# CHAPTER 6e. NUMERICAL INTERPOLATION

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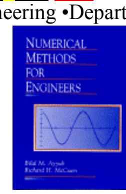


by

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**Spring 2001**

**ENCE 203 - Computation Methods in Civil Engineering II**

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# Lagrange Polynomials

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- Example 7

  Illustrate the use of Equations 31, 32, and 33 when $n$ is 2, 3, and 4.

$$f(x_0) = \sum_{i=1}^{n} w_i(x_0) f(x_i)$$

$$w_i(x_i) = \frac{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_0 - x_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_i - x_j)}$$

$$f(x_0) = w_1(x_0) f(x_1) + w_2(x_0) f(x_2) + w_3(x_0) f(x_3) + \ldots + w_n(x_0) f(x_n)$$

# Lagrange Polynomials



■ Example 7 (cont'd)

$$w_i(x_i) = \frac{\prod_{\substack{j=1\\j\neq i}}^{n-1}(x_0 - x_j)}{\prod_{\substack{j=1\\j\neq i}}^{n-1}(x_i - x_j)}$$

$$f(x_0) = \sum_{i=1}^{n} w_i(x_0) f(x_i)$$

$\underline{n = 2:}$

$$w_1(x_0) = \frac{x_0 - x_2}{x_1 - x_2}$$

$$w_2(x_0) = \frac{x_0 - x_1}{x_2 - x_1}$$

and

$$f(x_0) = w_1(x_0) f(x_1) + w_2(x_0) + f(x_2)$$

$$= \frac{x_0 - x_2}{x_1 - x_2} f(x_1) + \frac{x_0 - x_1}{x_1 - x_1} f(x_2)$$



# Lagrange Polynomials



■ Example 7 (cont'd)

$$w_i(x_i) = \frac{\prod_{\substack{j=1\\j\neq i}}^{n-1}(x_0 - x_j)}{\prod_{\substack{j=1\\j\neq i}}^{n-1}(x_i - x_j)}$$

$$f(x_0) = \sum_{i=1}^{n} w_i(x_0) f(x_i)$$

$\underline{n = 3:}$

$$w_1(x_0) = \frac{(x_0 - x_2)(x_0 - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

$$w_2(x_0) = \frac{(x_0 - x_1)(x_0 - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$

$$w_3(x_0) = \frac{(x_0 - x_1)(x_0 - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

and

$$f(x_0) = w_1(x_0) f(x_1) + w_2(x_0) f(x_2) + w_3(x_0) f(x_3)$$

$$= \frac{(x_0 - x_2)(x_0 - x_3)}{(x_1 - x_2)(x_1 - x_3)} f(x_1) + \frac{(x_0 - x_1)(x_0 - x_3)}{(x_2 - x_1)(x_2 - x_3)} f(x_2) + \frac{(x_0 - x_1)(x_0 - x_2)}{(x_3 - x_1)(x_3 - x_2)} f(x_3)$$

# Lagrange Polynomials



■ Example 7 (cont'd)

$n = 4$:

$$w_1(x_0) = \frac{(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

$$w_2(x_0) = \frac{(x_0 - x_1)(x_0 - x_3)(x_0 - x_4)}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)}$$

$$w_3(x_0) = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_4)}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)}$$

$$w_4(x_0) = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)}$$

$$w_i(x_i) = \frac{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_0 - x_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_i - x_j)}$$

$$f(x_0) = \sum_{i=1}^{n} w_i(x_0) f(x_i)$$

and

$$f(x_0) = w_1(x_0)f(x_1) + w_2(x_0)f(x_2) + w_3(x_0)f(x_3) + w_4(x_0)f(x_4)$$





# Lagrange Polynomials

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■ Example 8

Suppose that the sine function is given in tabulated form as provided in the following table. Use a Lagrange interpolating polynomial to estimate the sine of $12^0$.

| $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $x$ | 10 | 11 | 13 |
| $f(x)$ | 0.17365 | 0.19081 | 0.22495 |

# Lagrange Polynomials



■ Example 8 (cont'd)

The general form of the Lagrange polynomial is given by

$$f(x_0) = w_1(x_0)f(x_1) + w_2(x_0)f(x_2) + w_3(x_0)f(x_3) + \ldots + w_n(x_0)f(x_n)$$

For $n$ = 3, and $x_0 = 12^0$, it becomes

$$f(x_0) = w_1(12)f(x_1) + w_2(12)f(x_2) + w_3(12)f(x_3) \quad (34)$$



# Lagrange Polynomials

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■ Example 8 (cont'd)

Now we need to find the weights $w_i$'s:

$$w_i(x_i) = \frac{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_0 - x_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} (x_i - x_j)}$$

$$w_1 = \frac{(x_0 - x_2)(x_0 - x_3)}{(x_1 - x_2)(x_1 - x_3)} = \frac{(12-11)(12-13)}{(10-11)(10-13)} = -\frac{1}{3}$$

$$w_2 = \frac{(x_0 - x_1)(x_0 - x_3)}{(x_2 - x_1)(x_2 - x_3)} = \frac{(12-10)(12-13)}{(11-10)(11-13)} = 1$$

$$w_3 = \frac{(x_0 - x_1)(x_0 - x_2)}{(x_3 - x_1)(x_3 - x_2)} = \frac{(12-10)(12-11)}{(13-10)(13-11)} = \frac{1}{3}$$

# Lagrange Polynomials

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- Example 8 (cont'd)

  Substituting the weights and the tabulated values of $f(x) = \sin x$ into Eq. 34, yields

$$
\begin{aligned}
f(x_0) &= w_1(12)f(x_1) + w_2(12)f(x_2) + w_3(12)f(x_3) \\
&= -\frac{1}{3}(0.17365) + (1)(0.19081) + \frac{1}{3}(0.22495) \\
&= 0.20791
\end{aligned}
$$



# Finite Difference Interpolation

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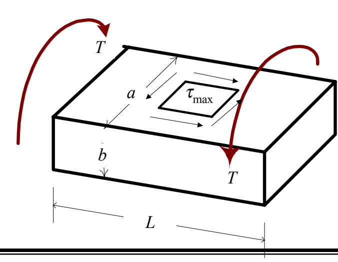
- Example 9

  Use the Newton's interpolation formula to find the largest torque which may be applied to the noncircular brass bar as shown. Assume $\tau = 40\times 10^6$ Pa, $a$ = 0.064 m, and $b$ = 0.025 m.

$$\tau_{\max} = \frac{T}{c_1 a b^2}$$



# Finite Difference Interpolation

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- Example 9 (cont'd)

The following table gives the constant $c_1$ and $c_2$ values

$$\tau_{\max} = \frac{T}{c_1 a b^2}$$

| *i* | *a/b* | $c_1$ | $c_2$ |
|---|---|---|---|
| 1 | 1.2 | 0.219 | 0.1661 |
| 2 | 1.5 | 0.231 | 0.1958 |
| 3 | 2.0 | 0.246 | 0.229 |
| 4 | 2.5 | 0.258 | 0.249 |
| 5 | 3.0 | 0.267 | 0.263 |
| 6 | 4.0 | 0.282 | 0.281 |



# Lagrange Polynomials

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- Example 9 (cont'd)

First find the ratio $a/b$:

$$\frac{a}{b} = \frac{0.064}{0.025} = 2.56$$

The general form of the Lagrange polynomial is given by

$$f(x_0) = w_1(x_0) f(x_1) + w_2(x_0) f(x_2) + w_3(x_0) f(x_3) + \ldots + w_n(x_0) f(x_n)$$

# Lagrange Polynomials



- Example 9 (cont'd)
  - For $n$ = 6, and $x_0 = 2.56$, it becomes

$$f(x_0) = w_1(2.56)f(x_1) + w_2(2.56)f(x_2) + w_3(2.56)f(x_3) + w_4(2.56)f(x_4) \; w_5(2.56)f(x_5) + w_6(2.56)f(x_6)$$



# Lagrange Polynomials

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- Example 9 (cont'd)

  Now we need to find the weights $w_i$'s:

$$w_1 = \frac{(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)(x_0 - x_5)(x_0 - x_6)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_1 - x_5)(x_1 - x_6)}$$

$$w_1 = \frac{(2.56 - 1.5)(2.56 - 2)(2.56 - 2.5)(2.56 - 3)(2.56 - 4)}{(1.2 - 1.5)(1.2 - 2)(1.2 - 2.5)(1.2 - 3)(1.2 - 4)}$$

$$w_1 = -0.014351$$

# Lagrange Polynomials



- Example 9 (cont'd)

$$w_2 = \frac{(x_0 - x_1)(x_0 - x_3)(x_0 - x_4)(x_0 - x_5)(x_0 - x_6)}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)(x_2 - x_5)(x_2 - x_6)}$$

$$w_2 = \frac{(2.56-1.2)(2.56-2)(2.56-2.5)(2.56-3)(2.56-4)}{(1.5-1.2)(1.5-2)(1.5-2.5)(1.5-3)(1.5-4)}$$

$$w_2 = 0.051472$$



# Lagrange Polynomials

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- Example 9 (cont'd)

$$w_3 = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_4)(x_0 - x_5)(x_0 - x_6)}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)(x_3 - x_5)(x_3 - x_6)}$$

$$w_3 = \frac{(2.56-1.2)(2.56-1.5)(2.56-2.5)(2.56-3)(2.56-4)}{(2.0-1.2)(2.0-1.5)(2.0-2.5)(2.0-3)(2.0-4)}$$

$$w_3 = -0.137010$$

# Lagrange Polynomials



■ Example 9 (cont'd)

$$w_4 = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_5)(x_0 - x_6)}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)(x_4 - x_5)(x_4 - x_6)}$$

$$w_4 = \frac{(2.56-1.2)(2.56-1.5)(2.56-2.0)(2.56-3)(2.56-4)}{(2.5-1.2)(2.5-1.5)(2.5-2.0)(2.5-3)(2.5-4)}$$

$$w_4 = 1.049236$$





# Lagrange Polynomials

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■ Example 9 (cont'd)

$$w_5 = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)(x_0 - x_6)}{(x_5 - x_1)(x_5 - x_2)(x_5 - x_3)(x_5 - x_4)(x_5 - x_6)}$$

$$w_5 = \frac{(2.56-1.2)(2.56-1.5)(2.56-2.0)(2.56-2.5)(2.56-4)}{(3.0-1.2)(3.0-1.5)(3.0-2.0)(3.0-2.5)(3.0-4)}$$

$$w_5 = 0.051667$$

# Lagrange Polynomials



■ Example 9 (cont'd)

$$w_6 = \frac{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)(x_0 - x_5)}{(x_6 - x_1)(x_6 - x_2)(x_6 - x_3)(x_6 - x_4)(x_6 - x_5)}$$

$$w_6 = \frac{(2.56-1.2)(2.56-1.5)(2.56-2.0)(2.56-2.5)(2.56-3)}{(4.0-1.2)(4.0-1.5)(4.0-2.0)(4.0-2.5)(4.0-3)}$$

$$w_6 = -0.001015$$



# Lagrange Polynomials

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■ Example 9 (cont'd)

– Therefore,

$$\begin{aligned} f(2.56) = & (-0.14351)(0.219) + (0.051472)(0.231) \\ & + (-0.13701)(0.246) + (1.049236)(0.258) \\ & (0.05167)(0.267) + (-0.001015)(0.282) \end{aligned}$$

Hence,

$$c_1(2.56) = f(2.56) = 0.259254$$

# Lagrange Polynomials



- Example 9 (cont'd)
  - The maximum torque is computed as follows:

$$\tau_{\max} = \frac{T}{c_1 a b^2}$$

$$40\times10^6 = \frac{T}{0.259254(0.064)(0.025)^2} \Rightarrow T = 414.81\ \text{N.m}$$