CHAPTER

4b

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Reinforced Concrete Design — Fifth Edition

SHEAR IN BEAMS

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Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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Shear Analysis Procedure

- The shear analysis procedure involves the following:
  – Checking the shear strength in an existing member
  – Verifying that the various ACI code requirements have been satisfied and met.
- Note that the member may reinforced or plain.

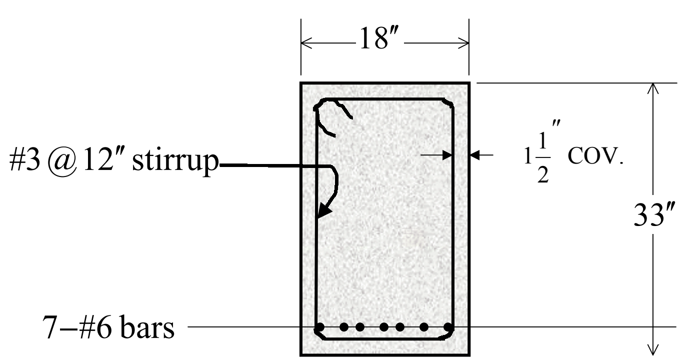
Shear Analysis Procedure

- Example 1

  A reinforced concrete beam of rectangular cross section shown is reinforced with seven No. 6 bars in a single layer. Beam width b = 18 in., d = 33 in., single-loop No. 3 stirrups are placed 12 in. on center, and typical cover is 1 ½ in. Find V_c, V_s, and the maximum factored shear force permitted on this member. Use = 4,000 psi and f_y = 60,000 psi.

Shear Analysis Procedure

- Example 1 (cont'd)



Shear Analysis Procedure

- Example 1 (cont'd)
  - The force that can be resisted by concrete alone is

$$V_c = 2\sqrt{f'_c}b_w d = \frac{2\sqrt{4{,}000}(18)(33)}{1000} = 75.1 \text{ kips}$$

  - The nominal shear force provided by the steel is

$$V_s = \frac{A_v f_y d}{s} = \frac{(2\times 0.11)(60)(33)}{12} = 36.3 \text{ kips}$$

  - The maximum factored shear force is

$$\text{maximum } V_u = \phi V_c + \phi V_s = 0.85(75.1 + 36.3)$$

$$= 94.7 \text{ kips}$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - According to the ACI Code, the design of beams for shear is based on the following relation:

$$\phi V_n \geq V_u \qquad (1)$$

Where

ϕ = strength reduction factor (= 0.85 for shear)

$V_n = V_c + V_s$

V_s = nominal shear strength provided by reinforcement

$$= \frac{A_v f_y d}{s} \quad \text{for inclined stirrups} \qquad (2)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Symbols

A_v = total cross-sectional area of web reinforcement within a distance s, for single loop stirrups, $A_v = 2A_s$
A_s = cross-sectional area of the stirrup bar (in^2)
b_w = web width = b for rectangular section (in.)
s = center-to-center spacing of shear reinforcement in a direction parallel to the longitudinal reinforcement (in.)
f_y = yield strength of web reinforcement steel (psi)

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - For inclined stirrups, the expression for nominal shear strength provided by reinforcement is

$$V_s = \frac{A_v f_y d(\sin\alpha + \cos\alpha)}{s} \qquad (3)$$

  - For $\alpha = 45^0$, the expression takes the form

$$V_s = \frac{1.414 A_v f_y d}{s} \qquad (4)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - The design for stirrup spacing can be determined from

$$\text{required } s = \frac{A_v f_y d}{V_s} \quad \text{(for vertical stirrups)} \tag{5}$$

and

$$\text{required } s = \frac{1.414 A_v f_y d}{V_s} \quad \text{(for } 45^0 \text{ stirrups)} \tag{6}$$

where

$$V_s = \frac{V_u - \phi V_c}{\phi} \tag{7}$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - According to the ACI Code, the maximum spacing of stirrups is the smallest value of

$$\left.\begin{aligned} s_{\max} &= \frac{A_v f_y}{50 b_w} \\ s_{\max} &= \frac{d}{2} \\ s_{\max} &= 24 \text{ in.} \end{aligned}\right. \tag{8}$$

If V_s exceeds $4\sqrt{f'_c}\, b_w d$, the maximum spacing must not exceed $d/4$ or 12 in.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - It is not usually good practice to space vertical stirrups closer than 4 in.
  - It is generally economical and practical to compute spacing required at several sections and to place stirrups accordingly in groups of varying spacing. Spacing values should be made to not less than 1-in. increments.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section
    - The maximum shear usually occurs in this section near the support.
    - For stirrup design, the section located a distance d from the face of the support is called the "***critical section***"
    - Sections located less than a distance d from the face of the support may be designed for the same V_u as that of the critical section.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section

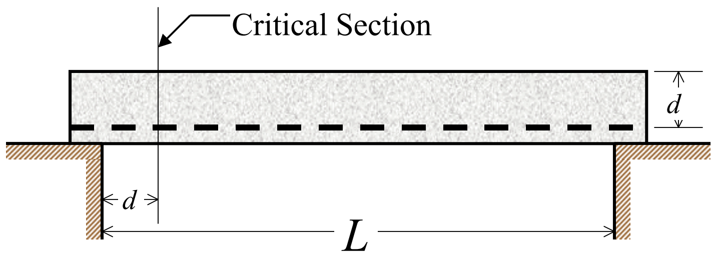


Figure 1

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Design
  - Critical Section (cont'd)
    - The stirrup spacing should be constant from the critical section back to the face of the support based on the spacing requirements at the critical section.
    - The first stirrup should be placed at a maximum distance of $s/2$ from the face of the support, where s equals the immediately adjacent required spacing (a distance of 2 in. is commonly used.

Stirrup Design Procedure

- The design of stirrups for shear reinforcement involves the determination of stirrup size and spacing pattern.
- A general procedure is as follows:
  1. Determine the shear values based on clear span and draw a shear diagram for V_u.
  2. Determine if stirrups are required.

Stirrup Design Procedure

3. Determine the length of span over which stirrups are required.
4. On the V_u diagram, determine the area representing “required ϕV_s.” This will display the required strength of the stirrups to be provided.
5. Select the size of the stirrups. Find the spacing required at the critical section (a distance d from the face of the support.

Stirrup Design Procedure

6. Establish the ACI Code maximum spacing requirements.
7. Determine the spacing requirements based on shear strength to be furnished by web reinforcing.
8. Establish the spacing pattern and show detailed sketches.

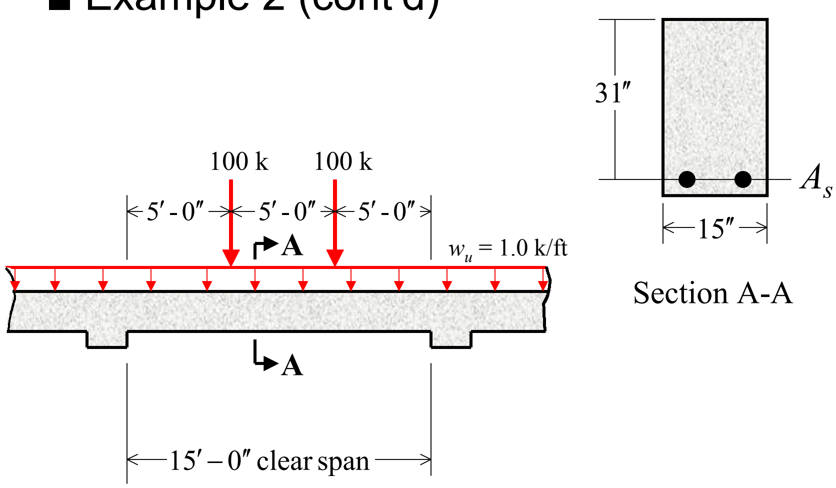
Stirrup Design Procedure

- Example 2

  A continuous reinforced concrete beam shown in the figure is 15 in. wide and has an effective depth of 31 in. The factored loads are shown, and the factored uniform load includes the weight of the beam. Design the web reinforcement if f'_c = 4000 psi and f_y = 60,000 psi.

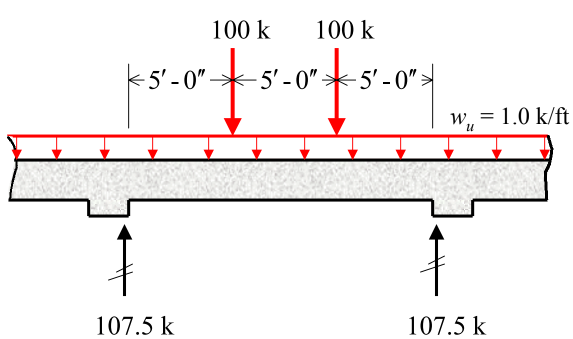
Stirrup Design Procedure

- Example 2 (cont'd)

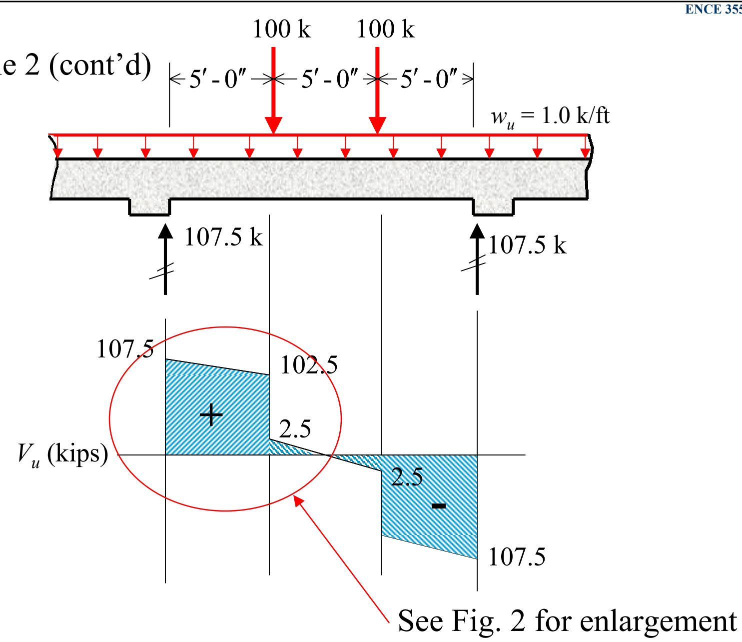


Stirrup Design Procedure

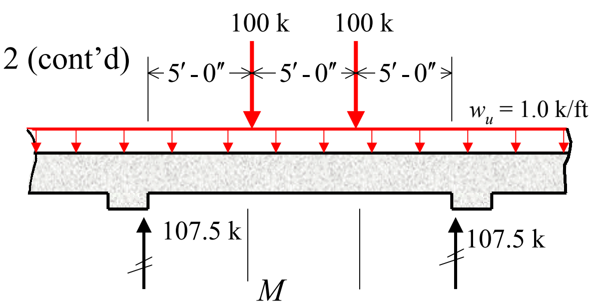
- Example 2 (cont'd) $R_1 = R_2 = \frac{2(100)+1(15)}{2} = 107.5\text{ k}$
  - Establish the shear force diagram for V_u:

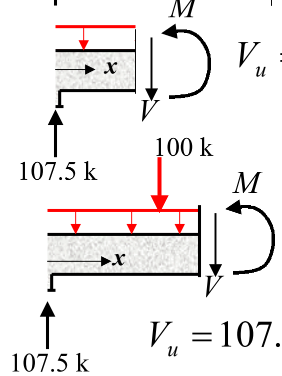


Example 2 (cont'd)



Example 2 (cont'd)





$$V_u = 107.5 - x \quad \text{for } 0 \leq x \leq 5$$

$$d = 31'' = 2.58'$$

$$V_u^* = V_u(2.58) = 107.5 - 2.58 = 104.9 \text{ k}$$

$$V_u = 107.5 - 100 - x \quad \text{for } 5 \leq x \leq 10$$

Stirrup Design Procedure

- Example 2 (cont'd)
  - Because of the symmetry, we will focus on the left half of the shear diagram as shown in Fig. 2.
  - Determine if stirrups required:

$$\phi V_c = \phi 2\sqrt{f_c'}\, b_w d = \frac{0.85(2)\sqrt{4{,}000}(15)(31)}{100} = 50 \text{ kips}$$

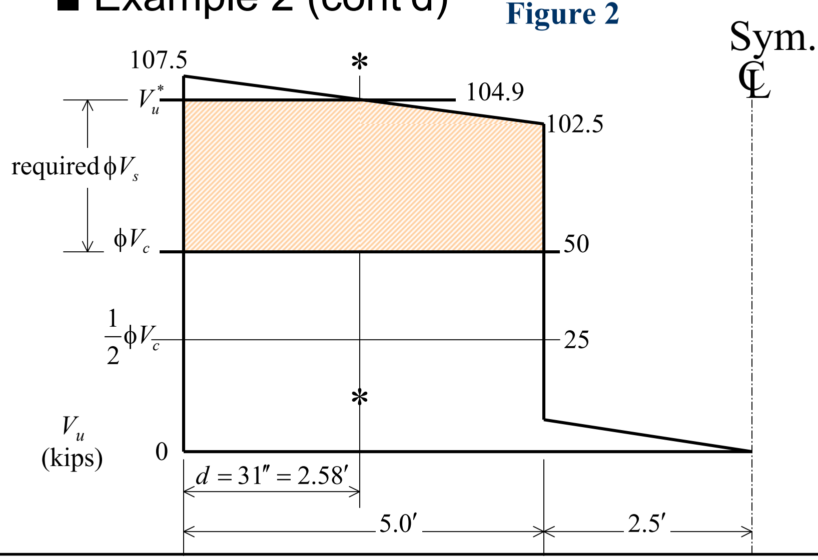
$$\frac{1}{2}\phi V_c = \frac{1}{2}(50) = 25 \text{ kips}$$

Since (V_u^* =104.9 k) > (1/2 ϕV_c = 25 k), stirrups are required.

Stirrup Design Procedure

- Example 2 (cont'd)

Figure 2



Stirrup Design Procedure

- Example 2 (cont'd)
  - Stirrups are required to the point where

$$V_u = \frac{1}{2}\phi V_c = 25\text{ kips}$$

From Fig. 2, this point is located at the first concentrated load and it is at distance 5 ft from the face of the support.

  - Determine the "required ϕV_s" on the V_u diagram:

$$\begin{aligned}\text{required}\,\phi V_s &= \max V_u - \phi V_c - wx \\ &= 107.5 - 50 - x \\ \text{required}\,\phi V_s &= 57.5 - x \quad \text{for } 2.58 \le x \le 5\end{aligned}$$

Stirrup Design Procedure

- Example 2 (cont'd)
  - Assume No. 3 vertical stirrups (A_v = 0.22 in^2):

$$\text{required } s^* = \frac{\phi A_v f_y d}{\text{required}\,\phi V_s^*} = \frac{0.85(0.22)(60)(31)}{104.9-50} = 6.3\text{ in.}$$

use 6 in.

  - Establish ACI Code maximum spacing requirements:

$$4\sqrt{f_c'}b_w d = \frac{4\sqrt{4000}(15)(31)}{1000} = 117.6\text{ kips}$$

$$V_s^* = \frac{\phi V_s^*}{\phi} = \frac{104.9-50}{0.85} = 64.6\text{ kips}$$

Stirrup Design Procedure

- Example 2 (cont'd)
  - Since 64.6 kips < 117.6 kips, the maximum spacing shall be the smallest of the following values (see Eq. 8):

$$s_{\max} = \frac{A_v f_y}{50 b_w} = \frac{0.22(60{,}000)}{50(15)} = 17.6 \text{ in.}$$

$$s_{\max} = \frac{d}{2} = \frac{31}{2} = 15.5 \text{ in.} \longleftarrow \text{controls}$$

$$s_{\max} = 24 \text{ in.}$$

Therefore, use a maximum spacing of 15 in.

Stirrup Design Procedure

- Example 2 (cont'd)
  - Determine the spacing requirements between the critical section and the first concentrated load:

$$\text{required } s = \frac{\phi A_v f_y d}{\text{required } \phi V_s} = \frac{0.85(0.22)(60)(31)}{57.5 - x}$$

  - The results of applying above equation for values of *x* range from 3 to 5 are tabulated as shown

x (ft)	Required s (in)
3	6.4
4	6.5
5	6.6

Stirrup Design Procedure

- Example 2 (cont'd)
  - Since no stirrups are required in the distance between the first concentrated load , it is clear that the maximum spacing of 15 in. need not be used in that distance.
  - A spacing of 6 in. will be used between the face of the support and the concentrated load.
  - The center part of the beam will be reinforced with stirrups at a spacing slightly less than the maximum spacing of 15 in.

Stirrup Design Procedure

- Example 2 (cont'd)

Final Sketch for Shear Reinforcement: