Reinforced Concrete Design Fifth Edition

CHAPTER 2a

RECTANGULAR R/C CONCRETE BEAMS: TENSION STEEL ONLY

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Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

Department of Civil and Environmental Engineering

University of Maryland, College Park

Prentice Hall

Introduction

- Bending moment produces bending strains on a beam, and consequently compressive and tensile stresses.
- Under positive moment (as normally the case), compressive stresses are produced in the top of the beam and tensile stresses are produced in the bottom.
- Bending members must resist both compressive and tensile stresses.

Introduction

- Stresses in Beam

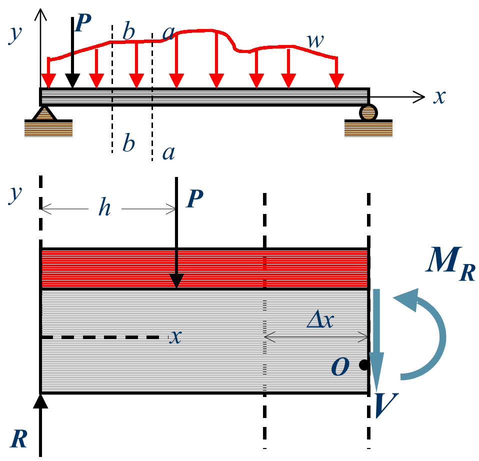
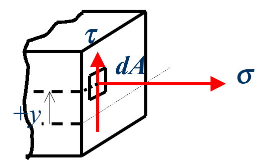


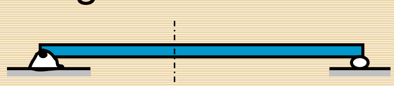
Figure 1

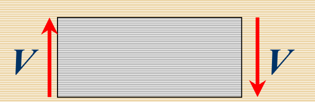


Introduction

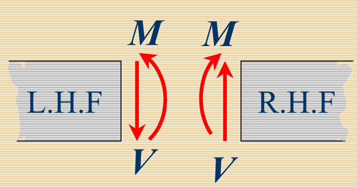
- Sign Convention

Figure 2

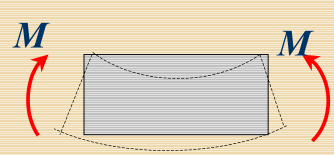




(b) Positive Shear (clockwise)



(a) Positive Shear & Moment



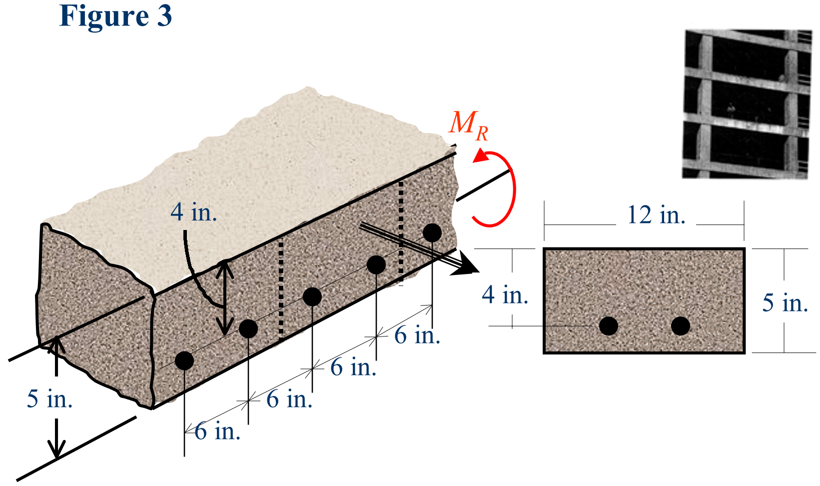
(c) Positive Moment
(concave upward)

Introduction

- Concrete Flexural Members
  - Types:
    - Beam
    - Wall
    - Slab
    - Etc.
  - These concrete members must resist both tensile and compressive stresses.
  - Because concrete is weak in tension, embedded steel bars are placed in the tension zone.

Introduction

Figure 3



Methods of Analysis and Design

- Elastic Design
  - Elastic design is considered valid for the homogeneous plain concrete beam as long as the tensile stress does not exceed the modulus of rupture f_r.
  - Elastic design can also be applied to a reinforced concrete beam using the working stress design (WSD) approach.

Methods of Analysis and Design

- WSD Assumptions
  1. A plain section before bending remains plane after bending.
  2. Stress is proportional to strain (Hooke's Law).
  3. Tensile stress for concrete is considered zero and reinforcing steel carries all the tension.
  4. The bond between the concrete and steel is perfect, so no slip occurs.

Methods of Analysis and Design

- Strength Design Method
  - This method is the modern approach for the analysis and design of reinforced concrete.
  - The assumption are similar to those outlined for the WSD with one exception:
    - Compressive concrete stress is approximately proportional to strain up to moderate loads. As the load increases, the approximate proportionality ceases to exit, and the stress diagram takes a shape similar to the concrete stress-strain curve of the following figure.

Methods of Analysis and Design

- Concrete Compressive Strength

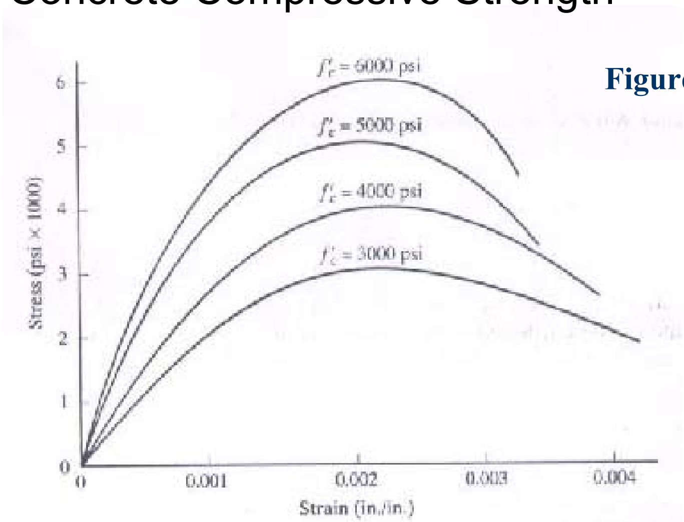


Figure 1

Methods of Analysis and Design

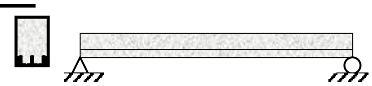
■ Comparison between the Two Methods

WSD or ASD	USD
■ Working (service) loads are used and a member is designed based on an allowable compressive bending stress, normally 0.45 f'_c ■Compressive stress pattern is assumed to vary linearly from zero at the neutral axis. ■Formula: $\frac{R_n}{\text{FS}} \geq \sum_{i=1}^{m} L_i$ ASD	■Service loads are amplified using partial safety factors. ■A member is design so that its strength is reduced by a reduction safety factor. ■The strength at failure is commonly called the ultimate strength ■Formula: $\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_i$ LRFD

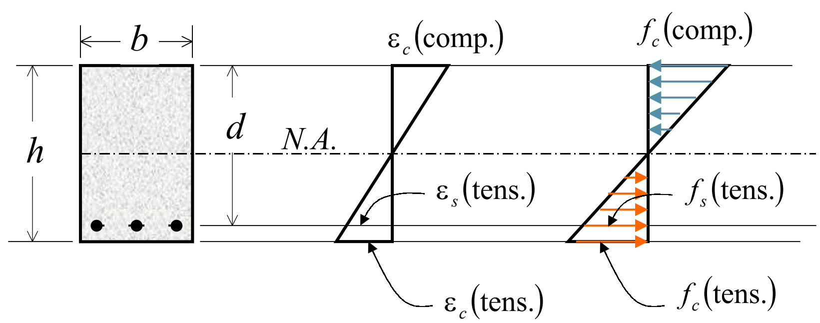
Behavior Under Load

(1) At very small loads:

Stresses Elastic and Section Uncracked



Reinforced Concrete Beam

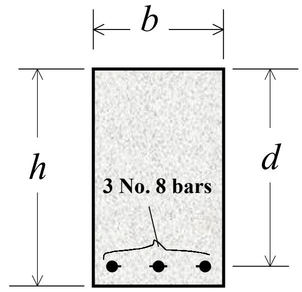


Stresses are below modulus of rupture.

Behavior Under Load

- Example 1

A rectangular beam, as shown in Fig. 1, has the dimensions *b* = 10 in.,*h* = 25 in., and *d* = 23 in., and is reinforced with three No. 8 bars. The concrete cylinder strength f_c' is 4000 psi, and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y is 60,000 psi. Determine the stresses caused by a bending moment *M* = 45 ft-kips. Assume the unit weight for concrete is 144 lb/ft^3.



Behavior Under Load

- Example 1 (cont'd)

$$\text{Area for No. 8 bar} = 0.79\ \text{in}^2 \ \text{(see Table 1)}$$

$$\text{Therefore, } A_s = 3(0.79) = 2.37\ \text{in}^2$$

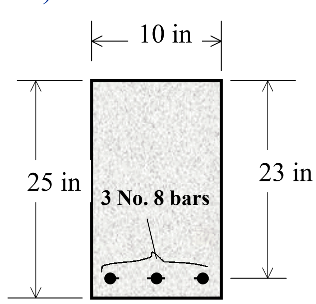
The modulus of elasticity for Concrete can be calculated from

$$E_c = w_c^{1.5} 33\sqrt{f_c'}$$

$$= (144)^{1.5}(33)\sqrt{4,000} = 3,606,514\ \text{psi}$$

Therefore,

$$n = \frac{E_s}{E_c} = \frac{29,000,000}{3,606,514} = 8.04 \approx 8$$



Behavior Under Load

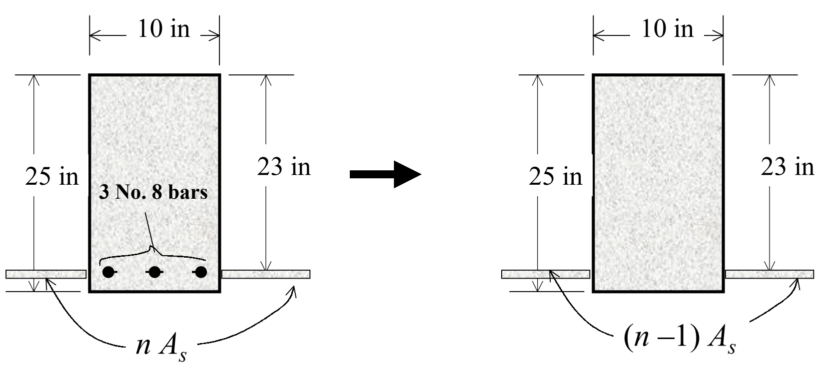
Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Behavior Under Load

- Example 1 (cont'd)
  - Transformed Section



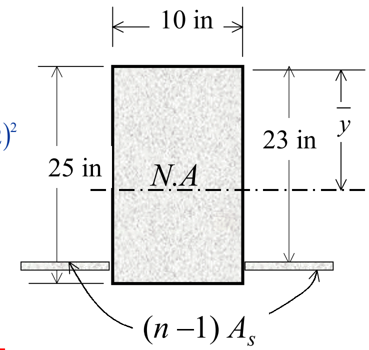
$A_c - A_s = n A_s - A_s = (n-1) A_s = (8\text{-}1)(2.37) = 16.59 \text{ in}^2$

Behavior Under Load

- Example 1 (cont'd)
  - Neutral axis location & moment of inertia

$$\bar{y} = \frac{(25)(10)\frac{25}{2} + 16.59(23)}{(25)(10) + 16.59} = 13.15 \text{ in}$$

$$I = \frac{10(13.15)^3}{3} + \frac{10(25-13.15)^3}{3} + 16.59(25-13.15-2)^2$$
$$= 14{,}736.1 \text{ in}^4$$



  - Stresses

$$f_c = \frac{Mc}{I} = \frac{(45\times12\times1000)(13.15)}{14{,}736.1} = \boxed{481.9 \text{ psi}}$$

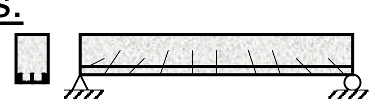
$$f_{ct} = \frac{Mc}{I} = \frac{(45\times12\times1000)(25-13.15)}{14{,}736.1} = \boxed{434.2 \text{ psi}} < 475 \text{ psi OK}$$

$$f_s = n\frac{Mc}{I} = 8\frac{(45\times12\times1000)(25-13.15-2)}{14{,}736.1} = \boxed{2{,}887.6 \text{ psi}}$$

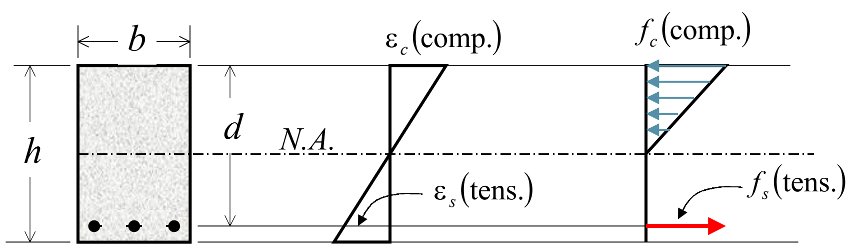
Behavior Under Load

(2) At moderate loads:

Stresses Elastic and Section Cracked



Reinforced Concrete Beam



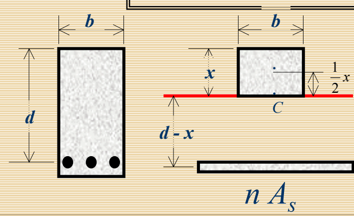
- Tensile stresses of concrete will be exceeded.
- Concrete will crack (hairline crack), and steel bars will resist tensile stresses.
- This will occur at approximately $0.5 f_c'$.

Behavior Under Load

- Reinforced Concrete Beam Formula

  The neutral axis for a concrete beam is found by solving the quadratic equation:

$$\boxed{\frac{1}{2}bx^2 + nA_s x - nA_s d = 0} \qquad (1)$$



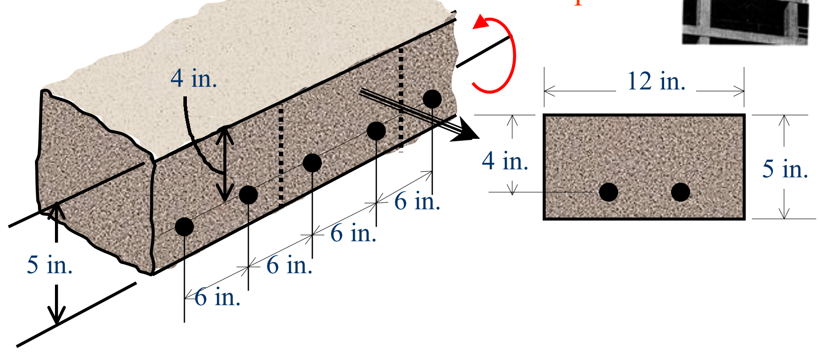
Behavior Under Load

- Example 2

  A concrete floor slab is reinforced by diameter steel rods placed 1 in. above the lower face of the slab and spaced 6 in. on centers. The modulus of elasticity is 3×10^6 psi for concrete used and 30×10^6 psi for steel. Knowing that a bending moment of 35 kip·in is applied to each 1-ft width of the slab, determine (a) the maximum stress in concrete and (b) the stress in the steel.

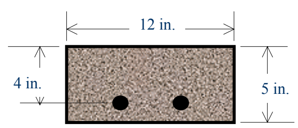
Behavior Under Load

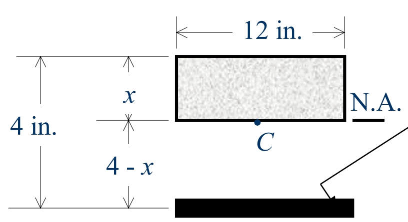
- Example 2 (cont'd)



Behavior Under Load

- Example 2 (cont'd)
  - Transformed Section
    - Consider a portion of the slab 12 in. wide, in which there are two $\frac{5}{8}$-in diameter rods having a total cross-sectional area





$$A_s = 2\frac{\left[\pi\left(\frac{5}{8}\right)^2\right]}{4} = 0.614\ \text{in}^2$$

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$nA_s = 10(0.614) = 6.14\ \text{in}^2$$

Behavior Under Load

- Example 2 (cont'd)
  - Neutral Axis
    - The neutral axis of the slab passes through the centroid of the transformed section. Using Eq. 1:

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x_1 = 1.575$ take

$x_2 = -2.599$

$$\frac{1}{2} b x^2 + nA_s x - nA_s d = 0$$

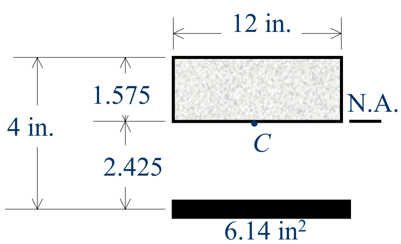
$$\frac{1}{2}(12)x^2 + 6.14x - 6.14(4) = 0$$

$$6x^2 + 6.14x - 24.56 = 0$$

$$\Longrightarrow x = 1.575 \text{ in}$$

Behavior Under Load

- Example 2 (cont'd)
  - Moment of Inertia
    - The centroidal moment of inertia of the transformed section is



$$I = \frac{12(1.575)^3}{3} + 6.14(2.425)^2 = 51.7 \text{ in}^4$$

Behavior Under Load

- Example 2 (cont'd)

Maximum stress in concrete:

$$\sigma_c = -\frac{My}{I} = -\frac{35(1.575)}{51.7} = \boxed{-1.066 \text{ ksi (C)}}$$

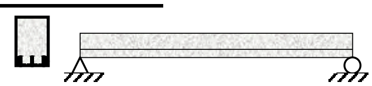
Stress in steel:

$$\sigma_s = -n\frac{My}{I} = -(10)\frac{35(-2.425)}{51.7} = \boxed{+16.42 \text{ ksi (T)}}$$

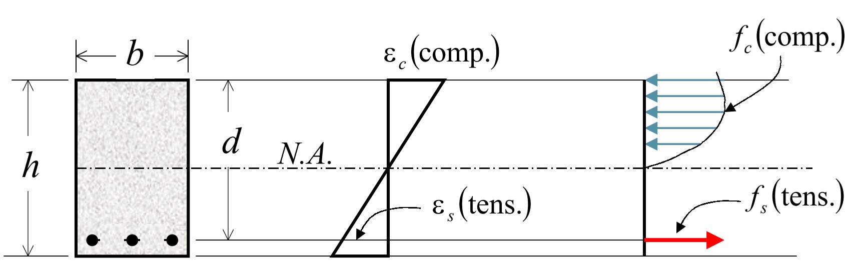
Behavior Under Load

(3) With further load increase:

Flexural Strength ACI Approach



Reinforced Concrete Beam



- Stress curve above N.A. will be similar to the stress-strain curve of Fig. 1.
- Concrete has cracked, and the process is irreversible.
- Steel bar has yielded and will not return to its original length.

Behavior Under Load

■ Concrete Compressive Strength

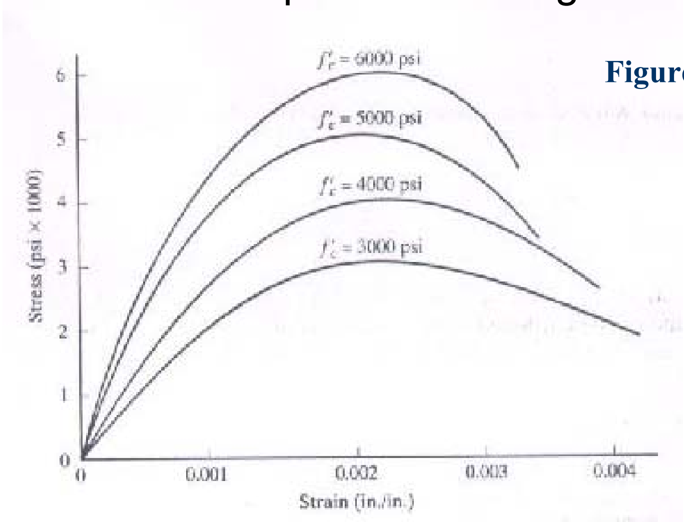


Figure 1

Strength Design Method Assumptions

■ Strength Design

- If the distribution of concrete compression stresses at or near ultimate load (Fig. 2), had a well- defined and invariable shape- parabolic – it would be possible to derive a completely rational theory of ultimate bending stress.
- This theory has been well established and incorporated in the ACI Manual.
- The basic assumptions follows.

Strength Design Method Assumptions

**Flexural Strength
ACI Approach**

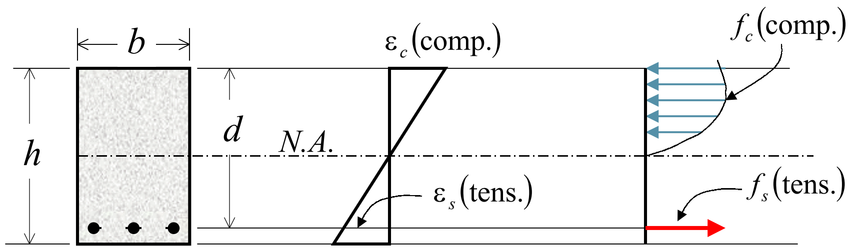


Figure 2

Strength Design Method Assumptions

- Basic Assumption:
  1. A plane section before bending remains plane after bending.
  2. Stresses and strain are approximately proportional up to moderate loads (concrete stress $\leq 0.5\ f_c'$). When the load is increased, the variation in the concrete stress is no longer linear.
  3. Tensile strength of concrete is neglected in the design of reinforced concrete beams.

Strength Design Method

Assumptions

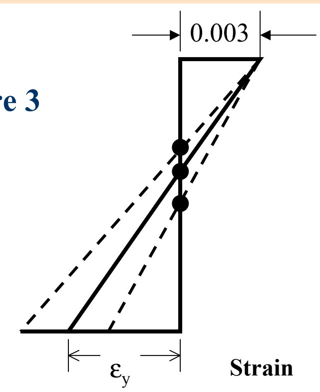
- Basic Assumption (cont'd):
  4. The maximum usable concrete compressive strain at the extreme fiber is assumed equal to 0.003 (Fig. 3)
  5. The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel. Also if the strain in the steel ε_s is less than the yield strain of the steel ε_y, the stress in the steel is $E_s\, \varepsilon_s$. If $\varepsilon_s \geq \varepsilon_y$, the stress in steel will be equal to f_y (Fig. 4)

Strength Design Method

Assumptions

- Basic Assumption (cont'd):
  6. The bond between the steel and concrete is perfect and no lip occurs.

Figure 3



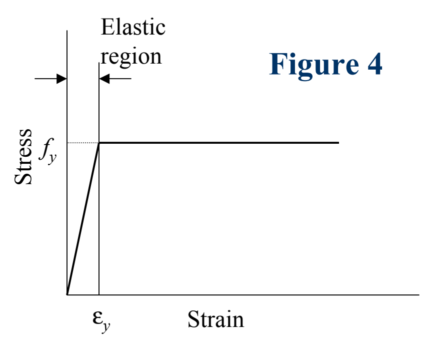


Figure 4

Idealized Stress-Strain Curve