University of Maryland at College Park
Department of Civil & Environmental Engineering

Quiz 1, **Closed Book & Notes, for 15 minutes**
February 12, 2001

ENCE 203 - Computation Methods in Civil Engineering II **Name:**_____

Problem 1

Develop a Taylor series expansion for the following function for three terms:

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

Use $x_0 = 1$ as the base (or starting) point and h as the increment. Evaluate the series for $x = 1.2$ and 2.0, and compare your results with the true value for both cases. Discuss the accuracy of the approximation. Note that Taylor series expansion is given by:

$$f(x_0 + h) = f(x_0) + hf^1(x_0) + \frac{h^2}{2!} f^2(x_0) + \frac{h^3}{3!} f^3(x_0) + ... + \frac{h^n}{n!} f^n(x_0)$$

***** SOLUTION *****

$f(x) = 25x^3 - 6x^2 + 7x - 88 \quad \Rightarrow \quad f(1) = 25 - 6 + 7 - 88 = -62$

$f'(x) = 75x^2 - 12x + 7 \quad \Rightarrow \quad f'(1) = 75(1)^2 - 12(1) + 7 = 70$

$f''(x) = 150x - 12 \quad \Rightarrow \quad f(1) = 150(1) - 12 = 138$

For three terms (2nd order approximation), the Taylor series expansion is given by

$$f(x) \approx f(x_0 + h) = -62 + 70h + 138\frac{h^2}{2} = -62 + 70h + 69h^2$$

For $x = 1.2$, $h = x - x_0 = 1.2 - 1 = 0.2$. Hence,

$$f(1.2) \approx -62 + 70(0.2) + 69(0.2)^2 = -45.24$$ **Ans.**

For $x = 2$, $h = x - x_0 = 2 - 1 = 1.0$. Hence,

$$f(2) \approx -62 + 70(1) + 69(1)^2 = 77$$ **Ans.**

True Values:

For $x = 1.2$, $f(1.2) = 25(1.2)^3 - 6(1.2)^2 + 7(1.2) - 88 = -45.0400$

For $x = 2.0$, $f(2) = 25(2)^3 - 6(2)^2 + 7(2) - 88 = 102.0000$

Comparison:

x	h	Taylor Series	True	Absolute Error
1.2	0.2	-45.2400	-45.0400	0.2000
2.0	1.0	77.0000	102.0000	25.0000

Conclusions:

The accuracy improves as the step size h decreases.