Reinforced Concrete Design — Fifth Edition

CHAPTER 5a

# DEVELOPMENT, SPLICES, AND SIMPLE SPAN BAR CUTOFFS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part I – Concrete Design and Analysis

FALL 2002


By
Dr. Ibrahim. Assakkaf

**ENCE 355 - Introduction to Structural Design**

Department of Civil and Environmental Engineering
University of Maryland, College Park

Prentice Hall

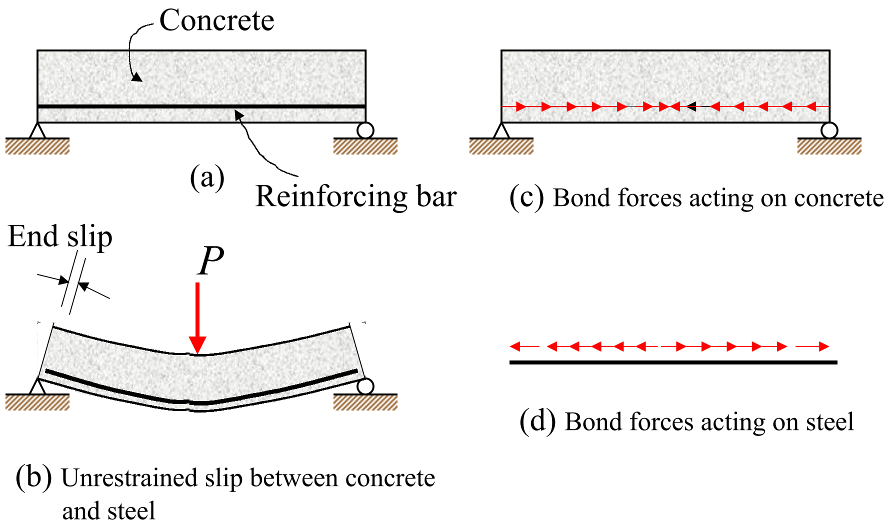


## Introduction

- If the reinforced concrete beam shown in Fig. 1 were constructed using plain round reinforcing bars, and in addition, if those bars were to be greased or otherwise lubricated before the concrete were poured, the beam would be as strong as it was made of plain concrete, without reinforcement.

# Introduction

Figure. 1. Bond Stresses due to Flexure






# Introduction

- If a load is applied as shown Fig. 1b, the bars would tend to maintain its original length as the beam deflects.
- The bars would slip longitudinally with respect to adjacent concrete, which would experience tensile strain due to flexure.
- The assumption that the strain in an embedded reinforcing bar is the same as that in surrounding concrete, would

# Introduction

Not be valid.

- In order for reinforced concrete to behave as intended, it is essential that "***bond forces***" be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at the interface.
- It is through the action of these interface bond forces that the slip of Fig. 5b is prevented.



# Introduction

- The assumptions for the design of reinforced concrete include:
  1. Perfect bonding between the concrete and steel exist, and
  2. No slippage occur.
- Based on these assumptions, it follows that some form of bond stress exists at the contact surface between the concrete and steel bars.

# Introduction

- In beams, this bond stress is caused by the change in bending moment along the length of the beam and the accompanying change in the tensile stress in the bars (flexural bond).
- The actual distribution of bond stresses along the reinforcing steel is highly complex, due mainly to the presence of concrete cracks.



# Introduction

- Large local variations in bond stress are caused by flexural and diagonal cracks.
- High bond stresses have been measured adjacent to these cracks.
- The high bond stress may result in:
  - Small local slips adjacent to the crack
  - Increased deflection
- In general, this is harmless as long as failure does not propagate all along the bar with complete loss of bond.

# Introduction

- Development Length
  - End anchorage may be considered reliable if the bar is embedded into concrete a prescribed distance known as the "***development length***" of the bar.
  - In a beam, if the the actual extended length of the bar is equal or greater than this required development length, then no bond failure will occur.



# Introduction

- Development Length

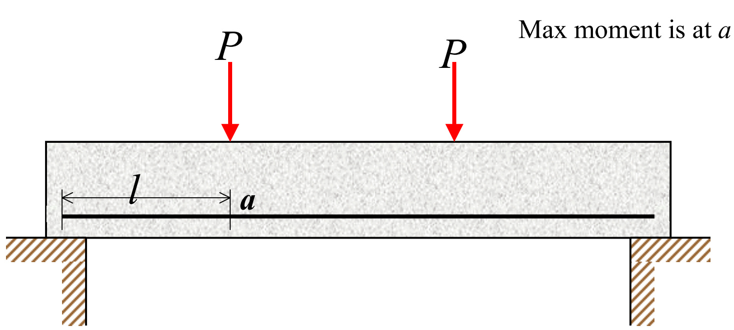
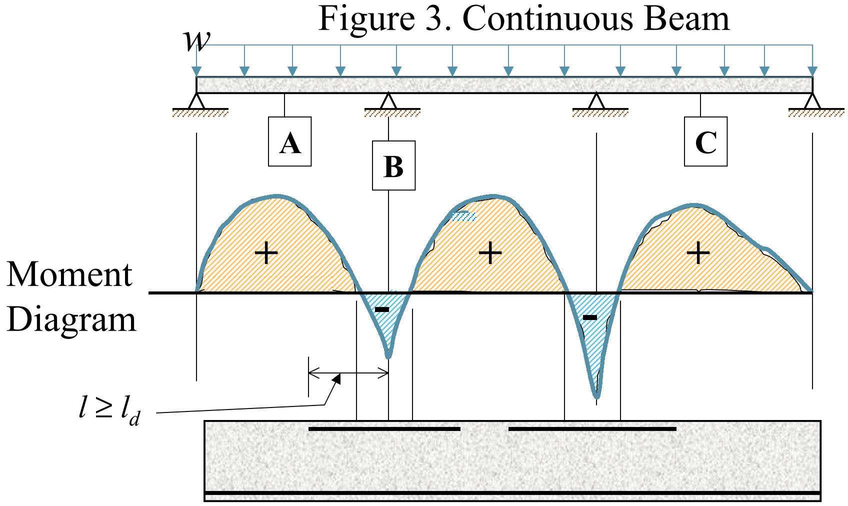



Figure 2. Development length
$l$ should be at least equal to $l_b$

# Introduction

- Need for Development Length

Figure 3. Continuous Beam






# Introduction

- Anchorages Versus Development Length

  If the actual available length is inadequate for full development, special anchorages ,such as hooks, must be provided to ensure adequate strength.

# Introduction

- ACI Code
  - The provisions of the ACI Code are directed toward providing adequate length of embedment, past the location at which the bar is fully stressed, which will ensure development of the full strength of the bar.
  - Therefore, the current method based on ACI disregard high localized bond stress even though it may result in localized slip between steel and concrete adjacent to the cracks.



# Development Length: Tension Bars

- Methods for Determining the Development Length, $l_d$
  - The ACI allows the determination of the development length by two methods:
    1. Tabular criteria (ACI Section 12.2.2)
    2. General equation (ACI Section 12.2.3)
  - In either case, $l_d$ shall not be less than 12 in.
  - The general equation of the ACI Code offers a simple approach that allows the user to see the effect of all variables controlling the development length.

# Development Length: Tension Bars

- Methods for Determining the Development Length, $l_d$ (cont'd)

  This equation (ACI Eq. 12-1) is provided in Section 12.2.3 of the ACI Code, and it is as follows:

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f_c'}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b \qquad (1)$$



# Development Length: Tension Bars

- Notations of Eq. 1:

  $(c + k_{tr})/d_b$: shall not be taken greater than 2.5

  $l_d$ = development length (in.)

  $f_y$ = yield strength of nonprestressed reinforcement (psi)

  $f_c'$ = compressive strength of concrete (psi); the value of $\sqrt{f_c'}$ shall not exceed 100 psi (ACI Code, Section 12.1.2)

  $d_b$ = nominal diameter of bar or wire (in.)

# Development Length: Tension Bars

- Comments for Eq. 1:
  1. $\alpha$ is a reinforcement location factor that accounts for the position of the reinforcement in freshly place concrete.

     **$\alpha$ = 1.3** (ACI Code, Section 12.2.4) where horizontal reinforcement is so placed that more than 12 in. of fresh concrete is cast in member below the development length or splice.

     **$\alpha$ = 1.0** for other reinforcement.
  2. $\beta$ is a coating factor reflecting the effects of epoxy coating.

     For epoxy-coated reinforcement having cover less than $3d_b$ or clear spacing between bars less than $6d_b$, use **$\beta$ = 1.5**




# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

     For all other conditions, use **$\beta$ = 1.2**

     For uncoated reinforcement, use **$\beta$ = 1.0**

     The product of $\alpha$ and $\beta$ need not be taken greater than **1.7** (ACI Code, Section 12.2.4)
  3. $\gamma$ is a reinforcement size factor.

     Where No. 6 and smaller bars are used, **$\gamma$ = 0.8**

     Where No. 7 and larger bars used, **$\gamma$ = 0.1**
  4. $\lambda$ is a lightweight-aggregate concrete factor.

     For lightweight-aggregate concrete when the average splitting tensile strength $f_{ct}$ is not specified, use **$\lambda$ = 1.3**

# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

When $f_{ct}$ is specified, use

$$\lambda = 6.7\frac{\sqrt{f'_c}}{f_{ct}} \geq 1.0$$

When normal-weight concrete is used, $\lambda$ **= 1.0** (ACI Code, Section 12.2.4)

5. $c$ represents a spacing or cover dimension (in.)

   The value of $c$ will be the smaller of either the distance from the center of the bar to the nearest concrete cover (surface) or one-half the center-to-center spacing of the bars being developed (spacing).



# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

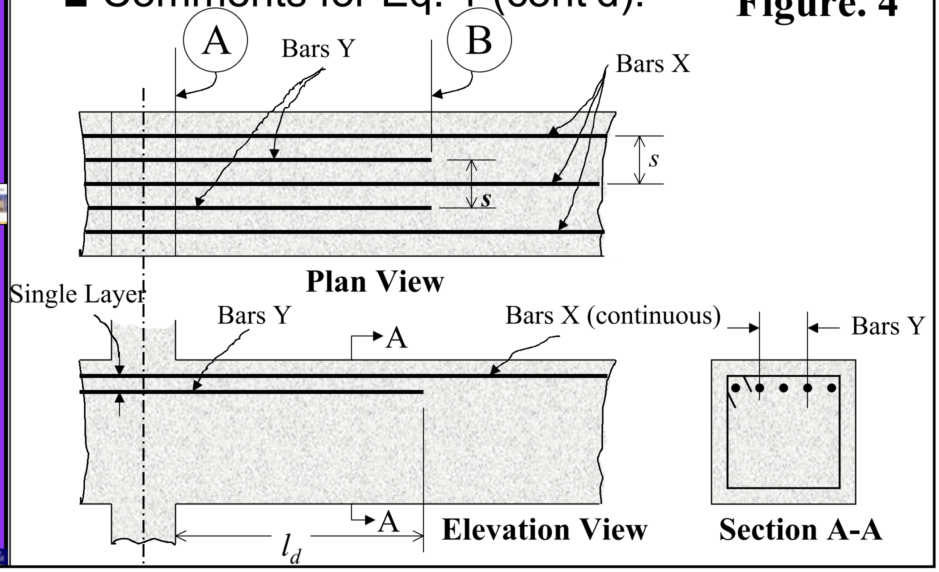
The bar spacing will be the actual center-to-center spacing between the bars if adjacent bars are all being developed at the same location. If, however, an adjacent bar has been developed at another location, the spacing to be used will be greater than the actual spacing to the adjacent bar.

Note in Fig. 4 that the spacing for bars Y may be taken the same as for bars X, since bars Y are developed in length AB, whereas bars X are developed at a location other than AB.

# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

**Figure. 4**






# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

6. The transverse reinforcement index $K_{tr}$ is to be calculated from

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

where

$A_{tr}$ = total cross-sectional area of all transverse reinforcement that is within the spacing $s$ and that crosses the potential plane of splitting through the reinforcement being developed (in$^2$)

$f_{yt}$ = yield strength of transverse reinforcement (psi)

# Development Length: Tension Bars

- Comments for Eq. 1 (cont'd):

$s$ = maximum center-to-center spacing of transverse reinforcement within the development length $l_d$ (in.)

$n$ = number of bars or wires being developed along the plane of splitting.



# Development Length: Tension Bars

- Reduction in Development Length
  - A reduction in the development length $l_d$ is permitted where reinforcement is in excess of that required by analysis (except where anchorage or development for $f_y$ is specifically required or where the design includes provisions for seismic considerations).
  - The reduction factor $K_{ER}$ is given by

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}} \qquad (2)$$

# Procedure for Calculation of $l_d$

1. Determine multiplying factors (use 1.0 unless otherwise determined).
   a. Use $\alpha$ = 1.3 for top reinforcement, when applicable.
   b. Coating factor $\beta$ applies to epoxy-coated bars. Determine cover and clear spacing as multiples of $d_b$. Use $\beta$ = 1.5 if cover < $3d_b$ or clear space < $6d_b$. Use $\beta$ = 1.2 otherwise.
   c. Use $\gamma$ = 0.8 for No. 6 bars and smaller.
   d. Use $\lambda$ = 1.3 for lightweight concrete with $f_{ct}$ not specified. Use $\lambda = 6.7\dfrac{\sqrt{f'_c}}{f_{ct}} \geq 1.0$ if $f_{ct}$ specified.



# Procedure for Calculation of $l_d$

2. Check $\alpha\beta \leq 1.7$.
3. Determine $c$, the smaller of cover or half-spacing (both referenced to the center of the bar).
4. Calculate

$$K_{tr} = \frac{A_{tr} f_y}{1500sn}, \text{ or use } K_{tr} = 0 \text{ (conservative)}$$

5. Check

$$\frac{c + K_{tr}}{d_b} \leq 2.5$$

# Procedure for Calculation of $l_d$

6. Calculate $K_{ER}$ if applicable:

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}}$$

7. Calculate $l_d$ from Eq. 1 (ACI Code Eq. 12-1):

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$



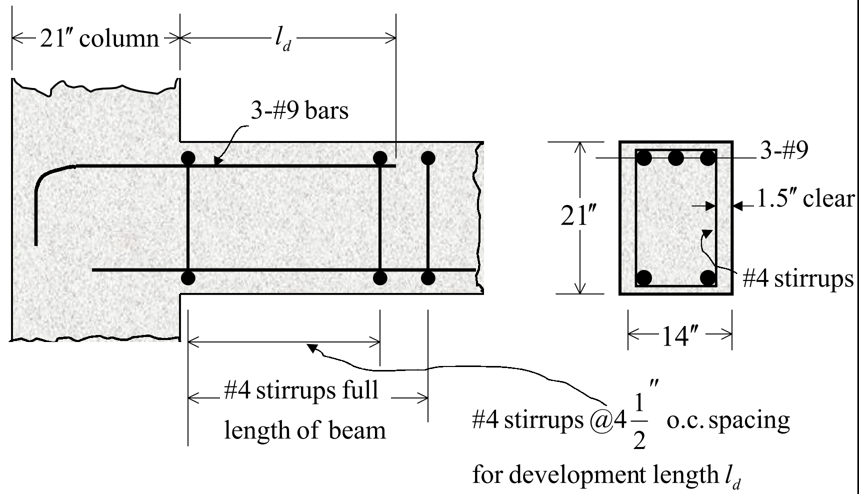
# Procedure for Calculation of $l_d$

- Example 1

  Calculate the required development length $l_d$ into the beam for the negative moment steel shown so as to develop the tensile strength of the steel at the face of the column. Required $A_s$ = 2.75 in$^2$, $f'_c$ =4,000 psi, and $f_y$ = 60,000 psi. Assume normal-weight concrete.

# Procedure for Calculation of $l_d$

- Example 1 (cont'd)






# Procedure for Calculation of $l_d$

- Example 1 (cont'd)

3#9 bars: $d_b = 1.128''$ ← From Table 1

(1) $\alpha = 1.3,\ \beta = 1.0,\ \gamma = 1.0,\ \text{and}\ \lambda = 1.0$

(2) $\alpha\beta = (1.3)(1) = 1.3 < 1.7$ OK

(3) cover: $c = 1.5 + 0.5 + \dfrac{1.128}{2} = 2.56''$

Dia. #4 stirrup

Half-spacing: $c = \dfrac{14 - 2(1.5) - 2(0.5) - 1.128}{2(2)} = 2.22''$ ← Controls

(4) $K_{tr} = \dfrac{A_{tr} f_{yt}}{1500sn} = \dfrac{0.4(60{,}000)}{1500(4.5)(3)} = 1.185$

Area of 2 #4 stirrups

# Procedure for Calculation of $l_d$

- Example 1 (cont'd)

Table 1. ASTM Standard - English Reinforcing Bars

| Bar Designation | Diameter in | Area in² | Weight lb/ft |
|---|---|---|---|
| #3 [#10] | 0.375 | 0.11 | 0.376 |
| #4 [#13] | 0.500 | 0.20 | 0.668 |
| #5 [#16] | 0.625 | 0.31 | 1.043 |
| #6 [#19] | 0.750 | 0.44 | 1.502 |
| #7 [#22] | 0.875 | 0.60 | 2.044 |
| #8 [#25] | 1.000 | 0.79 | 2.670 |
| #9 [#29] | 1.128 | 1.00 | 3.400 |
| #10 [#32] | 1.270 | 1.27 | 4.303 |
| #11 [#36] | 1.410 | 1.56 | 5.313 |
| #14 [#43] | 1.693 | 2.25 | 7.650 |
| #18 [#57] | 2.257 | 4.00 | 13.60 |

Note: Metric designations are in brackets



# Procedure for Calculation of $l_d$

- Example 1 (cont'd)

Table 2. Areas of Multiple of Reinforcing Bars (in$^2$)

| Number of bars | Bar number #3 | #4 | $5 | #6 | #7 | #8 | #9 | #10 | #11 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.11 | 0.20 | 0.31 | 0.44 | 0.60 | 0.79 | 1.00 | 1.27 | 1.56 |
| 2 | 0.22 | 0.40 | 0.62 | 0.88 | 1.20 | 1.58 | 2.00 | 2.54 | 3.12 |
| 3 | 0.33 | 0.60 | 0.93 | 1.32 | 1.80 | 2.37 | 3.00 | 3.81 | 4.68 |
| 4 | 0.44 | 0.80 | 1.24 | 1.76 | 2.40 | 3.16 | 4.00 | 5.08 | 6.24 |
| 5 | 0.55 | 1.00 | 1.55 | 2.20 | 3.00 | 3.95 | 5.00 | 6.35 | 7.80 |
| 6 | 0.66 | 1.20 | 1.86 | 2.64 | 3.60 | 4.74 | 6.00 | 7.62 | 9.36 |
| 7 | 0.77 | 1.40 | 2.17 | 3.08 | 4.20 | 5.53 | 7.00 | 8.89 | 10.92 |
| 8 | 0.88 | 1.60 | 2.48 | 3.52 | 4.80 | 6.32 | 8.00 | 10.16 | 12.48 |
| 9 | 0.99 | 1.80 | 2.79 | 3.96 | 5.40 | 7.11 | 9.00 | 11.43 | 14.04 |
| 10 | 1.10 | 2.00 | 3.10 | 4.40 | 6.00 | 7.90 | 10.00 | 12.70 | 15.60 |

Table A-2 Textbook

# Procedure for Calculation of $l_d$

■ Example 1 (cont'd)

(5) $\dfrac{c+K_{tr}}{d_b} = \dfrac{2.22+1.185}{1.128} = 3.02 > 2.5$, Therefore, use 2.5

(6) $K_{ER} = \dfrac{A_s \text{ required}}{A_s \text{ provided}} = \dfrac{2.75}{3.00} = 0.917$

(7) Calculate the development length $l_d$ using Eq. 1:

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c+k_{tr}}{d_b}\right)}\right]d_b$$



# Procedure for Calculation of $l_d$

■ Example 1 (cont'd)

Reduction factor

$$l_d = K_{ER} \times \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c+k_{tr}}{d_b}\right)}\right]d_b$$

$$l_d = 0.917 \times \frac{3}{40}\left(\frac{60{,}000}{\sqrt{4{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.5}\right](1.128) = 38.3''$$

38.3 in. > 12 in OK