CHAPTER 8. DIFFERENTIAL EQUATIONS

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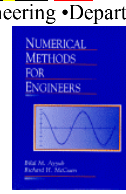

by

Dr. Ibrahim A. Assakkaf

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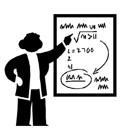
ENCE 203 - Computation Methods in Civil Engineering II

Department of Civil and Environmental Engineering

University of Maryland, College Park

Introduction



- Differential equations are used extensively in engineering and science to represent physical phenomena of a problem (problems).
- A differential equation is any equation containing one or more derivative terms.
- An ordinary differential equation is that involves a single independent variable.

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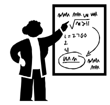
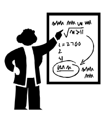
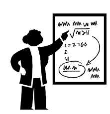


- Differential equations involving two or more independent variables are referred to as partial differential equations.
- The analytical solutions of both ordinary and partial differential equations is called "*closed-form solution*".
- This solution requires the constant of integration be evaluated by using prescribed values of the independent variable(s).

Introduction



- Classification of Differential Equations
  - Ordinary Differential Equations
    - First-order
    - Higher-order
    - Linear
    - Nonlinear
  - Partial Differential Equations
    - These equations are usually classified according to their mathematical form.



Introduction

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■ Ordinary Differential Equations

– The general forms of an ordinary differential equation is given by one of the following expression:

$$C_0(x)+\sum_{i=1}^{n}C_i(x)\frac{d^i}{dx^i}=0 \qquad (1)$$

$$C_0(x)+\sum_{i=1}^{n}C_i(x)\left(\frac{d^i}{dx^i}\right)^m=0 \quad (m\neq 0) \qquad (2)$$

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■ Ordinary Differential Equations

– Note that Eq. 1 is a linear ordinary differential equation, while Eq. 2 is a nonlinear differential equation.

– Furthermore, if the coefficient $C_0(x)$ is zero, the equation is called homogenous, otherwise nonhomogenous.

– ODE's of all types have many applications in engineering and science.

Introduction

- Examples Ordinary Differential Equations

$$\frac{dy}{dx} = 5x$$

$$\frac{dy}{dx} - \frac{1}{2} = 0$$

$$\frac{d^2y}{dx^2} - x + y = 0$$

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- Partial Differential Equations
  - Differential equations involving two or more independent variables are called pertial differential equations.
  - These equations may have only boundary conditions, in which they are referred to as *Boundary Value Problems (BVP)* or steady-state equations.

Introduction

- Partial Differential Equations
  - In some applications both boundary and initial conditions are specified.
  - In these cases, they are called *transient problems.*
  - In practice, very few partial differential equations have closed-form analytical solution. Therefore, numerical techniques are required in most cases.

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- Examples Partial Differential Equations

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$$C_1 \frac{\partial^2 w}{\partial x^2} + C_2 \frac{\partial^2 w}{\partial x \partial y} + C_3 \frac{\partial^2 w}{\partial y^2} + C_4 = 0$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{F_z}{D} - \frac{k}{D} w$$

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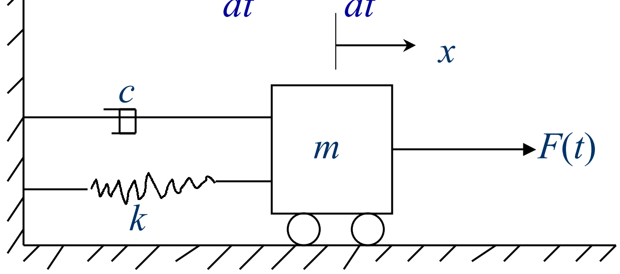
Introduction

- Engineering Examples
  - Mechanical System

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t)$$



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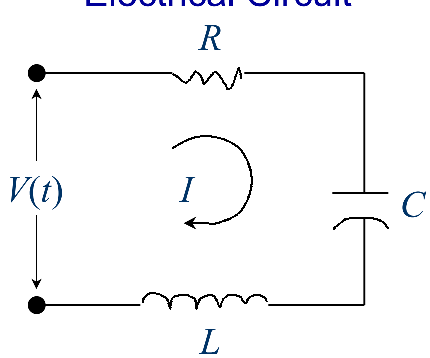
- Engineering Examples
  - Electrical Circuit



$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{1}{C}I=\frac{dv}{dt}$$

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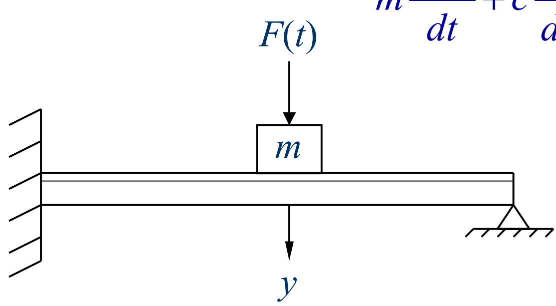
- Engineering Examples
  - Vibrating Beam

$$m\frac{d^2y}{dt}+c\frac{dy}{dt}+ky=F(t)$$



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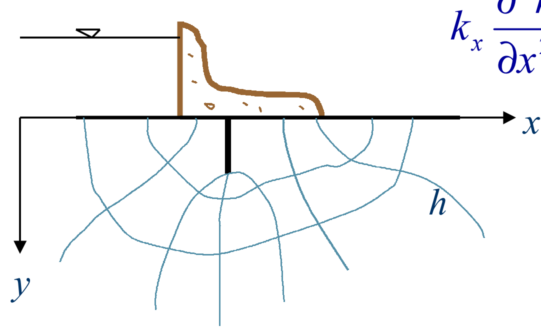
- Engineering Examples
  - Steady-state Fluid Flow under Dams

$$k_x\frac{\partial^2h}{\partial x^2}+k_y\frac{\partial^2h}{\partial y^2}=0$$



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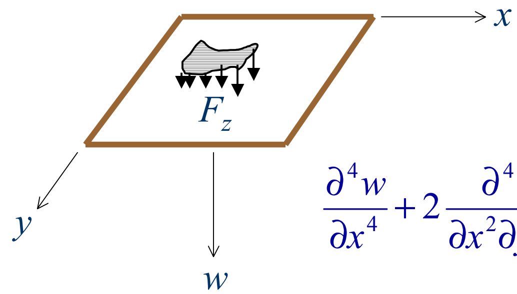
- Engineering Examples
  - Plate on Elastic Foundation



$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{F_z}{D} - \frac{k}{D} w$$

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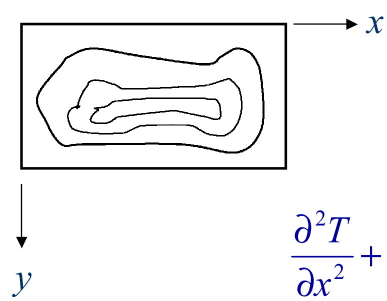
- Engineering Examples
  - Transient Temperature Distribution



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = C\frac{\partial T}{\partial t}$$

Introduction

- General Expressions for Differential Equations

$$\frac{dy}{dx} = f(x, y) \qquad \frac{dy}{dx} = 2 + x^2 - y$$

$$\frac{dy}{dx} = f(x) \qquad \frac{dy}{dx} = 3 - \frac{1}{x}$$

$$\frac{dy}{dx} = f(y) \qquad \frac{dy}{dx} = e^y - 2$$

$$\frac{dy}{dx} = C \qquad \frac{dy}{dx} = -4$$

Introduction

- General Expressions for Differential Equations

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right) \qquad \frac{d^2y}{dx^2} = 1 - x + 2y - \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = f\left(x, \frac{dy}{dx}\right) \quad \Longrightarrow \quad \frac{d^2y}{dx^2} = 3x + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = f\left(y, \frac{dy}{dx}\right) \qquad \frac{d^2y}{dx^2} = 2\frac{dy}{dx} + \frac{1}{y}$$

$$\frac{d^2y}{dx^2} = f(x, y) \qquad \frac{d^2y}{dx^2} = f(x, y) = 2x + \frac{2}{y}$$

Introduction

■ General Expressions for Differential Equations

$$\frac{dy}{dx} = f(x_1, x_2, y) \qquad \frac{dy}{dx} = x_1^2 - x_2^{-1} - 2y^2$$

$$\frac{dy}{dx} = f(x_1, x_2) \longrightarrow \frac{dy}{dx} = f(x_1, x_2) = \frac{1}{x_1} - x_2^2$$

$$\frac{dy}{dx} = f(x_1, y) \qquad \frac{dy}{dx} = f(x_1, y) = 3x_1 + 3y^2 - 1$$

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■ Origin of Differential Equations

- They can originate from either geometric or physical problems.
- For geometric case, consider the slop of a function, which is usually a relationship between *y* and *x*.
- This slope illustrates the geometric case, and is given by

$$\frac{dy}{dx} = c(y - x) \qquad (3)$$

Introduction

■ Origin of Differential Equations

– The solution of Eq. 3 would a relationship of the form

$$y = g(x) \qquad (4)$$

– Equation 4 may be subject to one or more boundary conditions.

– Physical problems can also be defined by differential equations.

Introduction

■ Origin of Differential Equations

– As we saw earlier, simple problems in electrical circuits and heat transfer involve differential equations.

– Simple problems motions can also be expressed by differential equations. For example the gravitational equation is

$$F = ma = m\frac{dV}{dt}$$