Reinforced Concrete Design — Fifth Edition

CHAPTER 3d

REINFORCED CONCRETE BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

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Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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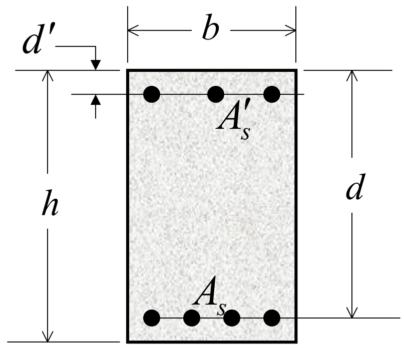
Doubly Reinforced Beams

- Introduction
  - If a beam cross section is limited because of architectural or other considerations, it may happen that concrete cannot develop the compression force required to resist the given bending moment.
  - In this case, reinforcing steel bars are added in the compression zone, resulting in a so-called ***doubly reinforced beam***, that is one with compression as well as tension reinforcement. (Fig. 1)

Doubly Reinforced Beams

■ Introduction (cont'd)

Figure 1. Doubly Reinforced Beam



Doubly Reinforced Beams

■ Introduction (cont'd)

– The use of compression reinforcement has decreased markedly with the use of strength design methods, which account for the full strength potential of the concrete on the compressive side of the neutral axis.

– However, there are situations in which compressive reinforcement is used for reasons other than strength.

Doubly Reinforced Beams

- Introduction (cont'd)
  - It has been found that the inclusion of some compression steel has the following advantages:
    - It will reduce the long-term deflections of members.
    - It will set a minimum limit on bending loading
    - It act as stirrup-support bars continuous through out the beam span.

Doubly Reinforced Beams

- Introduction (cont'd)
  - Another reason for placing reinforcement in the compression zone is that when beams span more than two supports (continuous construction), both positive and negative moments will exist as shown in Fig. 2.
  - In Fig. 2, positive moments exist at A and C; therefore, the main tensile reinforcement would be placed in the bottom of the beam.

Doubly Reinforced Beams

■ Introduction (cont'd)

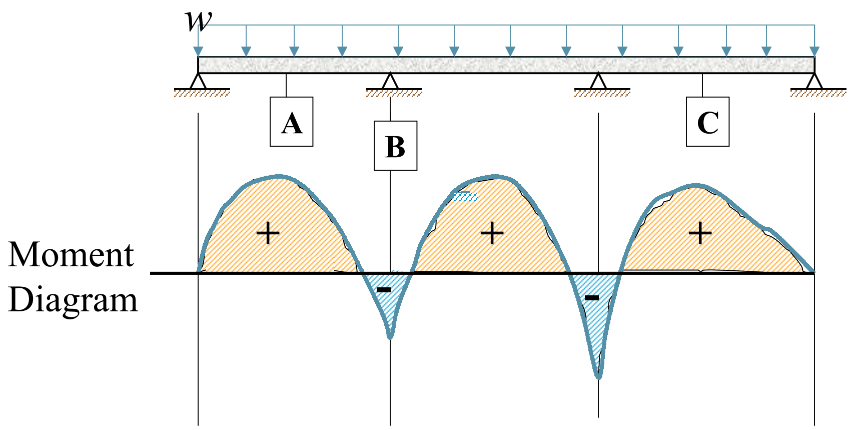


Figure 2. Continuous Beam

Doubly Reinforced Beams

■ Introduction (cont'd)

- At B, however, a negative moment exists and the bottom of the beam is in compression. The tensile reinforcement, therefore, must be placed near the top of the beam.

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress
  - The basic assumption for the analysis of doubly reinforced beams are similar to those for tensile reinforced beams.
  - The steel will behave elastically up to the point where the strain exceeds the yield strain ε_y. As a limit $f_s' = f_y$ when the compression strain $\varepsilon_s' \geq \varepsilon_y$.
  - If $\varepsilon_s' < \varepsilon_y$, the compression steel stress will be $f_s' = \varepsilon_s' E_s$.

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - If, in a doubly reinforced beam, the tensile steel ratio ρ is equal to or less than ρ_b, the strength of the beam may be approximated within acceptable limits by disregarding the compression bars.
  - The strength of such a beam will be controlled be tensile yielding, and the lever arm of the resisting moment will be little affected by the presence of comp. bars.

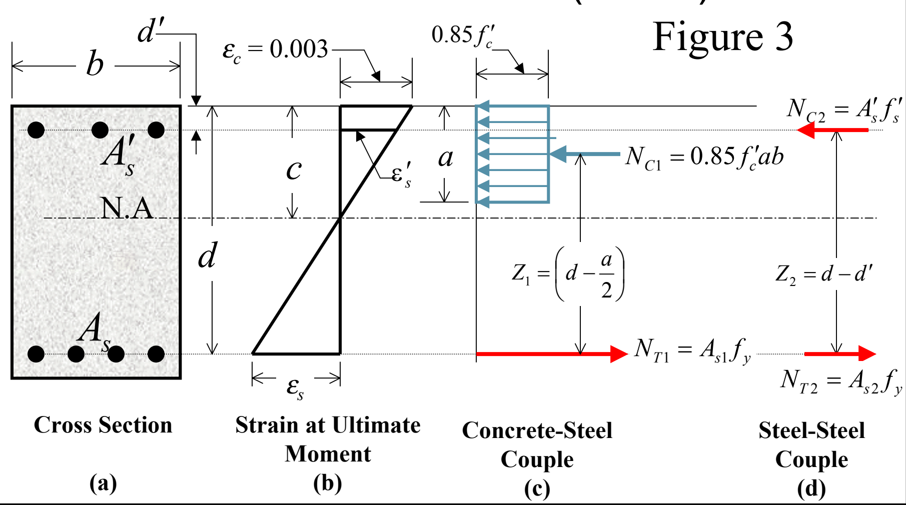
Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - If the tensile steel ratio ρ is larger than ρ_b, a somewhat elaborate analysis is required.
  - In Fig. 3a, a rectangular beam cross section is shown with compression steel A'_s placed at distance d' from the compression face and with tensile steel A_s at the effective depth d.

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

Figure 3



Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Notation for Doubly Reinforced Beam:

A'_s = total compression steel cross-sectional area
d = effective depth of tension steel
d' = depth to centroid of compressive steel from compression fiber
A_{s1} = amount of tension steel used by the concrete-steel couple
A_{s2} = amount of tension steel used by the steel-steel couple
A_s = total tension steel cross-sectional area ($A_s = A_{s1} + A_{s2}$)
M_{n1} = nominal moment strength of the concrete-steel couple
M_{n2} = nominal moment strength of the steel-steel couple
M_n = nominal moment strength of the beam
ε_s = unit strain at the centroid of the tension steel
ε'_s = unit strain at the centroid of the compressive steel

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Method of Analysis:
    - The total compression will now consist of two forces

      N_{C1}, the compression resisted by the concrete

      N_{C2}, the compression resisted by the steel
    - For analysis, the total resisting moment of the beam will be assumed to consist of two parts or two internal couples: The part due to the resistance of the compressive concrete and tensile steel and the part due to the compressive steel and additional tensile steel.

Doubly Reinforced Beam Analysis

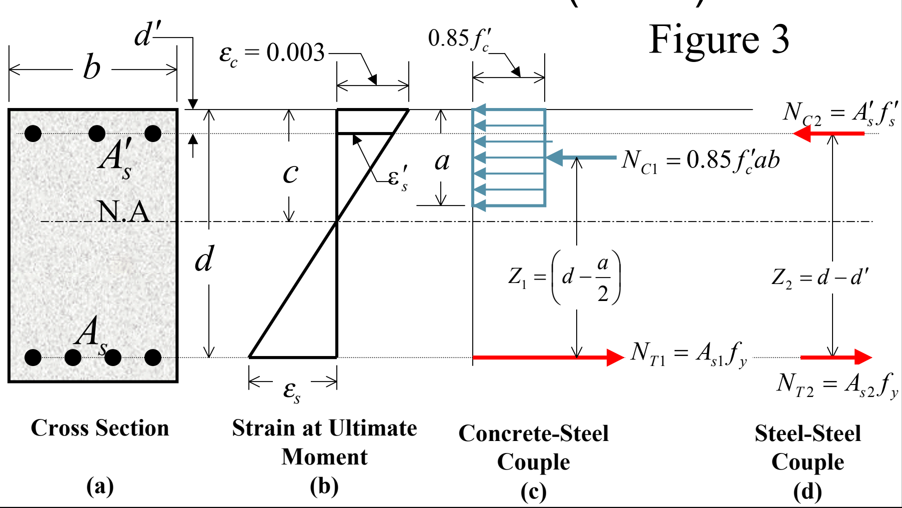
- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - The total nominal capacity may be derived as the sum of the two internal couples, neglecting the concrete that is displaced by the compression steel.
  - The strength of the steel-steel couple is given by (see Fig. 3)

$$M_{n2} = N_{T2} Z_2 \qquad (1)$$

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

Figure 3



Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

$M_{n2} = A_{s2} f_y (d - d')$ assuming $f_s = f_y$

$N_{C2} = N_{T2} \Rightarrow A'_s f'_s = A_{s2} f_y \Rightarrow A'_s = A_{s2}$

Therefore,

$$M_{n2} = A'_s f_y (d - d') \qquad (2)$$

– The strength of the concrete-steel couple is given by

$$M_{n1} = N_{T1} Z_1 \qquad (3)$$

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)

$M_{n1} = A_{s1} f_y \left(d - \frac{a}{2} \right)$ assuming $f_s = f_y$

$A_s = A_{s1} + A_{s2} \Rightarrow A_{s1} = A_s - A_{s2}$

since $A_{s2} = A'_s$, then

$$A_{s1} = A_s - A'_s$$

Therefore

$$M_{n1} = (A_s - A'_s) f_y \left[d - \frac{a}{2} \right] \qquad (4)$$

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Nominal Moment Capacity

    From Eqs. 2 and 4, the nominal moment capacity can be evaluated as

$$
\begin{aligned}
M_n &= M_{n1} + M_{n2} \\
&= \left(A_s - A_s'\right) f_y \left[d - \frac{a}{2}\right] + A_s' f_y \left(d - d'\right)
\end{aligned}
\quad (5)
$$

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  - Determination of the Location of Neutral Axis:

$$c = \frac{a}{\beta_1}$$

$$N_T = N_{C1} + N_{C2}$$

$$A_s f_y = \left(0.85 f_c'\right) ab + A_s' f_y$$

Therefore,

$$a = \frac{\left(A_s - A_s'\right) f_y}{0.85 f_c' b} = \frac{A_{s1} f_y}{0.85 f_c' b}$$

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  – Location of Neutral Axis *c*

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{A_{s1} f_y}{0.85 f_c' b} \quad (6)$$

$$c = \frac{a}{\beta_1} = \frac{(A_s - A_s')f_y}{0.85 \beta_1 f_c' b} \quad (7)$$

NOTE: if $f_c' \leq 4{,}000$ psi, then $\beta_1 = 0.85$, otherwise see next slide

Doubly Reinforced Beam Analysis

- Condition I: Tension and Compression Steel Both at Yield Stress (cont'd)
  – The value of β_1 may determined by

$$\beta_1 = \begin{cases} 0.85 & \text{for } f_c' \leq 4{,}000 \text{ psi} \\ 1.05 - 5\times10^{-5} f_c' & \text{for } 4{,}000 \text{ psi} < f_c' \leq 8{,}000 \text{ psi} \\ 0.65 & \text{for } f_c' > 8{,}000 \text{ psi} \end{cases} \quad (8)$$

Doubly Reinforced Beam Analysis

- ACI Code Ductility Requirements
  - The ACI Code limitation on ρ applies to doubly reinforced beams as well as to singly reinforced beams.
  - Steel ratio ρ can be determined from

$$\rho = \frac{A_{s1}}{bd} \qquad (9)$$

  - This value of ρ shall not exceed $0.75\rho_b$ as provided in Table 1 (Table A-5, Textbook)

Doubly Reinforced Beam Analysis

(Table A-5 Text)

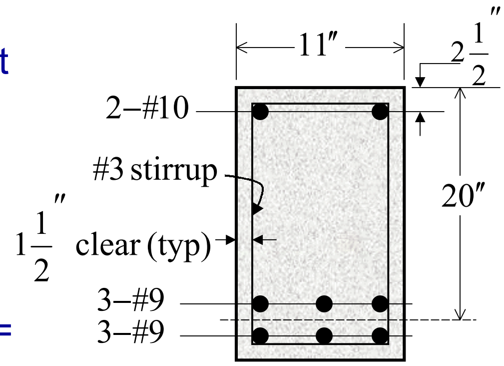
Table 1.
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\,\rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
F_y = 40,000 psi				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = 50,000 psi				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = 60,000 psi				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = 75,000 psi				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Doubly Reinforced Beam Analysis

- Example 1

Compute the practical moment capacity ϕM_n for the beam having a cross section as shown in the figure. Use f'_c= 3,000 psi and f_y = 60,000 psi.



Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

Determine the values for A'_s and A_s:

From Table 2 (A-2, Textbook),

$$A'_s = \text{area of } 2\ \#10 = 2.54\ \text{in}^2$$

$$A_s = \text{area of } 6\ \#9 = 6.0\ \text{in}^2$$

We assume that all the steel yields:

$$f'_s = f_y \quad \text{and} \quad f_s = f_y$$

Therefore,

$$A_{s2} = A'_s = 2.54\ \text{in}^2$$

$$A_{s1} = A_s - A_{s2} = 6.0 - 2.54 = 3.46\ \text{in}^2$$

Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

Table 2. Areas of Multiple of Reinforcing Bars (in^2)

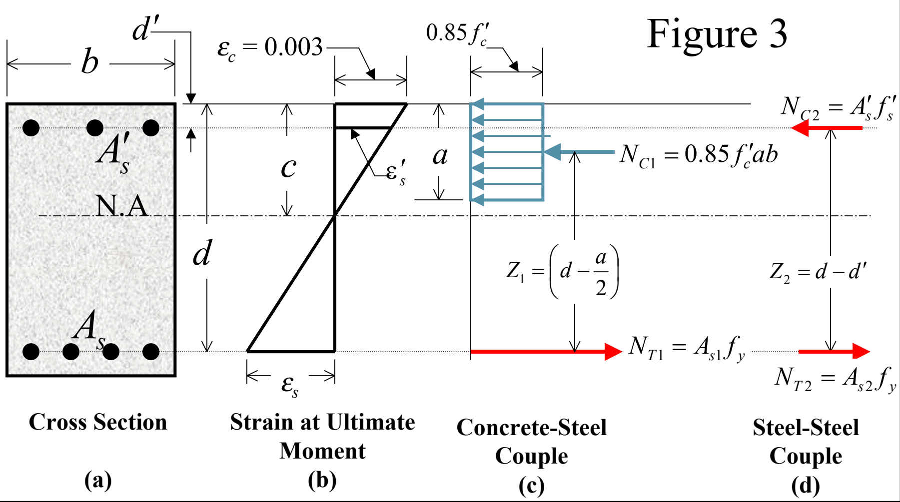
Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Table A-2 Textbook

Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

Figure 3



Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

From Eq. 6 (concrete-steel couple):

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{A_{s1} f_y}{0.85 f_c' b} = \frac{3.46(60)}{0.85(3)(11)} = 7.40 \text{ in.}$$

From Eq. 7 (note that f_c'< 4,000 psi, thus β_1 = 0.85):

$$c = \frac{a}{\beta_1} = \frac{7.40}{0.85} = 8.71 \text{ in.}$$

Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

Check assumptions for yielding of both the compressive and tensile steels:

From Fig. 3b:

$$\frac{\varepsilon_s'}{c - d'} = \frac{0.003}{c} \Rightarrow \varepsilon_s' = \frac{0.003(c - d')}{c} = \frac{0.003(8.71 - 2.5)}{8.71} = 0.00214$$

Also

$$\frac{\varepsilon_s}{d - c} = \frac{0.003}{c} \Rightarrow \varepsilon_s = \frac{0.003(d - c)}{c} = \frac{0.003(20 - 8.71)}{8,71} = 0.00389$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60,000}{29 \times 10^6} = 0.00207 > [\varepsilon_s' = 0.00214 \text{ and } \varepsilon_s = 0.00389] \quad \text{OK}$$

Therefore, the assumptions are valid

Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

From Eq. 8:

$$M_n = M_{n1} + M_{n2}$$
$$= \left(A_s - A'_s\right) f_y \left[d - \frac{a}{2}\right] + A'_s f_y \left(d - d'\right)$$
$$= 3.46(60)\left[20 - \frac{7.4}{2}\right] + 2.54(60)(20 - 2.5) = 6{,}050.9 \text{ in - k}$$

$$M_n = \frac{6{,}050.9}{12} \text{ ft - kips} = 504.2 \text{ ft - kips}$$

Doubly Reinforced Beam Analysis

- Example 1 (cont'd)

The practical moment capacity is evaluated as follows:

$$\phi M_u = 0.9(504.2) = 454 \text{ ft - kips}$$

Check ductility according to ACI Code:

$$\rho = \frac{A_{s1}}{bd} = \frac{3.46}{11(20)} = 0.0157$$

From Table 1

$$\text{Since } (\rho = 0.0157) < \left(\rho_{max} = 0.0161\right) \quad \text{OK}$$

Doubly Reinforced Beam Analysis

- Condition II: Compression Steel Below Yield Stress
  - The preceding equations are valid only if the compression steel has yielded when the beam reaches its ultimate strength.
  - In many cases, however, such as for wide, shallow beams reinforced with higher-strength steels, the yielding of compression steel may not occur when the beam reaches its ultimate capacity.

Doubly Reinforced Beam Analysis

- Condition II: Compression Steel Below Yield Stress
  - It is therefore necessary to to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.
  - The development of these equations will be based on

$$\varepsilon_s' < \varepsilon_y \qquad (10)$$

Doubly Reinforced Beam Analysis

- **Condition II:** Compression Steel Below Yield Stress
  - Development of the Equations for Condition II
    - Referring to Fig. 3,

$$N_T = N_{C1} + N_{C2} \qquad (11)$$

$$A_s f_y = (0.85 f'_c) ba + f'_s A'_s$$

- But

$$a = \beta_1 c \qquad (12)$$

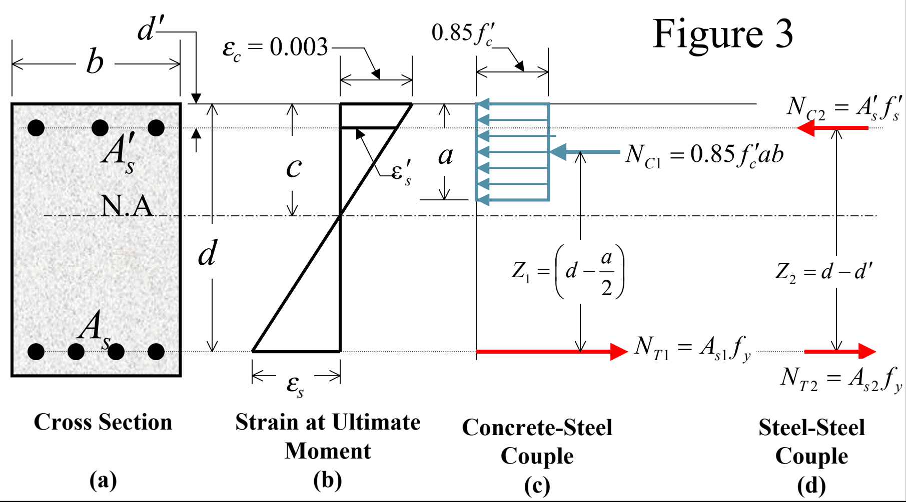
- and

$$f'_s = \varepsilon'_s E_s = \left[\frac{0.003(c - d')}{c}\right] E_s \qquad (13)$$

Doubly Reinforced Beam Analysis

- **Condition II:** Compression Steel Below Yield Stress

Figure 3



Doubly Reinforced Beam Analysis

- Condition II: Compression Steel Below Yield Stress
  - Substituting Eqs 12 and 13 into Eq. 11, yields

$$A_s f_y = \left(0.85 f'_c\right) b \beta_1 c + \left[\frac{0.003\left(c - d'\right)}{c}\right] E_s A'_s \qquad (14)$$

  - Multiplying by *c*, expanding, and rearranging, yield

$$\left(0.85 f'_c b \beta\right)_1 c^2 + \left(0.003 E_s A'_s - A_s f_y\right) c - 0.003 d' E_s A'_s = 0 \qquad (15)$$

  - If E_s is taken as 29×10^3 ksi, Eq. 15 will take the following form:

Doubly Reinforced Beam Analysis

- Condition II: Compression Steel Below Yield Stress

  The following quadratic equation can be used to find *c* when $\varepsilon'_s < \varepsilon_y$:

$$\underbrace{\left(0.85 f'_c b \beta_1\right)}_{a} c^2 + \underbrace{\left(87 A'_s - A_s f_y\right)}_{b} c - \underbrace{87 d' A'_s}_{c} = 0 \qquad (16)$$

Analogous to:

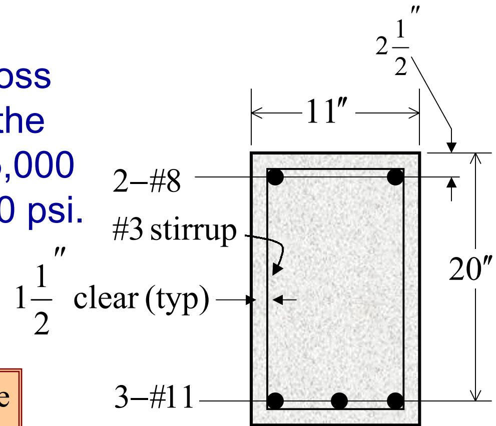
$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Doubly Reinforced Beam Analysis

■ Example 2

Compute the practical moment ϕM_n for a beam having a cross section shown in the figure. Use f'_c = 5,000 psi and f_y = 60,000 psi.



See Textbook for complete solution for this example.