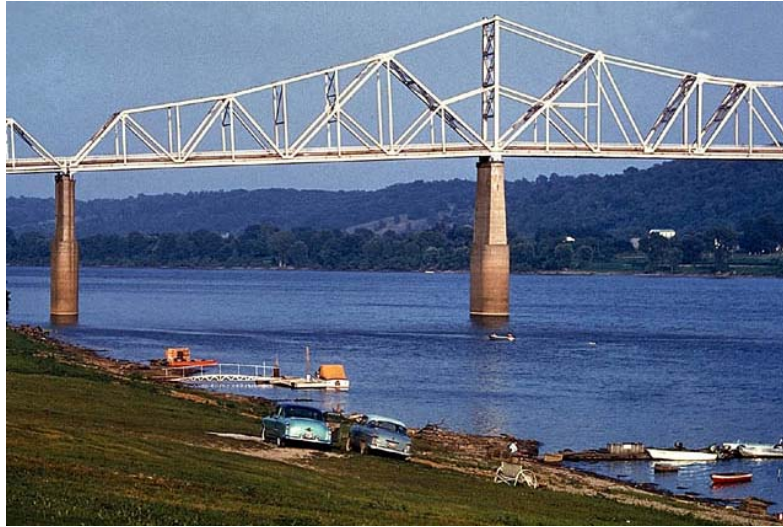


*Pont-du-Gard. Roman aqueduct built in 19 B.C. to carry water across the Gardon Valley to Nimes. Spans of the first and second level arches are 53-80 feet. (Near Remoulins, France)*





*Ohio River Bridge. Typical cantilever and suspended span bridge, showing the truss geometry in the end span and cantilevered portion of the main span. (Madison, Indiana)*



## Introduction







# Introduction



# Introduction

## ■ Column load transfer from beams and slabs

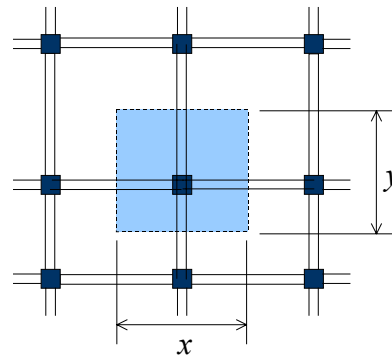
### 1) Tributary area method:

*Half distance to adjacent columns*

Load on column = area  $\times$  floor load

Floor load = DL + LL

DL = slab thickness  $\times$  conc. unit wt.



**Example:**  $x = 16.0$  ft,  $y = 13.0$  ft, LL = 62.4 lb/ft<sup>2</sup>, slab thickness = 4.0 in.

Floor load = 4.0 (150)/12 + 62.4 = 112.4 lb/ft<sup>2</sup>

Load on column = (16.0)(13.0)(112.4) = 10,800 kg = 23.4 kips

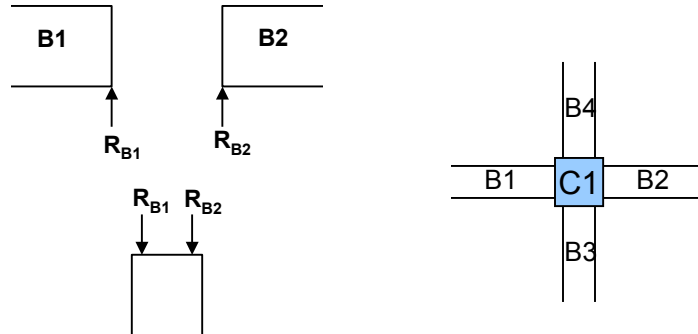


# Introduction

## ■ Column load transfer from beams and slabs

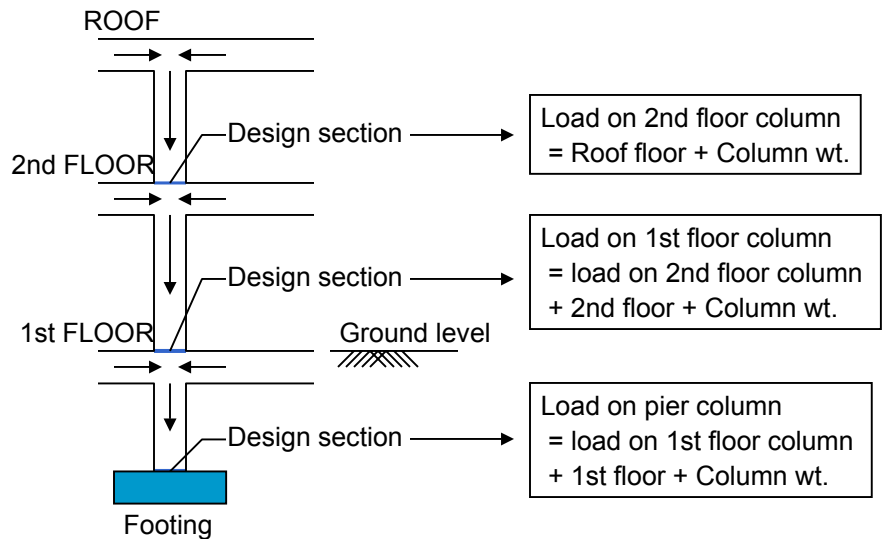
### 2) Beams reaction method:

Collect loads from adjacent beam ends



# Introduction

## ■ Load summation on column section for design





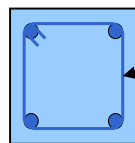
## Introduction

- Types of Reinforced Concrete Columns
  1. Members reinforced with longitudinal bars and lateral ties.
  2. Members reinforced with longitudinal bars and continuous spirals.
  3. Composite compression members reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement.

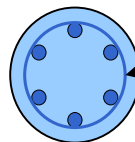


## Introduction

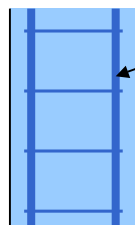
- Types of Reinforced Concrete Columns



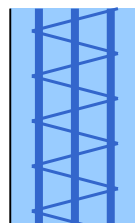
Tie



Spiral



Longitudinal steel



$s = \text{pitch}$

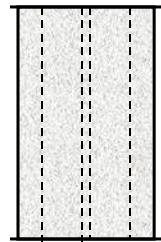
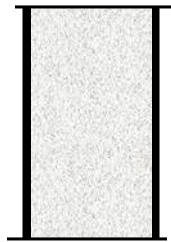
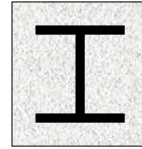
Tied column

Spirally reinforced column



## Introduction

### ■ Types of Reinforced Concrete Columns



Composite columns



## Introduction

### ■ Types of Columns in Terms of Their Strengths

#### 1. Short Columns

A column is said to be short when its length is such that lateral buckling need not be considered. Most of concrete columns fall into this category.

#### 2. Slender Columns

When the length of the column is such that buckling need to be considered, the column is referred to as slender column. It is recognized that as the length increases, the usable strength of a given cross section is decreased because of buckling problem.





## Introduction

### ■ Buckling

– Buckling is a mode of failure generally resulting from structural instability due to compressive action on the structural member or element involved.

#### – Examples

- Overloaded metal building columns.
- Compressive members in bridges.
- Roof trusses.
- Hull of submarine.



## Introduction

### ■ Buckling

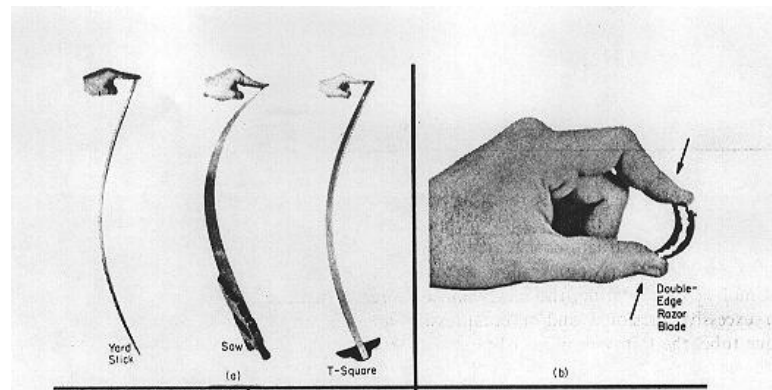


Figure 1a



## Introduction

### ■ Buckling

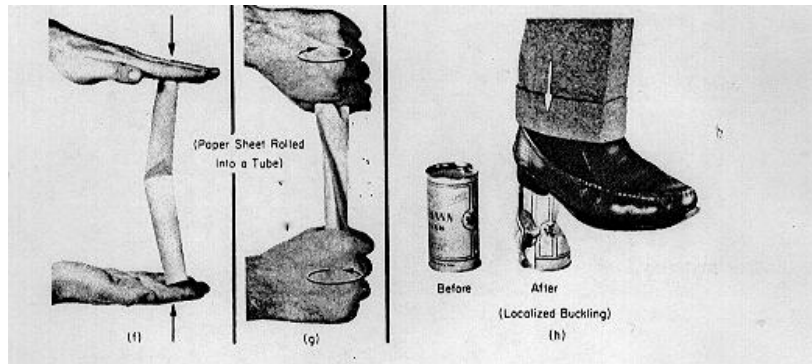


Figure 1b



## Introduction

### ■ The Nature of Buckling

#### Definition

***“Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased.”***



## Introduction

- Buckling Failure of Reinforced Concrete Columns



Figure 2



## Introduction

- Critical Buckling Load,  $P_{cr}$   
The critical buckling load (Euler Buckling) for a long column is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (1)$$

where

$E$  = modulus of elasticity of the material

$I$  = moment of inertia of the cross section

$L$  = length of column



## Strength of Reinforced Concrete Columns: Small Eccentricity

- If a compression member is loaded parallel to its axis by a load  $P$  without eccentricity, the load  $P$  theoretically induces a uniform compressive stress over the cross-sectional area.
- If the compressive load is applied a small distance  $e$  away from the longitudinal axis, however, there is a tendency for the column to bend due to the moment  $M = Pe$ .



## Strength of Reinforced Concrete Columns: Small Eccentricity

- Eccentric Axial Loading in a Plane of Symmetry
  - When the line of action of the axial load  $P$  passes through the centroid of the cross section, it can be assumed that the distribution of normal stress is uniform throughout the section.
  - Such a loading is said to be *centric*, as shown in Fig 3.



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Eccentric Axial Loading in a Plane of Symmetry

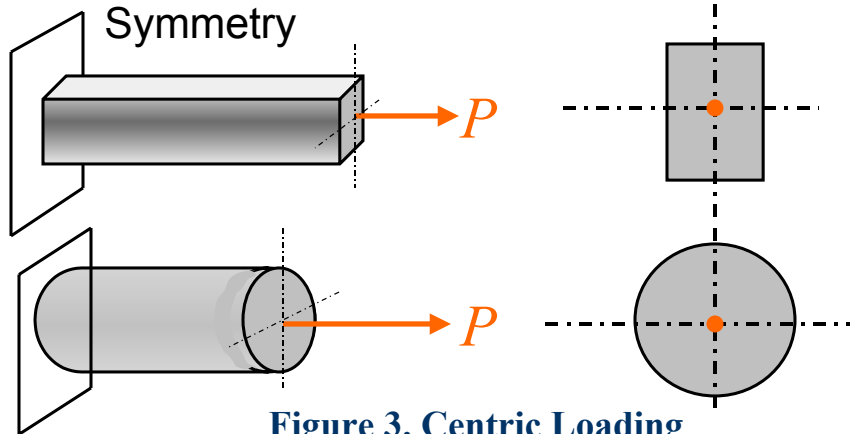


Figure 3. Centric Loading



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Eccentric Axial Loading in a Plane of Symmetry

- When the line of action of the concentrated load  $P$  does not pass through the centroid of the cross section, the distribution of normal stress is no longer uniform.
- Such loading is said to be eccentric, as shown in Fig 4.



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Eccentric Axial Loading in a Plane of Symmetry

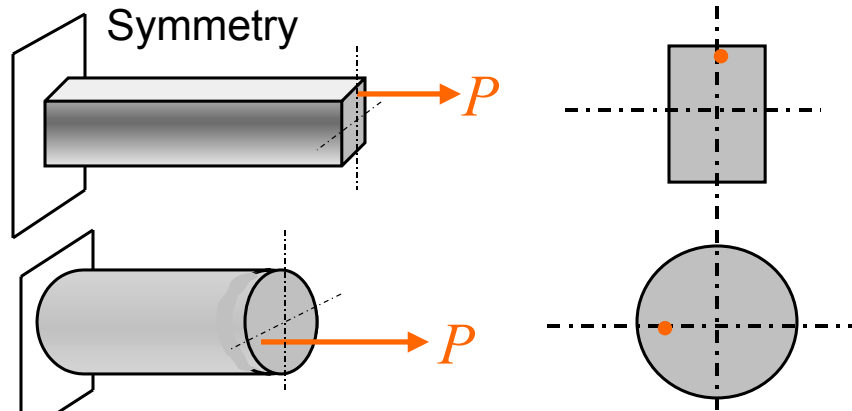


Figure 4. Eccentric Loading



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Eccentric Axial Loading in a Plane of Symmetry

The stress due to eccentric loading on a beam cross section is given by

$$f_x = \frac{P}{A} \pm \frac{My}{I} \quad (2)$$





## Strength of Reinforced Concrete Columns: Small Eccentricity

- Columns Loaded with Small Eccentricities
  - The concrete column that is loaded with a compressive axial load  $P$  at zero eccentricity is probably nonexistent, and even the axial/small eccentricity combination is relatively rare.
  - Nevertheless, the case of columns that are loaded with compressive axial loads at small eccentricity  $e$  is considered first. In this case we define the situation in which the induced small moments are of little significance.



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Notations Columns Loaded with Small Eccentricities

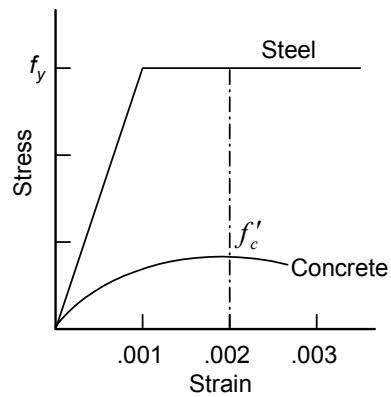
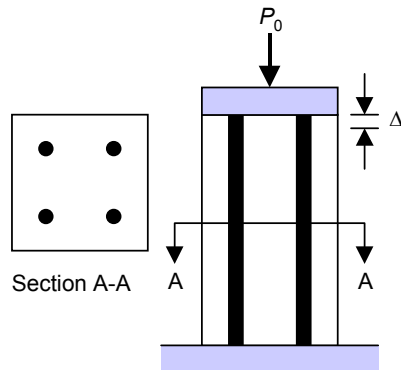
$A_g$  = gross area of the column section (in<sup>2</sup>)  
 $A_{st}$  = total area of longitudinal reinforcement (in<sup>2</sup>)  
 $P_0$  = nominal or theoretical axial load at zero eccentricity  
 $P_n$  = nominal or theoretical axial load at given eccentricity  
 $P_u$  = factored applied axial load at given eccentricity  
 $\rho_g$  = ratio of total longitudinal reinforcement area to cross-sectional area of column:

$$\rho_g = \frac{A_{st}}{A_g} \quad (3)$$



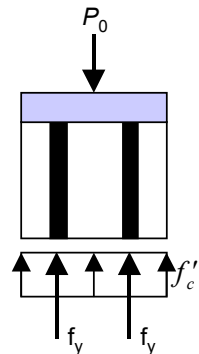
## Strength of Reinforced Concrete Columns: Small Eccentricity

### Strength of Short Axially Loaded Columns



## Strength of Reinforced Concrete Columns: Small Eccentricity

### Strength of Short Axially Loaded Columns



$$F_s = A_{st} f_y$$

$$F_c = (A_g - A_{st}) f'_c$$

$$[\Sigma F_y = 0]$$

$$P_0 = f'_c (A_g - A_{st}) + f_y A_{st}$$

From experiment (e.g., ACI):

$$P_0 = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$$

where

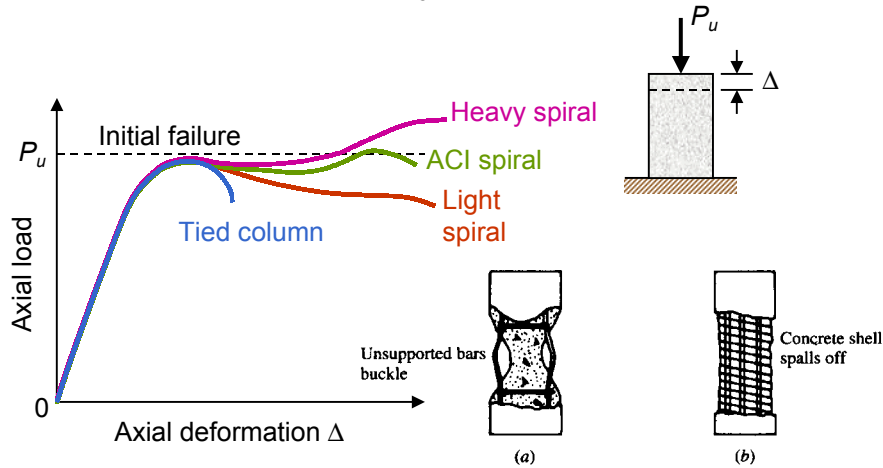
$A_g$  = Gross area of column section

$A_{st}$  = Longitudinal steel area



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ Column Failure by Axial Load



## Strength of Reinforced Concrete Columns: Small Eccentricity

### ■ ACI Code Requirements for Column Strength

$$\phi P_n \geq P_u \quad (4)$$

Spirally reinforced column:

$$\phi P_{n(\max)} = 0.85\phi [0.85f'_c(A_g - A_{st}) + f_y A_{st}], \quad \phi = 0.75 \quad (5)$$

Tied column:

$$\phi P_{n(\max)} = 0.80\phi [0.85f'_c(A_g - A_{st}) + f_y A_{st}], \quad \phi = 0.70 \quad (6)$$



## Code Requirements Concerning Column Details

### ■ Limits on percentage of reinforcement

$$0.01 \leq \left[ \rho_g = \frac{A_{st}}{A_g} \right] \leq 0.08 \quad (7)$$

Lower limit: To prevent failure mode of plain concrete

Upper limit: To maintain proper clearances between bars



## Code Requirements Concerning Column Details

### ■ Minimum Number of Bars

– The minimum number of longitudinal bars is

- four within rectangular or circular ties
- Three within triangular ties
- Six for bars enclosed by spirals

### ■ Clear distance between Bars

– The clear distance between longitudinal bars must not be less than 1.5 times the nominal bar diameter nor 1 ½ in.



# Code Requirements Concerning Column Details

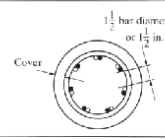
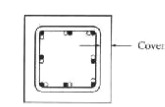
- Clear distance between Bars (cont'd)
  - Table 1 (Table A-14, Textbook) may be used to determine the maximum number of bars allowed in one row around the periphery of circular or square columns.
- Cover
  - Cover shall be 1 ½ in. minimum over primary reinforcement, ties or spirals.



# Code Requirements Concerning Column Details

Table A-14, Textbook

Table 1. Preferred Maximum Number of Column Bars in One Row

Recommended spiral or tie bar number	Core size (in.) = column size - 2 x cover	Circular area (in. <sup>2</sup> )	Bar number							Square area (in. <sup>2</sup> )	Bar number						
			#5	#6	#7	#8	#9	#10	#11		#5	#6	#7	#8	#9	#10	#11
3*	9	63.6	8	7	7	6	—	—	—	81	8	8	8	8	4	4	4
	10	78.5	10	9	8	7	6	—	—	100	12	8	8	8	8	4	4
	11	95.0	11	10	9	8	7	6	—	121	12	12	8	8	8	8	4
	12	113.1	12	11	10	9	8	7	6	144	12	12	12	8	8	8	8
	13	132.7	13	12	11	10	8	7	6	169	16	12	12	12	8	8	8
	14	153.9	14	13	12	11	9	8	7	196	16	16	12	12	12	8	8
15	176.7	15	14	13	12	10	9	8	225	16	16	16	12	12	12	8	
4	16	201.1	16	15	14	12	11	9	8	256	20	16	16	16	12	12	8
	17	227.0	18	16	15	13	12	10	9	289	20	20	16	16	12	12	8
	18	254.5	19	17	15	14	12	11	10	324	20	20	16	16	16	12	12
	19	283.5	20	18	16	15	13	11	10	361	24	20	20	16	16	12	12
	20	314.2	21	19	17	16	14	12	11	400	24	24	20	20	16	12	12
	21	346.4	22	20	18	17	15	13	11	441	28	24	20	20	16	16	12
22	380.1	23	21	19	18	15	14	12	484	28	24	24	20	20	16	12	
5	23	415.5	24	22	21	19	16	14	13	529	28	28	24	24	20	16	16
	24	452.4	25	23	21	20	17	15	13	576	32	28	24	24	20	16	16
	25	490.9	26	24	22	20	18	16	14	625	32	28	28	24	20	20	16
	26	530.9	28	25	23	21	19	16	14	676	32	32	28	24	24	20	16
	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

\*No. 4 tie for No. 11 or larger longitudinal reinforcement.



## Code Requirements Concerning Column Details

### ■ Tie Requirements

- According to Section 7.10.5 of ACI Code, the minimum is
  - No. 3 for longitudinal bars No. 10 and smaller
  - Otherwise, minimum tie size is No. 4 (see Table 1 for a suggested tie size)
- The center-to-center spacing of ties must not exceed the smaller of 16 longitudinal bar diameter, 48 tie-bar diameter, or the least column dimension.



## Code Requirements Concerning Column Details

### ■ Spiral Requirements

- According to Section 7.10.4 of ACI Code, the minimum spiral size is 3/8 in. in diameter for cast-in-place construction (5/8 is usually maximum).
- Clear space between spirals must not exceed 3 in. or be less than 1 in.





## Code Requirements Concerning Column Details

### ■ Spiral Requirements (cont'd)

- The spiral steel ratio  $\rho_s$  must not be less than the value given by

$$\rho_{s(\min)} = 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_y} \quad (8)$$

where

$$\rho_s = \frac{\text{volume of spiral steel in one turn}}{\text{volume of column core in height } (s)}$$

$s$  = center-to-center spacing of spiral (in.), also called pitch

$A_g$  = gross cross-sectional area of the column (in<sup>2</sup>)

$A_c$  = cross-sectional area of the core (in<sup>2</sup>) (out-to-out of spiral)

$f_y$  = spiral steel yield point (psi)  $\leq 60,000$  psi  
= compressive strength of concrete (psi)



## Code Requirements Concerning Column Details

### ■ Spiral Requirements (cont'd)

- An Approximate Formula for Spiral Steel Ratio

- A formula in terms of the physical properties of the column cross section can be derived from the definition of  $\rho_s$ .
- In reference to Fig. 5, the overall core diameter (out-to-out of spiral) is denoted as  $D_c$ , and the spiral diameter (center-to-center) as  $D_s$ .
- The cross-sectional area of the spiral bar or wire is given the symbol  $A_{sp}$ .



## Code Requirements Concerning Column Details

### ■ Spiral Requirements (cont'd)

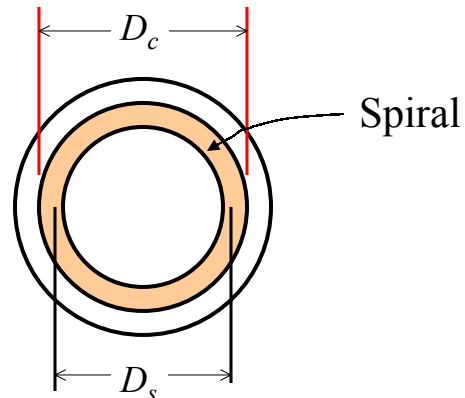


Figure 5. Definition of  $D_c$  and  $D_s$



## Code Requirements Concerning Column Details

### ■ Spiral Requirements (cont'd)

- From the definition of  $\rho_s$ , an expression may be written as follows:

$$\text{actual } \rho_s = \frac{A_{sp} \pi D_s}{(\pi D_c^2 / 4)(s)} \quad (9)$$

- If the small difference between  $D_c$  and  $D_s$  is neglected, then in terms of  $D_c$ , the actual spiral steel ratio is given by

$$\text{actual } \rho_s = \frac{4A_{sp}}{D_c s} \quad (10)$$