

University of Maryland, College Park
Department of Civil & Environmental Engineering

Quiz 5 Solution, Closed Book & Notes, for 15 minutes
 April 25, 2001

ENCE 203 - Computation Methods in Civil Engineering II Name: _____

Problem 1

Given the following values for x and $f(x)$:

x	1	2	3	4	5
$f(x)$	5	152	455	1676	6425

- (a) Construct a finite-difference table and from the table determine the first derivative of $f(x)$ with respect to x at $x = 3$ using the forward, backward, and two-step finite-difference approximation.
- (b) Evaluate $\int_2^4 f(x) dx$ using the trapezoidal rule.

*** SOLUTION ***

(a)

Finite-difference Table

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1	5				
		147			
2	152		156		
		303		762	
3	455		918		1848
		1221		2610	
4	1676		3528		
		4749			
5	6425				

From the table:

$$\left. \frac{df(x)}{dx} \right|_{x=3} \approx \frac{1221}{4-3} = 1221 \quad (\text{Forward})$$

$$\left. \frac{df(x)}{dx} \right|_{x=3} \approx \frac{303}{3-2} = 303 \quad (\text{Backward})$$

$$\left. \frac{df(x)}{dx} \right|_{x=3} \approx \frac{1221+303}{2} = 762 \quad (\text{Two - Step})$$

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(b)

$$\int_{x_1}^{x_n} f(x) dx \approx \sum_{i=1}^{n-1} (x_{i+1} - x_i) \frac{f(x_{i+1}) + f(x_i)}{2}$$

The limit of integration is from 2 to 4, therefore, we have three data points, that is $n = 3$.

Hence,

$$\int_2^4 f(x) dx \approx \sum_{i=1}^{3-1} (x_{i+1} - x_i) \frac{f(x_{i+1}) + f(x_i)}{2}$$

$$\begin{aligned} \int_2^4 f(x) dx &\approx (3-2) \frac{455+152}{2} + (4-3) \frac{1676+455}{2} = 303.5 + 1065.5 \\ &= 1369 \end{aligned}$$