CHAPTER

1b

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Prentice Hall **Reinforced Concrete Design** Fifth Edition

MATERIALS AND MECHANICS OF BENDING

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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Concrete in Tension

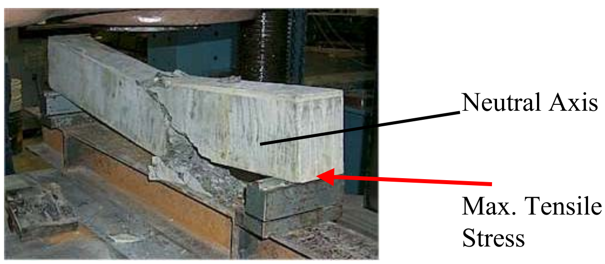
- Concrete tensile stresses occur as a result of shear, torsion, and other actions, and in most cases member behavior changes upon cracking.
- It is therefore important to be able to predict, with reasonable accuracy, the tensile strength of concrete.
- The tensile and compressive strengths of concrete are not proportional, and an increase in compressive strength is accompanied by smaller percentage increase in tensile strength.

Concrete in Tension

- The tensile strength of normal-weight concrete in flexure is about 10% to 15% of the compressive strength.
- There are considerable experimental difficulties in determining the true tensile strength of concrete.
- The true tensile strength of concrete is difficult to determine.

Concrete in Tension

- One common approach is to use the *modulus of rupture* f_r.
- The modulus of rupture is the maximum tensile bending stress in a plain concrete test beam at failure.



Concrete in Tension

- ACI Code Recommendation

  For normal-weight concrete, the ACI Code recommends that the modulus of rupture f_r be taken as

$$f_r = 7.5\sqrt{f'_c} \qquad (1)$$

  where f_r in psi.

Concrete in Tension

- Cracking Moment, M_{cr}
  - The moment that produces a tensile stress just equal to the modulus of rupture is called cracking moment M_{cr}.
- The Split-Cylinder Test
  - The split-cylinder test has also been used to determine the tensile strength of lightweight aggregate concrete.
  - It has been accepted as a good measure of the true tensile strength.

Concrete in Tension

- The Split-Cylinder Test (cont'd)
  - This test uses a standard 6-in.-diameter, 12 in.-long cylinder placed on its in a testing machine (see Fig. 1).
  - A compressive line load is applied uniformly along the length of the cylinder.
  - The compressive load produces a transverse tensile stress, and the cylinder will split in half along the diameter when it tensile strength is reached.

Concrete in Tension

- Schematic for Split-Cylinder Test

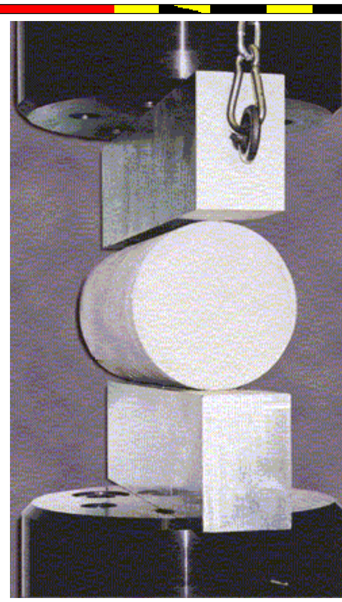


Figure 1

Concrete in Tension

- Splitting Tensile Strength, f_{ct}

  The tensile splitting stress can be calculated from the following formula:

$$f_{ct} = \frac{2P}{\pi LD} \qquad (2)$$

where

f_{cr} = splitting tensile strength of concrete (psi)
P = applied load at splitting (lb)
L = length of cylinder (in.)
D = diameter of cylinder (in.)

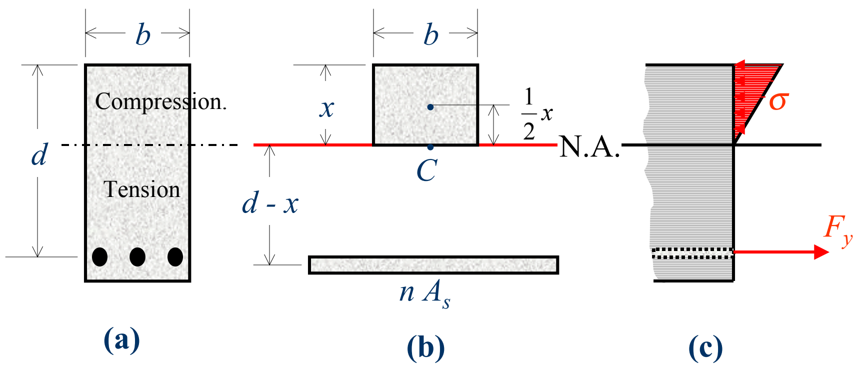
Reinforcing Steel

- Steel is a high-cost material compared with concrete.
- It follows that the two materials are best used in combination if the concrete is made to resist the compressive stresses and the steel the tensile stresses.
- Concrete cannot withstand very much tensile stress without cracking.

Reinforcing Steel

- Reinforced Concrete Beam

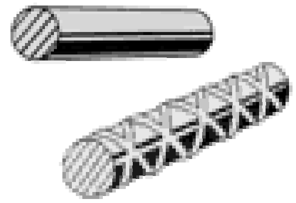
Figure 2



Reinforcing Steel

- It follows that tensile reinforcement must be embedded in the concrete to overcome the deficiency.
- Forms of Steel Reinforcement
  - Steel Reinforcing Bars
  - Welded wire fabric composed of steel wire.
  - Structural Steel Shapes
  - Steel Pipes.

Reinforcing Steel



Reinforcing Steel

- Reinforcing Bars (rebars)
  - The specifications for steel reinforcement published by the **American Society for Testing and Materials** (ASTM) are generally accepted for steel used in reinforced concrete construction in the United States and are identified in the ACI Code.

Reinforcing Steel

- Reinforcing Bars (rebars)
  - These bars are readily available in straight length of 60 ft.
  - The bars vary in designation from

No. 3 through No. 11

  - With additional bars:

No. 14 and No. 18

Reinforcing Steel

Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Reinforcing Steel

Table 2. ASTM Standard - Metric Reinforcing Bars

Bar Designation	Diameter mm	Area mm^2	Mass kg/m
#10 [#3]	9.5	71	0.560
#13 [#4]	12.7	129	0.994
#16 [#5]	15.9	199	1.552
#19 [#6]	19.1	284	2.235
#22 [#7]	22.2	387	3.042
#25 [#8]	25.4	510	3.973
#29 [#9]	28.7	645	5.060
#32 [#10]	32.3	819	6.404
#36 [#11]	35.8	1006	7.907
#43 [#14]	43.0	1452	11.38
#57 [#18]	57.3	2581	20.24

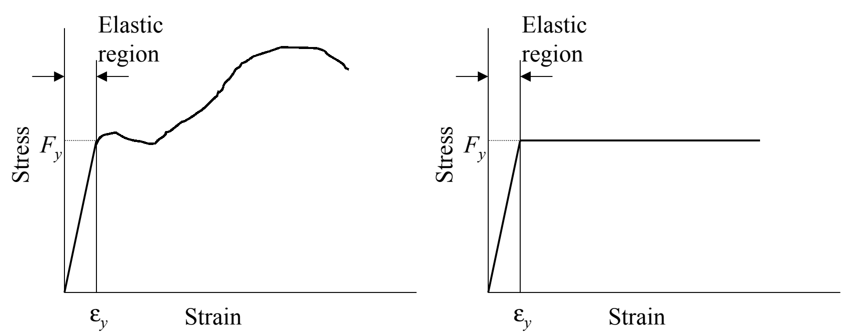
Note: English designations are in brackets

Reinforcing Steel

- Yield Stress for Steel
  - Probably the most useful property of reinforced concrete design calculations is the yield stress for steel, f_y.
  - A typical stress-strain diagram for reinforcing steel is shown in Fig. 3a.
  - An idealized stress-strain diagram for reinforcing steel is shown in Fig. 3b.

Reinforcing Steel

Figure 3



(a) As Determined by Tensile Test **(b) Idealized**

Reinforcing Steel

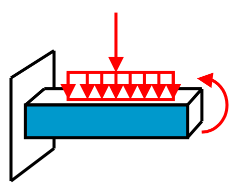
- Modulus of Elasticity for Steel
  - The modulus of elasticity for reinforcing steel varies over small range, and has been adopted by the ACI Code as

$$E = 29{,}000{,}000 \text{ psi} = 29{,}000 \text{ ksi}$$

Beams: Mechanics of Bending Review

- Introduction
  - The most common type of structural member is a beam.
  - In actual structures beams can be found in an infinite variety of
    - Sizes
    - Shapes, and
    - Orientations

Beams: Mechanics of Bending Review

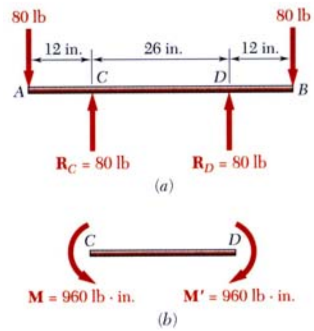


- Introduction

  Definition

A beam may be defined as a member whose length is relatively large in comparison with its thickness and depth, and which is loaded with transverse loads that produce significant bending effects as oppose to twisting or axial effects

Beams: Mechanics of Bending Review



Pure Bending: Prismatic members subjected to equal and opposite couples acting in the same longitudinal plane

Beams: Mechanics of Bending Review

- Flexural Normal Stress

For flexural loading and linearly elastic action, the neutral axis passes through the centroid of the cross section of the beam

Beams: Mechanics of Bending Review

- The elastic flexural formula for normal stress is given by

$$f_b = \frac{Mc}{I} \qquad (3)$$

where

f_b = calculated bending stress at outer fiber of the cross section

M = the applied moment

c = distance from the neutral axis to the outside tension or compression fiber of the beam

I = moment of inertia of the cross section about neutral axis

Beams: Mechanics of Bending Review

- By rearranging the flexure formula, the maximum moment that may be applied to the beam cross section, called the *resisting moment, M_R*, is given by

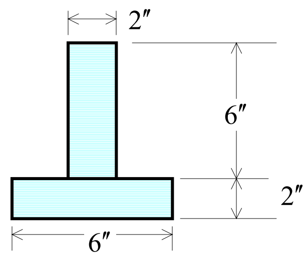
$$M_R = \frac{F_b I}{c} \qquad (4)$$

Where F_b = the allowable bending stress

Beams: Mechanics of Bending Review

- Example 1

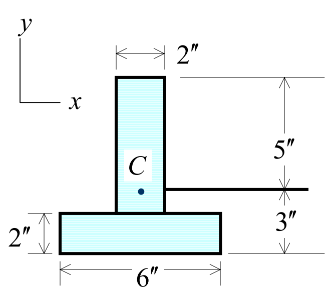
  Determine the maximum flexural stress produced by a resisting moment *M* of +5000 ft-lb if the beam has the cross section shown in the figure.



Beams: Mechanics of Bending Review

- Example 1 (cont'd)

  First, we need to locate the neutral axis from the bottom edge:



$$y_C = \frac{(1)(2\times 6) + (2+3)(2\times 6)}{2\times 6 + 2\times 6} = \frac{72}{24} = 3''$$

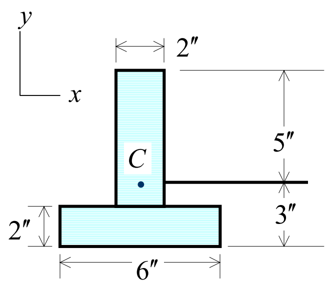
$$y_{\text{ten}} = 3'' \quad y_{\text{com}} = 6 + 2 - 3 = 5'' = y_{\max} = c$$

$$\text{Max. Stress} = f_b = \frac{Mc}{I}$$

Beams: Mechanics of Bending Review

- Example 1 (cont'd)

Find the moment of inertia I with respect to the x axis using parallel axis-theorem:



$$I = \frac{6(2)^3}{12} + (6\times 2)(2)^2 + \frac{2(6)^3}{12} + (2\times 6)(3-1)^2$$
$$= 4 + 48 + 36 + 48 = 136\ \text{in}^4$$

$$\text{Max. Stress (com)} = \frac{(5\times 12)(5)}{136} = \underline{2.21\ \text{ksi}}$$

Beams: Mechanics of Bending Review

- Internal Couple Method (cont'd)
  - The procedure of the flexure formula is easy and straightforward for a beam of known cross section for which the moment of inertia *I* can be found.
  - However, for a reinforced concrete beam, the use of the flexure formula can be somewhat complicated.
  - The beam in this case is not homogeneous and concrete does not behave elastically.

Beams: Mechanics of Bending Review

- Internal Couple Method (cont'd)
  - In this method, the couple represents an internal resisting moment and is composed of a compressive force *C* and a parallel internal tensile force *T* as shown in Fig. 4.
  - These two parallel forces *C* and *T* are separated by a distance *Z*, called the the moment arm. (Fig. 4)
  - Because that all forces are in equilibrium, therefore, *C* must equal *T*.

Beams: Mechanics of Bending Review

- Internal Couple Method (cont'd)

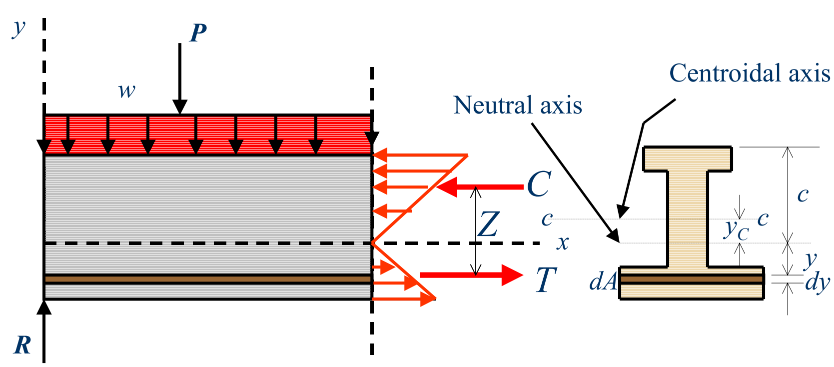


Figure 4

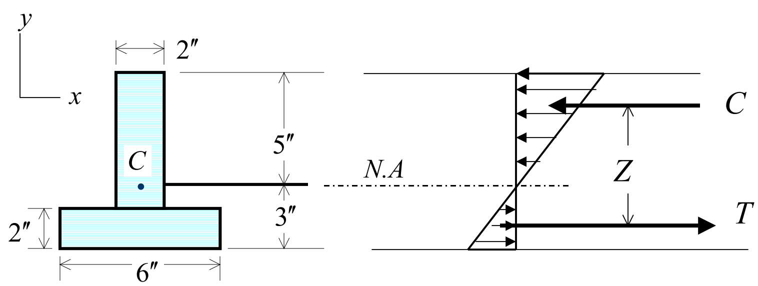
Beams: Mechanics of Bending Review

- Internal Couple Method (cont'd)
  - The internal couple method of determining beam stresses is more general than the flexure formula because it can be applied to homogeneous or non-homogeneous beams having linear or nonlinear stress distributions.
  - For reinforced concrete beam, it has the advantage of using the basic resistance pattern that is found in a beam.

Beams: Mechanics of Bending Review

- Example 2

  Repeat Example 1 using the internal couple method.



Beams: Mechanics of Bending Review

- Example 2 (cont'd)
  - Because of the irregular area for the tension zone, the tensile force T will be broken up into components T_1, T_2, and T_3.
  - Likewise, the moment arm distance Z will be broken up into components Z_1, Z_2, and Z_3, and calculated for each component tensile force to the compressive force C as shown in Fig. 5.

Beams: Mechanics of Bending Review

- Example 2 (cont'd)

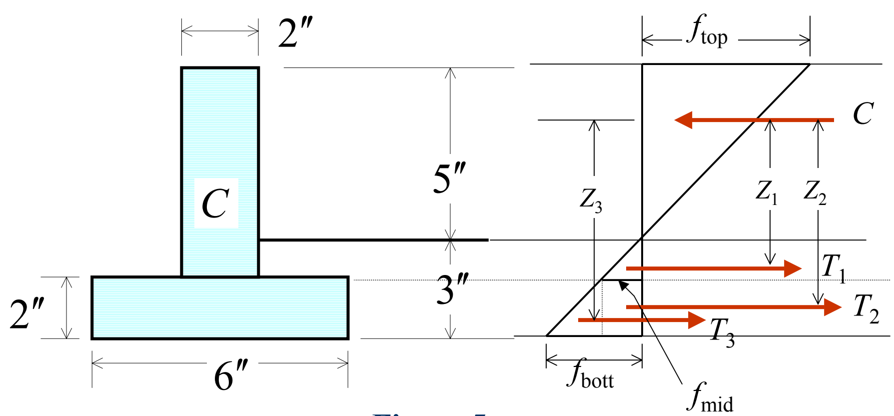
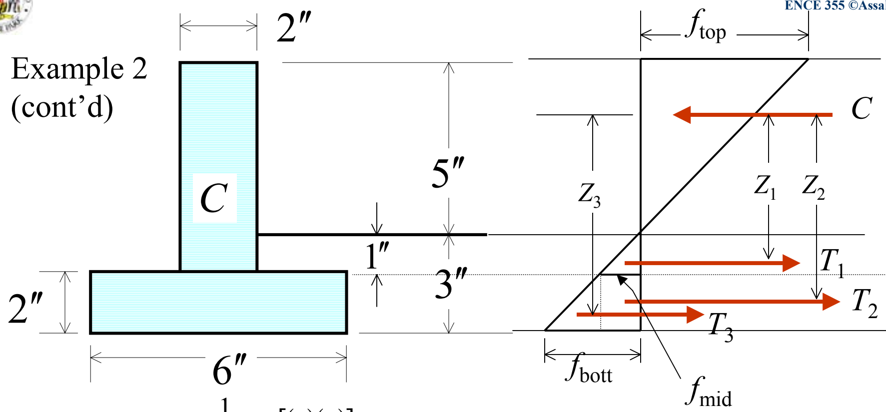


Figure 5

Example 2 (cont'd)



$$C = f_{avg} \times \text{area} = \frac{1}{2} f_{top}[(5)(2)] = 5 f_{top}$$

$$T_1 = f_{avg} \times \text{area} = \frac{1}{2} f_{mid}[(1)(2)] = f_{mid} = \frac{1}{3} f_{bott}$$

$$T_2 = f_{avg} \times \text{area} = f_{mid}[(2)(6)] = 12 f_{mid} = 4 f_{bott}$$

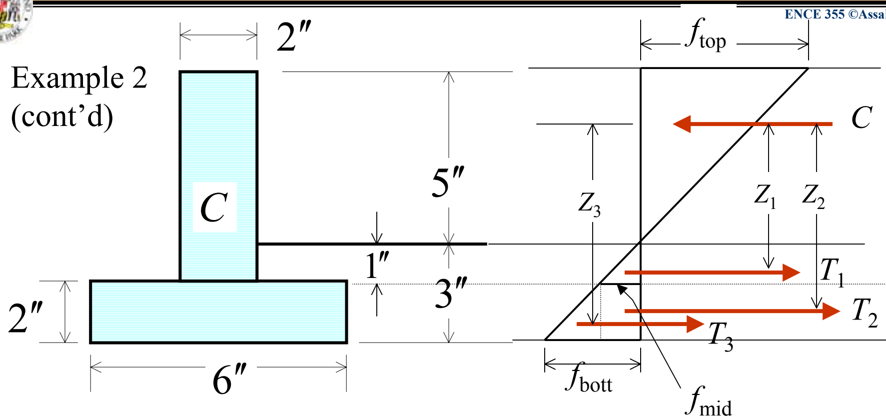
$$T_3 = f_{avg} \times \text{area} = \left(\frac{f_{bott} - f_{mid}}{2}\right)[(2)(6)] = 6 f_{bott} - 6 f_{mid}$$

From similar triangles:

$$\frac{f_{mid}}{f_{bott}} = \frac{1}{3}$$

$$\therefore f_{mid} = \frac{1}{3} f_{bott}$$

Example 2 (cont'd)



$$C = T = T_1 + T_2 + T_3$$

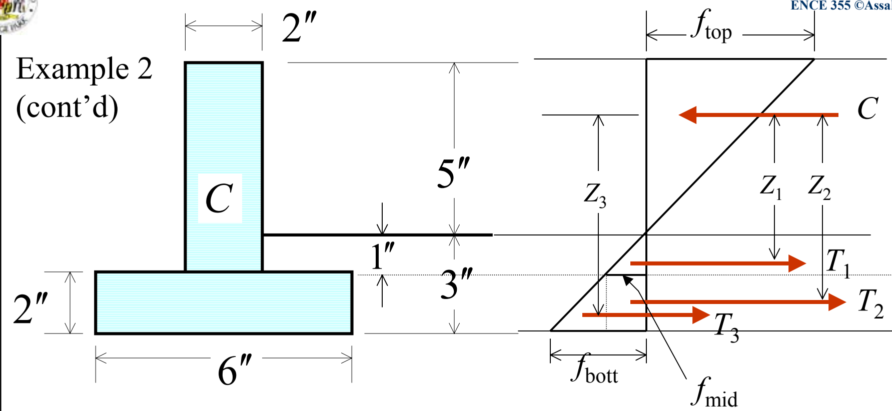
$$5 f_{top} = \frac{1}{3} f_{bott} + 4 f_{bott} + 6 f_{bott} - 6 f_{mid}$$

$$5 f_{top} = \frac{1}{3} f_{bott} + 4 f_{bott} + 6 f_{bott} - 2 f_{bott} = \frac{25}{3} f_{bott}$$

➔

$$f_{top} = \frac{5}{3} f_{bott}$$

Example 2 (cont'd)



$$Z_1 = \frac{2}{3}(5) + \frac{2}{3}(1) = 4 \text{ in.}$$

$$Z_2 = \frac{2}{3}(5) + 2 = \frac{16}{3} \text{ in.}$$

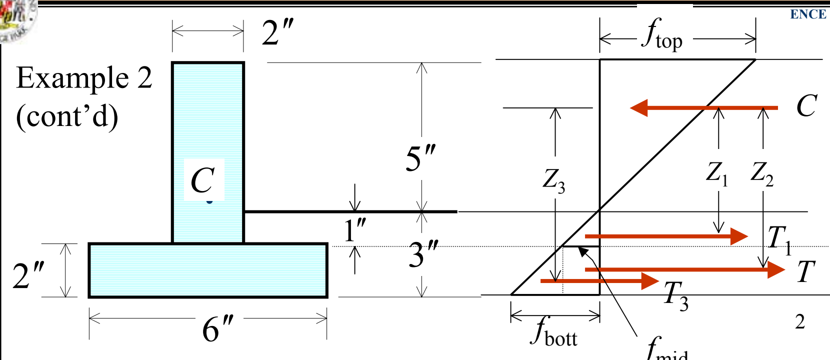
$$Z_3 = \frac{2}{3}(5) + 1 + \frac{2}{3}(2) = \frac{17}{3} \text{ in.}$$

$$M_{ext} = M_R$$

$$5000(12) = Z_1T_1 + Z_2T_2 + Z_3T_3$$

$$60{,}000 = Z_1T_1 + Z_2T_2 + Z_3T_3$$

Example 2 (cont'd)



$$60{,}000 = 4\left(\frac{1}{3}f_{bott}\right) + \frac{16}{3}(4f_{bott}) + \frac{17}{3}(4f_{bott}) = \frac{136}{3}f_{bott}$$

Therefore,

$$f_{bott} = 1{,}323.53 \text{ psi (Tension)}$$

The maximum Stress is compressive stress :

$$f_{max} = f_{top} = \frac{5}{3}f_{bott} = \frac{5}{3}(1{,}323.53) = 2{,}205.88 \text{ psi} = \boxed{2.21 \text{ ksi (Com)}}$$