CHAPTER

2b

Prentice Hall

Reinforced Concrete Design — Fifth Edition

RECTANGULAR R/C CONCRETE BEAMS: TENSION STEEL ONLY

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Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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Flexural Strength of Rectangular Beams

- Ultimate Moment (Strength)
  - The ultimate moment for a reinforced concrete beam can be defined as the moment that exists just prior to the failure of the beam.
  - In order to evaluate this moment, we have to examine the strains, stresses, and forces that exist in the beam.
  - The beam of Fig. 1 has a width of b, an effective depth d, and is reinforced with a steel area A_s.

Flexural Strength of Rectangular Beams

■ Ultimate Strength

Flexural Strength ACI Approach

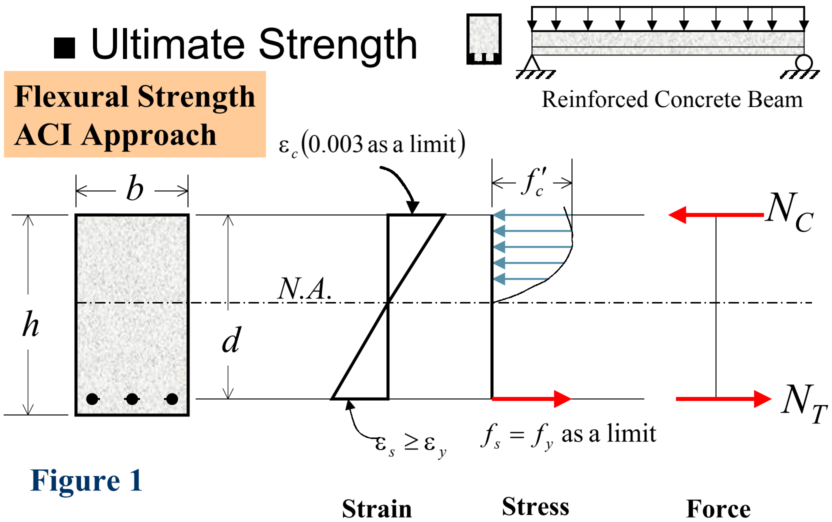


Figure 1

Flexural Strength of Rectangular Beams

■ Possible Values for Concrete Strains due to Loading (Modes of Failure)

1. Concrete compressive strain is less than 0.003 in./in. when the maximum tensile steel unit equal its yield stress f_y as a limit.
2. Maximum compressive concrete strain equals 0.003 in./in. and the tensile steel unit stress is less than its yield stress f_y.

Flexural Strength of Rectangular Beams

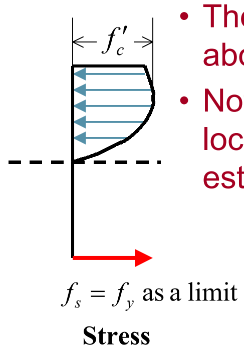
- Notes on Concrete Compressive Stresses
  - The ultimate compressive stress for concrete does not occur at the outer fiber.
  - The shape of the curve is not the same for different-strength concretes.
  - The shape of the curve will also depend on the size and dimensions of the beam.
  - The ultimate compressive stress of concrete develops at some intermediate level near, but not at, the extreme outer fiber.

Flexural Strength of Rectangular Beams

- Nominal Moment Strength
  - The forces N_C and N_T, and the distance Z separated them constitute an internal resisting couple whose maximum value is termed ***nominal moment*** *strength* of the bending member.
  - As a limit, this nominal strength must be capable of resisting the actual design bending moment induced by the applied loads.

Flexural Strength of Rectangular Beams

- Nominal Moment Strength (cont'd)
  - The determination of the moment strength is complex because of
    - The shape of the compressive stress diagram above the neutral axis
    - Not only is N_C difficult to evaluate but also its location relative to the tensile steel is difficult to establish



Flexural Strength of Rectangular Beams

- How to Determine the Moment Strength of Reinforced Concrete Beam?
  - To determine the moment capacity, it is necessary only to know
    1. **The total resultant compressive force N_C in the concrete, and**
    2. **Its location from the outer compressive fiber, from which the distance Z may be established.**

Flexural Strength of Rectangular Beams

- How to Determine the Moment Strength of Reinforced Concrete Beam? (cont'd)
  - These two values may easily be established by replacing the unknown complex compressive stress distribution by a fictitious (equivalent) one of simple geometrical shape (e.g., rectangle).
  - Provided that the fictitious distribution results in the same total N_C applied at the same location as in the actual distribution when it is at the point of failure.

Flexural Strength of Rectangular Beams

- Mathematical Motivation
  - Consider the function

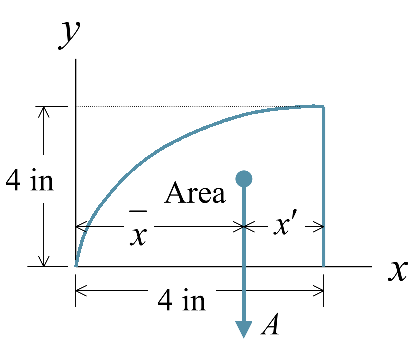
$$f(x) = y = 2\sqrt{x} \qquad (1)$$

  - Plot of this function is shown in Fig. 2 for x ranges from 0 to 4, and y from 0 to 4.
  - The area under the curve will be determined analytically.
  - Note that in real situation this area will be the equivalent,for example, to compressive force N_C for concrete per unit length.

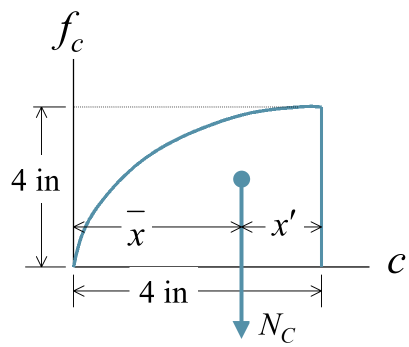
Flexural Strength of Rectangular Beams

- Mathematical Motivation (cont'd)

Area under the Curve



N_C per unit length



Flexural Strength of Rectangular Beams

- Mathematical Motivation (cont'd)

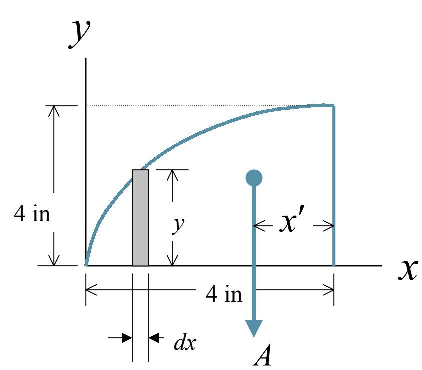
$$A = \int_0^4 y\,dx = \int_0^4 \left(2\sqrt{x}\right)dx = 2\int_0^4 x^{\frac{1}{2}}dx = 10.7 \text{ in}^2$$

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int \tilde{x}\,dA}{10.7}$$

$$\int \tilde{x}\,dA = \int_0^4 x(y\,dx)$$

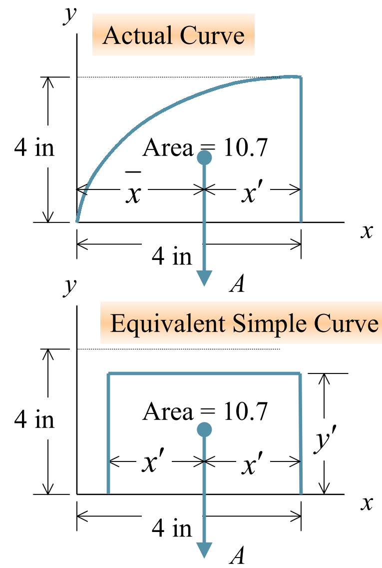
$$= \int_0^4 x\left(2\sqrt{x}\,dx\right) = 2\int_0^4 x^{\frac{3}{2}}dx = 25.6$$

$$\text{Therefore, } \bar{x} = \frac{25.6}{10.7} = 2.4 \text{ in.}$$



Flexural Strength of Rectangular Beams

- Mathematical Motivation (cont'd)
  - Objective
    - Our objective is to find a fictitious or equivalent curve results in the same total area A applied at the same location as the actual curve.
    - Find x' and y'



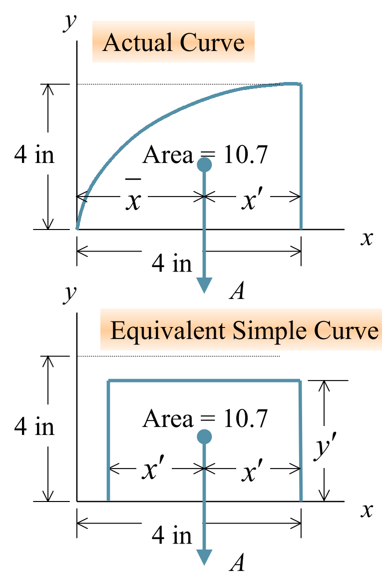
Flexural Strength of Rectangular Beams

- Mathematical Motivation (cont'd)
  - Calculations of x' and y'

$$x' = 4 - \bar{x} = 4 - 2.4 = 1.6 \text{ in.}$$

$$\text{Area} = 2x'y'$$

$$y' = \frac{\text{Area}}{2x'} = \frac{10.7}{2(1.6)} = 3.34 \text{ in.}$$



Flexural Strength of Rectangular Beams

- Mathematical Motivation (cont'd)
  - If we are dealing with a concrete compressive stress distribution and we let $x' = a/2$,then

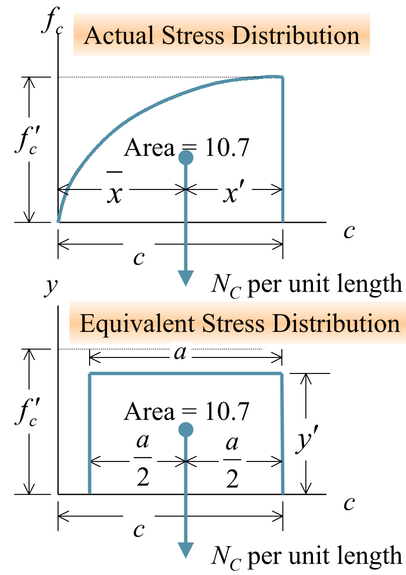
$$y' = 0.84 f_c'$$

and

$$a = 2x' = \beta_1 c = 2(1.6) = 3.2 \text{ in.}$$

Then,

$$\beta_1 = \frac{3.2}{4} = 0.80$$



Equivalent Stress Distribution

- As we saw in our previous mathematical example, any complicated function can be replaced with an equivalent or fictitious one to make the calculations simple and will give the same results.
- For purposes of simplification and practical application, a fictitious but equivalent rectangular concrete stress distribution was proposed.

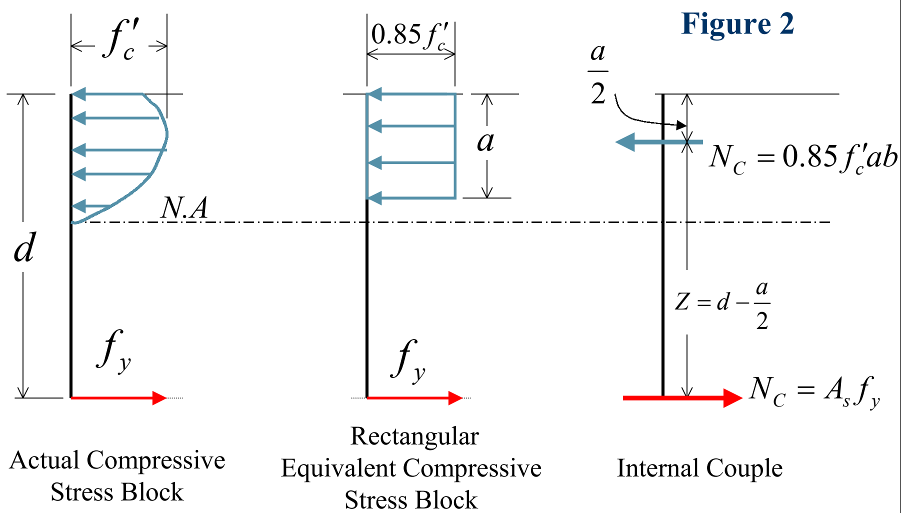
Equivalent Stress Distribution

- This rectangular stress distribution was proposed by Whiney (1942) and subsequently adopted by the ACI Code
- The ACI code also stipulates that other compressive stress distribution shapes may be used provided that they are in agreement with test results.
- Because of its simplicity, however, the rectangular shape has become the more widely stress distribution (Fig. 2).

Equivalent Stress Distribution

- Whitney's Rectangular Stress Distribution

Figure 2



Equivalent Stress Distribution

- Whitney's Rectangular Stress Distribution
  - According to Fig. 2, the average stress distribution is taken as

$$\text{Average Stress} = 0.85 f'_c$$

  - It is assumed to act over the upper area on the beam cross section defined by the width b and a depth a as shown in Fig. 3.

Equivalent Stress Distribution

- Whitney's Rectangular Stress Distribution

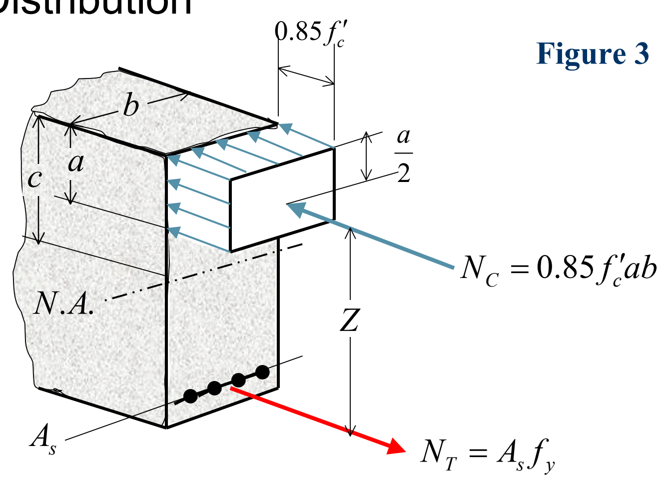


Figure 3

Equivalent Stress Distribution

- Whitney's Rectangular Stress Distribution
  - The magnitude of a may determined by

$$a = \beta_1 c \qquad (2)$$

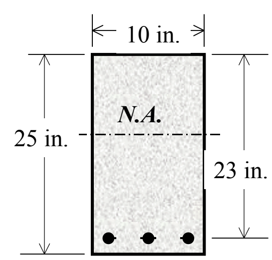
Where
C = distance from the outer fiber to the neutral axis
β_1 = a factor dependent on concrete strength, and is given by

$$\beta_1 = \begin{cases} 0.85 & \text{for } f'_c \leq 4{,}000 \text{ psi} \\ 1.05 - 5\times10^{-5} f'_c & \text{for } 4{,}000 \text{ psi} < f'_c \leq 8{,}000 \text{ psi} \\ 0.65 & \text{for } f'_c > 8{,}000 \text{ psi} \end{cases} \qquad (3)$$

Equivalent Stress Distribution

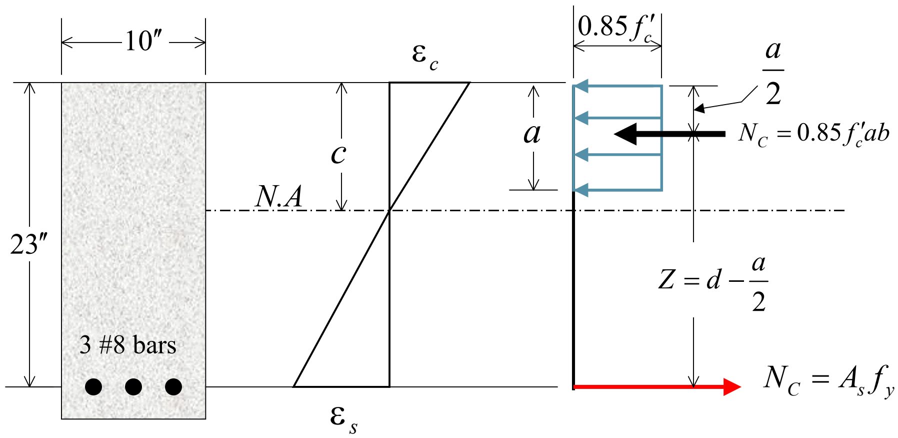
- Example 1

Determine the nominal moment M_n for a beam of cross section shown, where = 4,000 psi. Assume A615 grade 60 steel that has a yield strength of 60 ksi and a modulus of elasticity = 29×10^6 psi.



Equivalent Stress Distribution

- Example 1 (cont'd)



Equivalent Stress Distribution

- Example 1 (cont'd)

Area for No. 8 bar = 0.79 in^2 (see Table 1)

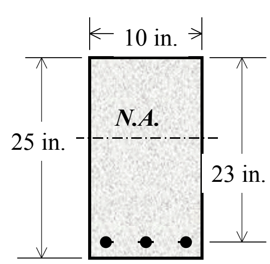
Therefore, $A_s = 3(0.79) = 2.37\text{ in}^2$ (Also see Table A-2 Text)

Assume that f_y for steel exists subject later check.

$$N_C = N_S$$

$$0.85 f'_c ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.37(60)}{0.85(4)(10)} = 4.18\text{ in.}$$



Equivalent Stress Distribution

Table 1. ASTM Standard - English Reinforcing Bars

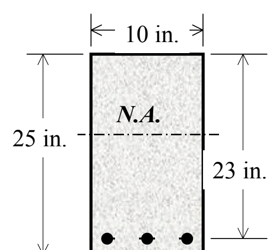
Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Equivalent Stress Distribution

■ Example 1 (cont'd)

Calculation of M_n



$$M_n = N_C\left(d - \frac{a}{2}\right) = N_T\left(d - \frac{a}{2}\right)$$

$$M_n = 0.85 f'_c ab\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

Based on steel:

$$M_n = 2.37(60)\left(23 - \frac{4.18}{2}\right) = 2{,}973.4 \text{ in.-kips}$$

$$= \frac{2{,}973.4}{12} = \boxed{247.8 \text{ ft-kips}}$$

Equivalent Stress Distribution

- Example 1 (cont'd)

  Check if the steel reaches its yield point before the concrete reaches its ultimate strain of 0.003:

  - Referring to the next figure (Fig. 4), the neutral axis can be located as follows:

Using Eqs. 2 and 3:

$$\beta_1 = 0.85$$

$$a = \beta_1 c$$

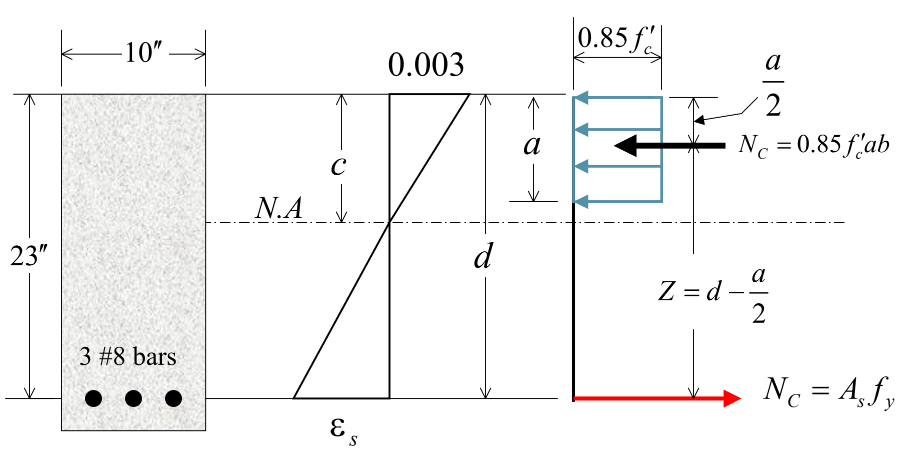
Therefore,

$$c = \frac{a}{\beta_1} = \frac{4.18}{0.85} = 4.92 \text{ in.}$$

Equivalent Stress Distribution

- Example 1 (cont'd)

Figure 4



Equivalent Stress Distribution

- Example 1 (cont'd)

By similar triangles in the strain diagram, the strain in steel when the concrete strain is 0.003 can be found as follows:

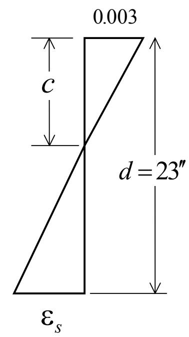
$$\frac{0.003}{c} = \frac{\varepsilon_s}{d-c}$$

$$\varepsilon_s = 0.003\frac{d-c}{c} = 0.003\frac{23-4.92}{4.92} = 0.011 \text{ in./in.}$$

The strain at which the steel yields is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29\times10^6} = 0.00207 \text{ in./in.}$$

Since $\varepsilon_s (= 0.011) > \varepsilon_y (= 0.00207)$ **OK**



Balanced, Overreinforced, and Underreinforced Beams

- Strain Distribution

Figure 5

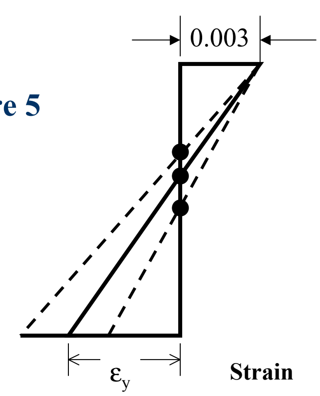
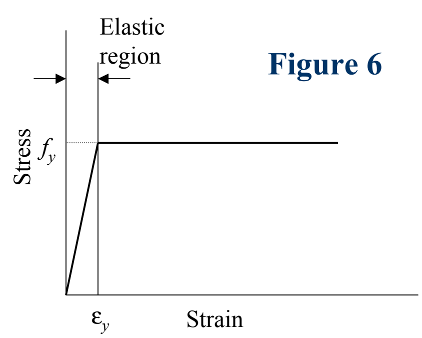


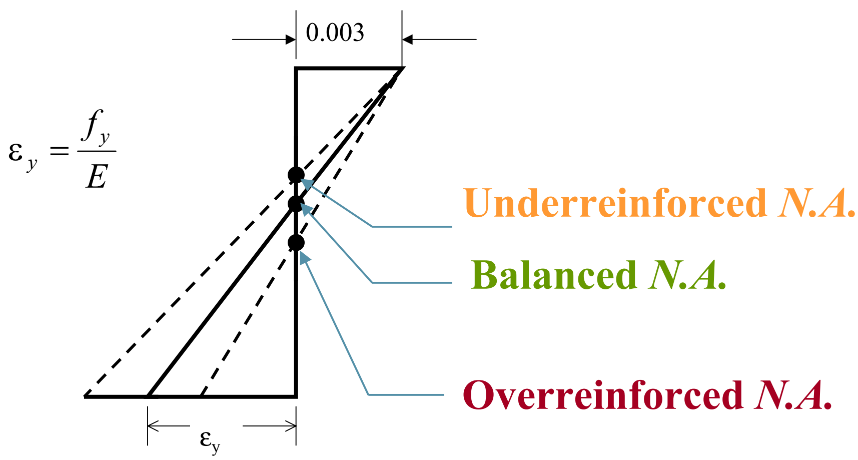
Figure 6



Idealized Stress-Strain Curve

Balanced, Overreinforced, and Underreinforced Beams

- Strain Distribution



Balanced, Overreinforced, and Underreinforced Beams

- Balanced Condition:

  $\varepsilon_s = \varepsilon_y$ and $\varepsilon_c = 0.003$

- Overreinforced Beam

  $\varepsilon_s < \varepsilon_y$, and $\varepsilon_c = 0.003$. The beam will have more steel than required to create the balanced condition. This is not preferable since will cause the concrete to crush suddenly before that steel reaches its yield point.

- Underreinforced Beam

  $\varepsilon_s > \varepsilon_y$, and $\varepsilon_c = 0.003$. The beam will have less steel than required to create the balanced condition. This is preferable and is ensured by the ACI Specifications.

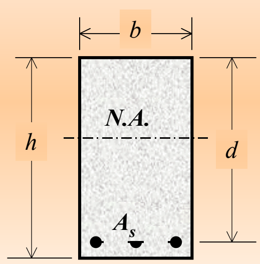
Reinforcement Ratio Limitations and Guidelines

- Although failure due yielding of the steel is gradual with adequate warning of collapse, failure due to crushing of the concrete is sudden and without warning.
- The first type (Underreinforced beam) is preferred and ensured by the specifications of the ACI.
- The ACI code stipulates that

$$A_s \leq 0.75 A_{sb} \quad (4)$$

Reinforcement Ratio Limitations and Guidelines

- Steel Ratio
  - The steel ratio (sometimes called reinforcement ratio) is given by



$$\rho = \frac{A_s}{bd} \quad (5)$$

ACI stipulates that

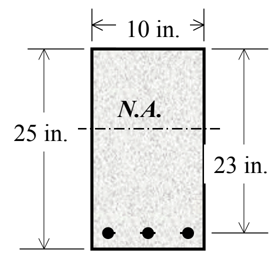
$$\rho_{\max} = 0.75\rho_b \quad (6)$$

$$\text{or} \quad A_{s_{\max}} = 0.75 A_{sb}$$

Reinforcement Ratio Limitations and Guidelines

- Example 2

Determine the amount of steel required to create a balanced condition for the beam shown, where = 4,000 psi. Assume A615 grade 60 steel that has a yield strength of 60 ksi and a modulus of elasticity = 29 × 10^6 psi. Also check the code requirement for ductile-type beam.



Reinforcement Ratio Limitations and Guidelines

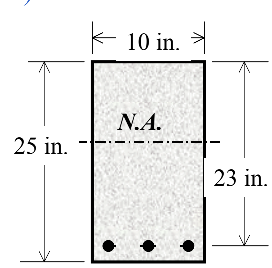
- Example 2 (cont'd)

Area for No. 8 bar = 0.79 in^2 (see Table 1)

Therefore, $A_s = 3(0.79) = 2.37 \text{ in}^2$

The strain at which the steel yields is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29\times 10^6} = 0.00207 \text{ in./in.}$$



In reference to the strain diagram of Fig. 7, and from similar triangles,

$$\frac{c_b}{0.003} = \frac{d - c_b}{0.00207}$$

Reinforcement Ratio Limitations and Guidelines

Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in²	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
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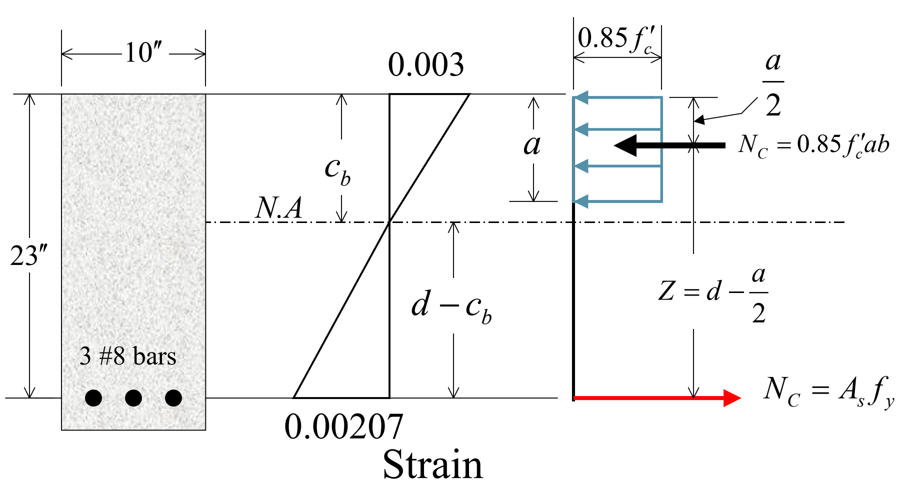
Reinforcement Ratio Limitations and Guidelines

- Example 2 (cont'd)

Figure 7



Reinforcement Ratio Limitations and Guidelines

- Example 2 (cont'd)

$$\frac{c_b}{0.003} = \frac{23 - c_b}{0.00207}$$

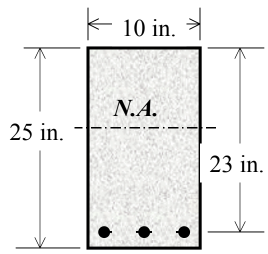
From which,

$$c_b = 13.6 \text{ in.}$$

Using Eqs. 2 and 3:

$$\beta_1 = 0.85 \text{ bbcause } f_c' = 4{,}000 \text{ psi}$$

$$a = \beta_1 c = 0.85(13.6) = 11.6 \text{ in.}$$



Reinforcement Ratio Limitations and Guidelines

- Example 2 (cont'd)

$$N_{Cb} = 0.85 f_c' a_b b = 0.85(4)(11.6)(10) = 394.4 \text{ kips}$$

$$N_{Cb} = N_{Tb} = A_{sb} f_y$$

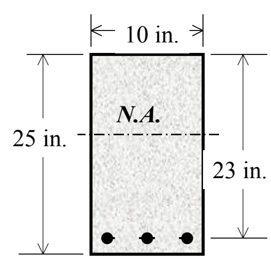
Therefore,

$$A_{sb} = \frac{N_{Cb}}{f_y} = \frac{394.4}{60} = 6.57 \text{ in}^2$$

Hence, required steel for balanced condition = 6.57 in^2

From Eq. 6,

$$A_{s_{\max}} = 0.75 A_{sb} = 0.74(6.57) = 4.93 \text{ in}^2 > A_s = 2.37 \text{ in}^2 \text{ OK}$$



Reinforcement Ratio Limitations and Guidelines

- Steel Ratio Formula for Balanced Beam

  Instead of using laborious techniques for determining the balanced steel of beam, the following formula can be used to determine the steel ratio ρ_b at the balance condition:

$$\rho_b = \frac{0.85 f_c' \beta_1}{f_y}\left(\frac{87{,}000}{f_y + 87{,}000}\right) \qquad (7)$$

where
f_c' = compressive strength of concrete (psi)
f_y = yield strength of steel (psi)
β_1 = factor that depends on f_c' as given by Eq. 3

Reinforcement Ratio Limitations and Guidelines

- Lower Limit for Steel Reinforcement
  - The ACI Code establishes a lower limit on the amount of tension reinforcement. The code states that where tensile reinforcement is required , the steel area A_s shall not be less than that given by

$$A_{s,\min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad (8)$$

Note that for rectangular beam $b_w = b$

Reinforcement Ratio Limitations and Guidelines

(Table A-5 Text)

Table 1. Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\ \rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
F_y = 40,000 psi				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = 50,000 psi				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = 60,000 psi				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = 75,000 psi				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657