



UNIVERSITY OF MARYLAND
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Reinforced Concrete Design

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CHAPTER

REINFORCED CONCRETE BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

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Part I – Concrete Design and Analysis

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3c






CHAPTER 3c. R/C BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

Slide No. 1

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Procedure for Analysis of T-Beams For Moments

1. Establish the effective flange width, b based on ACI criteria.
2. Check $A_{s,min}$. Use Table 1 (**Table A-5, Textbook**).
3. Check the ACI Code ductility requirements using the proper expression for $A_{s,max}$ from Eq. 1 or Table 2 (**Table 3-1, Textbook**). $A_{s,max}$ must be larger than actual A_s .





Procedure for Analysis of T-Beams For Moments

Table A-5 Textbook

Table 1
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y} \right]$	$\rho_{\max} = 0.75 \rho_b$	Recommended Design Values	
			ρ_b	\bar{k} (ksi)
$F_y = 40,000$ psi				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
$F_y = 50,000$ psi				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
$F_y = 60,000$ psi				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
$F_y = 75,000$ psi				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657



Procedure for Analysis of T-Beams For Moments

Table 2. Expressions for $A_{s,\max}$ (T-Beams)

f'_c (psi)	f_y (psi)	$A_{s,\max}$ (in ²)
3,000	40,000	$0.0478h_f \left\{ b + b_w \left[\frac{0.582}{h_f} d - 1 \right] \right\}$
	60,000	$0.0319h_f \left\{ b + b_w \left[\frac{0.503}{h_f} d - 1 \right] \right\}$
4,000	40,000	$0.0638h_f \left\{ b + b_w \left[\frac{0.582}{h_f} d - 1 \right] \right\}$
	60,000	$0.0425h_f \left\{ b + b_w \left[\frac{0.503}{h_f} d - 1 \right] \right\}$



Procedure for Analysis of T-Beams For Moments

- The check the ductility of a T-beam, the following equation can be used for various combinations of f'_c and f_y

$$A_{s,\max} = \frac{0.638}{f_y} f'_c h_f \left\{ b + b_w \left[\frac{\beta_b}{h_f} \left(\frac{87,000}{87,000 + f_y} d \right) - 1 \right] \right\} \quad (1)$$



Procedure for Analysis of T-Beams For Moments

4. Compute the total tension in the steel:

$$N_T = A_s f_y$$

5. Compute the magnitude of the compression that the flange its is capable of furnishing:

$$N_{Cf} = 0.85 f'_c b h_f$$

6. If $N_T > N_{Cf}$, the beam will behave as a **true T-beam**, and the remaining compression, which equals $N_T - N_{Cf}$



Procedure for Analysis of T-Beams For Moments

will be furnished by additional web area. If $N_T < N_{Cf}$, the beam will behave as a rectangular beam of width b .

Rectangular T-Beam

7. Compute the actual steel ratio in order to find \bar{k} :

$$\rho = \frac{A_s}{bd}$$



Procedure for Analysis of T-Beams For Moments

8. Consult the proper Table 3, (**Tables A-7 to A-11, Text**) and find the required \bar{k} for the ρ value from step 7.
9. Compute the practical moment capacity ϕM_n of the beam cross section:

$$\phi M_n = \phi b d^2 \bar{k}$$



Procedure for Analysis of T-Beams For Moments

Sample Values

Table 3.
Coefficient of Resistance

Table A-10 Textbook

ρ	\bar{k}
0.0010	0.0595
0.0011	0.0654
0.0012	0.0712
0.0013	0.0771
0.0014	0.0830
0.0015	0.0888
0.0016	0.0946
0.0017	0.1005
0.0018	0.1063
0.0019	0.1121
0.0020	0.1179
0.0021	0.1237



Procedure for Analysis of T-Beams For Moments

True T-Beam

- Determine the depth of the compressive stress block:

$$a = \frac{N_T - N_{Cf}}{0.85 f'_c b_w} + h_f$$

- (a) Locate the centroid of the total compressive area referenced to top of the flange using the relationship



Procedure for Analysis of T-Beams For Moments

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

From which

$$Z = d - \bar{y}$$

Compute the practical moment capacity ϕM_n of the beam:

$$\phi M_n = \phi N_C Z \quad \text{or} \quad \phi N_T Z$$



Procedure for Analysis of T-Beams For Moments

Or

(b) Calculate ϕM_n using a summation of internal couples contributed by the flange and the web:

$$\phi M_n = \phi \left\{ N_{cf} \left(d - \frac{h_f}{2} \right) + (N_T - N_{cf}) \left[d - h_f \left(\frac{a - h_f}{2} \right) \right] \right\}$$



Procedure for Analysis of T-Beams For Moments

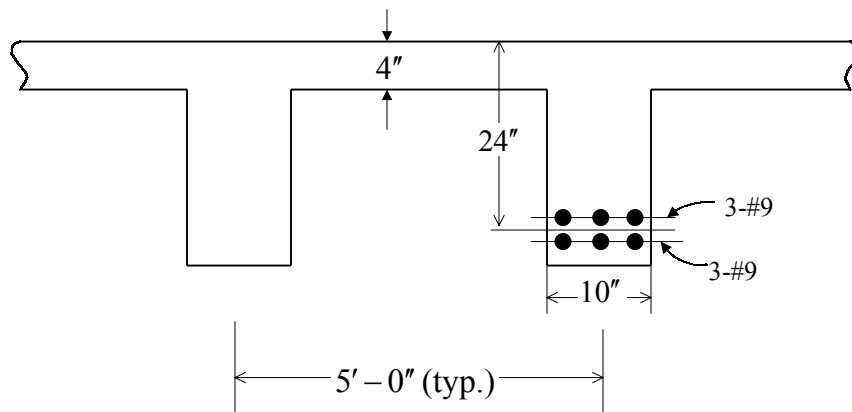
■ Example 1: T-Beam Analysis

Determine the practical moment capacity ϕM_n for the T-beam in the floor system shown. The beam span is 24 in. Use $f_y = 60,000$ psi and $f'_c = 3,000$ psi. Check the steel to ensure that it is within allowable limits according to the ACI Code.



Procedure for Analysis of T-Beams For Moments

■ Example 1: T-Beam Analysis (cont'd)





Procedure for Design of T-Beams

For Moments

1. Compute the design moment M_u .
2. Assume that the effective depth d is equal to $h - 3$ in.
3. Establish the effective flange width based on ACI criteria.
4. Compute the practical moment strength ϕM_{nf} assuming that the total effective flange is in compression:

$$\phi M_{nf} = \phi (0.85 f'_c) b h_f \left(d - \frac{h_f}{2} \right)$$



Procedure for Design of T-Beams

For Moments

5. If $\phi M_{nf} > M_u$ the beam will behave as **rectangular T-beam** of width b .
Otherwise, the beam will behave as a **true T-beam**.

Rectangular T-Beam

6. Design as a rectangular beam with b and d as known values. Compute the required \bar{k} :

$$\text{required } \bar{k} = \frac{M_u}{\phi b d^2}$$



Procedure for Design of T-Beams For Moments

- From the tables in Appendix A of textbook (see Table 3), determine the required ρ for the required \bar{k} of step 6.
- Compute the required A_s :
$$\text{required } A_s = \rho b d$$
- Select bars and check the beam width. Check the actual d and compare it with the assumed d . If the actual d is slightly in excess of the assumed d ,



Procedure for Design of T-Beams For Moments

the design will be slightly conservative. If the actual d is less than the assumed d , the design may be on the nonconservative side (depending on the steel provided) and should be more closely investigated for possible revision.

- Check $A_{s,\min}$. Use Table 1 (Table A-5, Textbook).



Procedure for Design of T-Beams For Moments

11. Check the ACI ductility requirement using the proper expression for $A_{s,max}$ from Table 2 (**Table 3-1 Text**) or Eq. 1. Note that $A_{s,max}$ must be larger than actual A_s .
12. Sketch the design.



Procedure for Design of T-Beams For Moments

True T-Beam

6. Using an estimated $d_f = h - 3$ in. and $Z_f = d_f - h_f/2$, determine the steel area A_s required for the flange couple:

$$\text{required } A_{sf} = \frac{\phi M_{nf}}{\phi f_y Z_f}$$

7. Design the web couple as a rectangular reinforced concrete beam



Procedure for Design of T-Beams

For Moments

having a total depth $h_w = h - h_f$ using an estimated $d_w = h_w - 3$ in. and a beam width of b_w . Design for an applied moment $M_u - \phi M_{nf}$. Determine required \bar{k} , required ρ , and required A_{sw} .

8. Total required $A_{sw} = A_{sf} + A_{sw}$.
9. Select the bars. Bars must fit into beam width b_w . Check d as in step 9



Procedure for Design of T-Beams

For Moments

of the rectangular T-beam design.

10. Check $A_{s,min}$. Use Table 1 (Table A-5, Textbook).
11. Check $A_{s,max}$. Use Table 2 (Table 3-1, Textbook) or Eq. 1.
12. Sketch the design.



Procedure for Design of T-Beams For Moments

■ Example: T-Beam Design

Design a T-beam having a cross section shown in the figure. Assume that the effective flange width given is acceptable. The T-beam will carry a total design moment M_u of 340 ft-kips. Use $f'_c = 3,000$ psi and $f_y = 60,000$ psi. Use 1.5-in. cover and No. 3 stirrups.



Procedure for Design of T-Beams For Moments

■ Example: T-Beam Design (cont'd)

