

University of Maryland, College Park
Department of Civil & Environmental Engineering

Quiz 4 Solution, Closed Book & Notes, for 15 minutes
April 6, 2001

ENCE 203 - Computation Methods in Civil Engineering II Name: _____

Problem 1

Given the following set of equations, find the solution using the method of determinants:

$$2X_1 - 7X_2 = 1$$

$$3X_1 + 5X_2 = 3$$

$$X_1 + X_2 - X_3 = 4$$

Note that $X_i = \frac{|A_i|}{|A|}$.

*** SOLUTION ***

$$A = \begin{bmatrix} 2 & -7 & 0 \\ 3 & 5 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -7 & 0 \\ 3 & 5 & 0 \\ 1 & 1 & -1 \end{vmatrix} = (-1)[2(5) - (-7)(3)] = -31$$

$$|A_1| = \begin{vmatrix} 1 & -7 & 0 \\ 3 & 5 & 0 \\ 4 & 1 & -1 \end{vmatrix} = (-1)[(1)(5) - (-7)(3)] = -26$$

$$|A_2| = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 3 & 0 \\ 1 & 4 & -1 \end{vmatrix} = (-1)[2(3) - (1)(3)] = -3$$

$$|A_3| = \begin{vmatrix} 2 & -7 & 1 \\ 3 & 5 & 3 \\ 1 & 1 & 4 \end{vmatrix} = 95$$

Therefore,

$$X_1 = \frac{|A_1|}{|A|} = \frac{-26}{-31} = 0.838710$$

$$X_2 = \frac{|A_2|}{|A|} = \frac{-3}{-31} = 0.096774$$

$$X_3 = \frac{|A_3|}{|A|} = \frac{95}{-31} = -3.064516$$

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Problem 2

A coefficient matrix of a set of simultaneous equations is decomposed into its lower and upper triangular matrices L and U , respectively. What is the set of these equations if

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 2 & 3 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 0 \\ 2.5 \\ 3 \end{bmatrix}$$

The system of equations can be given in matrix form as $[A] [X] = [C]$.

*** SOLUTION ***

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 10 \\ 3 & 10 & 22 \end{bmatrix}$$

Therefore, the set of these equations is

$$\begin{aligned} X_1 + 2X_2 + 3X_3 &= 0 \\ 2X_1 + 8X_2 + 10X_3 &= 2.5 \\ 3X_1 + 10X_2 + 22X_3 &= 3 \end{aligned}$$